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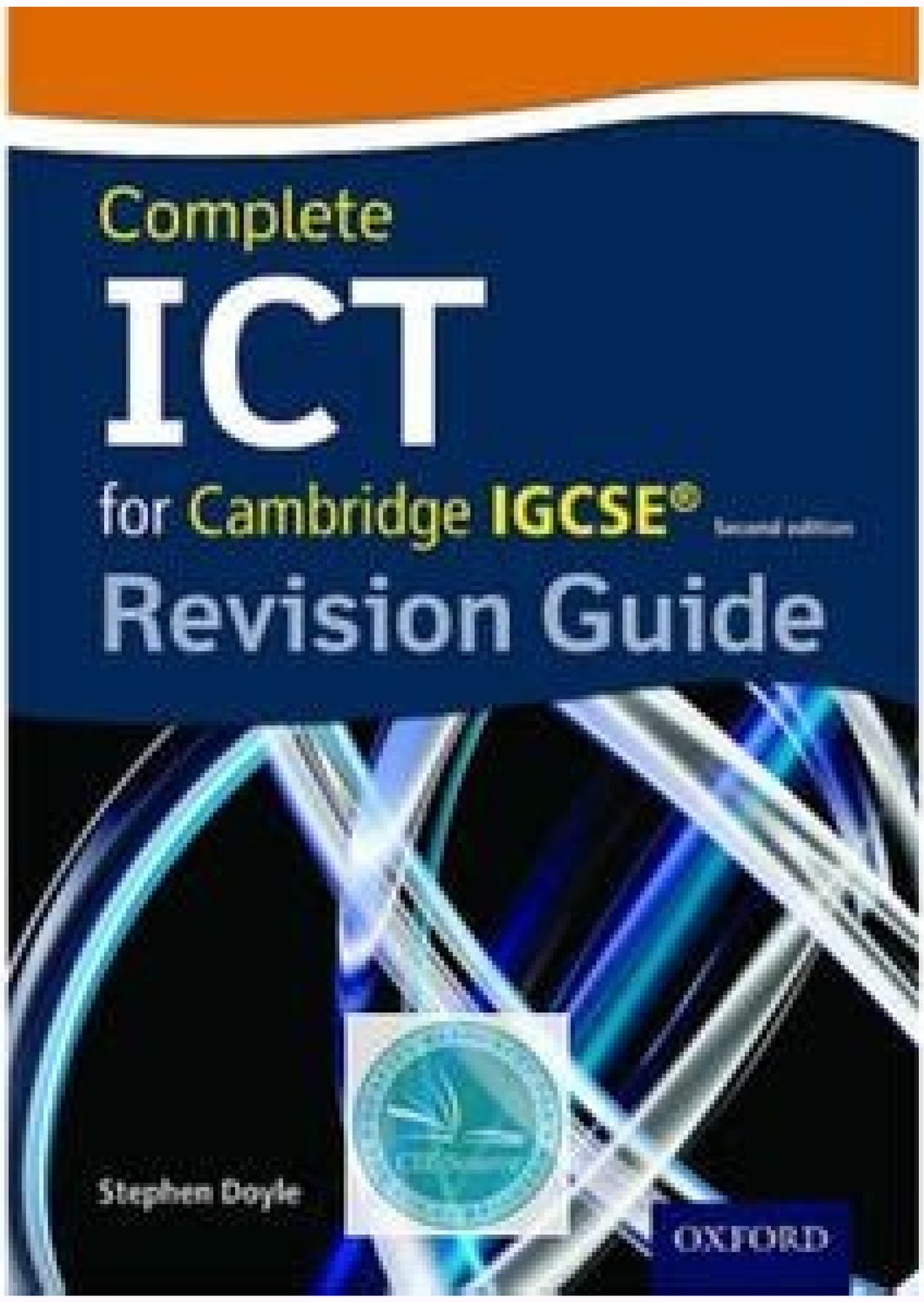

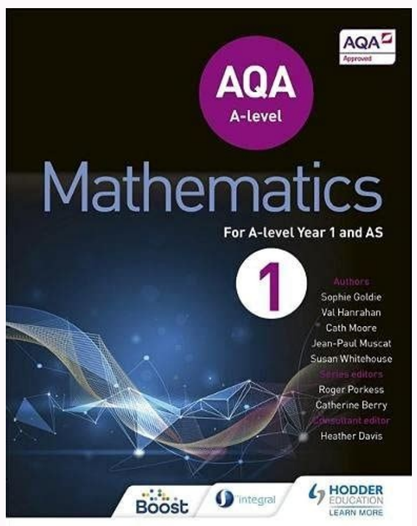

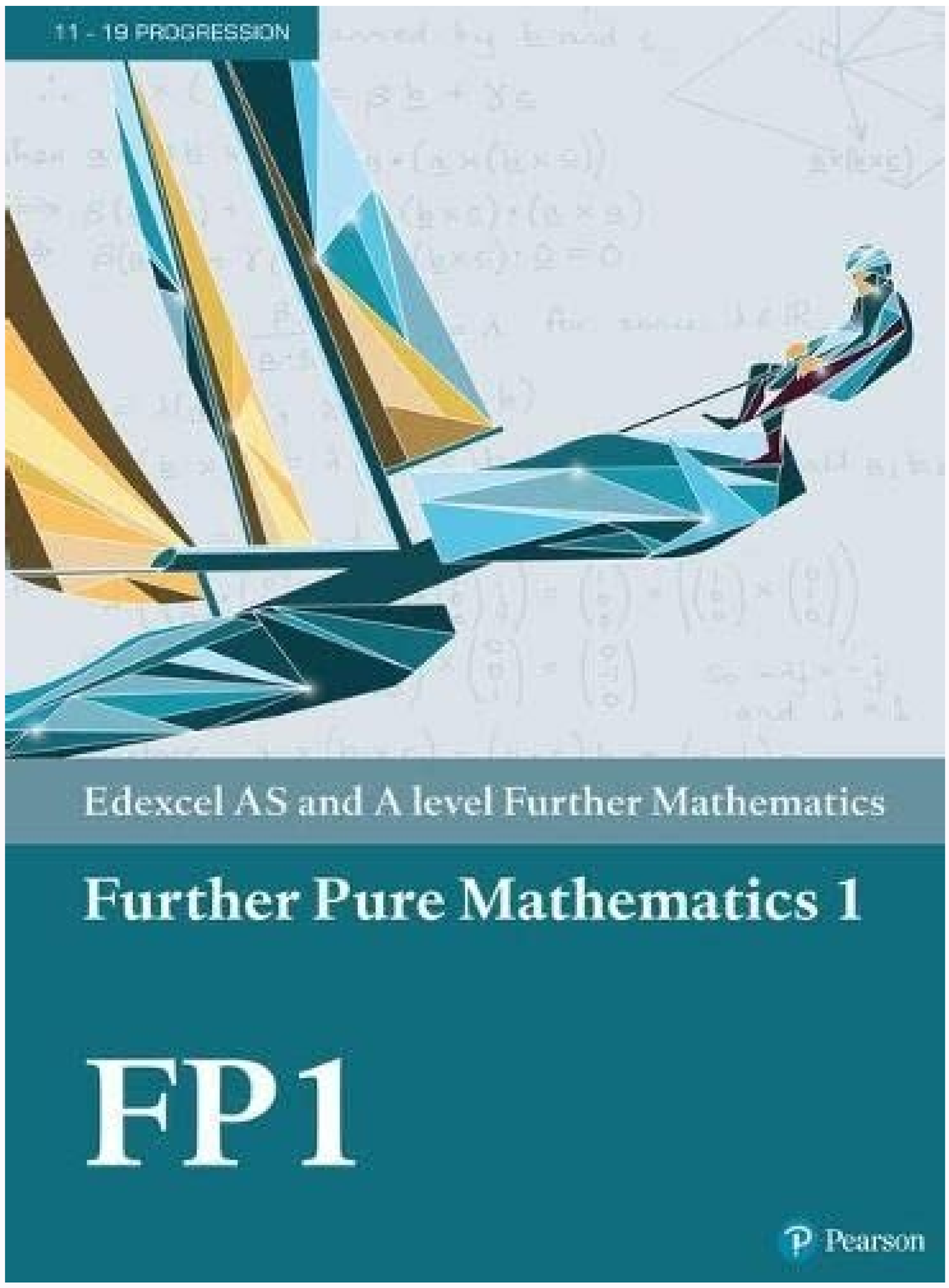

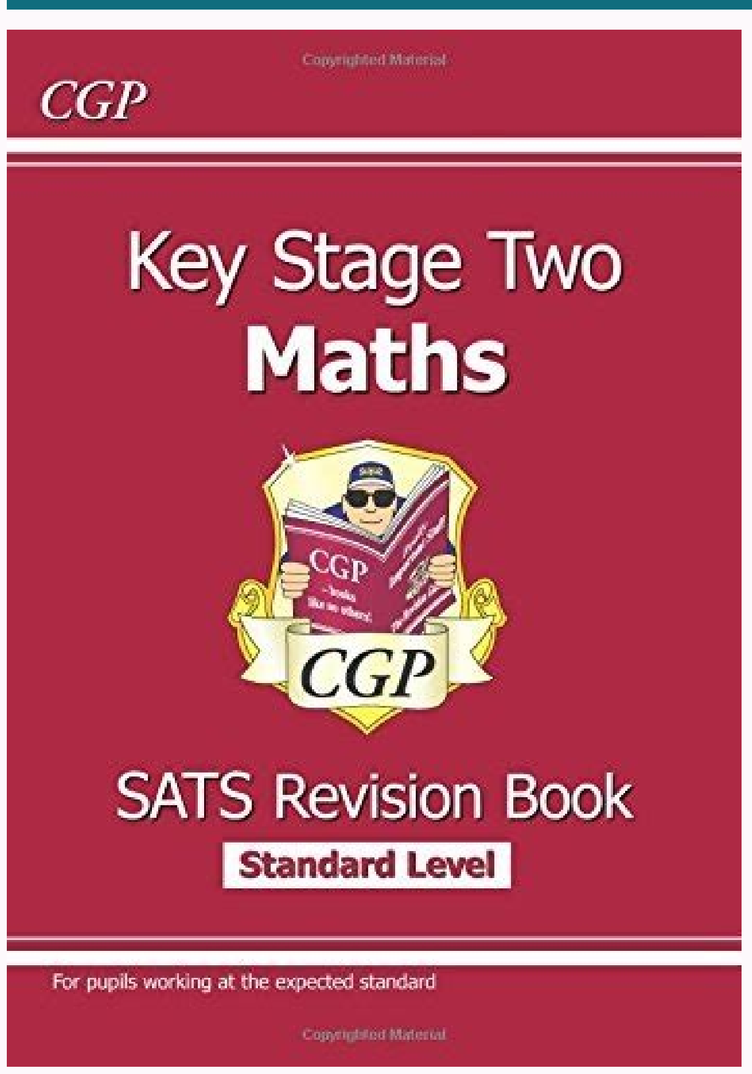

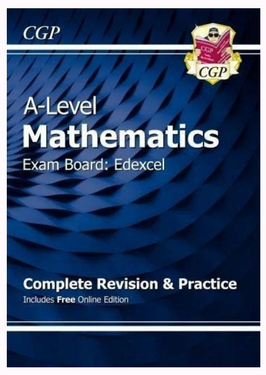

ff Now draw a horizontal line from this point to the line y = x. PROBLE M SOLVING Q6 Particle A is moving in a vertical plane. a) Find the acceleration of the particle at time t. 36t2 – 9 = 0 Tip: As always, ignore the negative value of t. – As t increases, N gets larger without any limit. Chapter 2 Algebra and Functions 7 Algebraic division Important terms There are a few words that come up a lot in algebraic division, so make sure you know what they all mean. To answer a projectiles question, split all the vector quantities you know — velocity, acceleration and displacement — into their horizontal and vertical components. There's also a proof for the quotient rule — again you won't need to know it for the exam, but you might find it helpful in understanding how it works. dy a) Find for this curve. Tip: Once you've found the ball's initial velocity, you can resolve horizontally and vertically and use the suvat equations as usual. ING Action film Comedy film Total Under 20 20 and over Total b) One of the cinema-goers is picked at random. c) Find the value of t for which the particle is directly above O. So the range is fg(x) > 1. a ff If 2 = b , then 2b = a ff Squaring both sides gives you 2b2 = a2 — so a2 is an even number. PROBLE M SOLVING Chapter 10 Vectors 257 Chapter 11 Probability 1. So p = 8 6561 = 3. For the normal approximation to a binomial distribution to work well, you need the following conditions to be true: Suppose the random variable X follows a binomial distribution, i.e. X ~ B(n, p). ff If you're given only one x-value (i.e. if you need to find P(X ≥ x) or P(X ≤ x) for some x) you might need to choose your own lower or upper bound — just pick a really large negative or positive number (see Example 1 below). Q3 The rate of increase of the variable V at time t satisfies the differential dV equation = a – bV, where a and b are positive constants. Parametric Equations of Curves 2. ff You can use the following formula to find the moment of a force about a point: Learning Objectives: ff Be able to find the moment of a force about a point. Exercise 2.1 Q1 a) Show that the equation x3 + 3x2 – 7 = 0 has a root in the interval (1, 2). The distribution of the sunflowers' heights is symmetrical about the mean, with the shortest sunflower measuring 5.8 ft and the tallest measuring 13.7 ft. If he walks to the bus stop, the probability that he catches the bus is 0.3. a) Draw a Venn diagram representing the events W, 'Vikram walks to the bus stop', and C, 'Vikram catches the bus'. Tip: If you used a continuity correction here, you'd work out P(320.5 < Y < 350.5) instead, which is 0.472 (3 s.f.). 2 Q5 y = (x – 3)5, so let y = u5 where u = x – 3 dy du fi = 5u4 = 5(x – 3)4, =1 du dx dy dy du = × = 5(x – 3)4 × 1 = 5(x – 3)4 dx du dx At the point (1, –32), gradient = 5(1 – 3)4 = 80 The equation of a straight line is y = mx + c fi –32 = (80 × 1) + c fi c = –112 So the equation of the tangent is y = 80x – 112. It might be useful to write the constant as ln k rather than C (see p.224). = 229 m (3 s.f.) c) E.g. The acceleration should take into account the increased drag from the water — vertical acceleration can no longer be assumed to be equal to g, and horizontal acceleration can no longer be assumed to be 0. Q11 Stationary points occur when the gradient is 0. + 4 = 1.7965... ff First use the quotient rule to find u = ln x fi du 1 = x dx dy : dx and v = x2 fi ln x (x > 0). Q2 A curve C is defined parametrically by x = t3 – 2t2, y = t3 – t2 + 5t. In general, these diagrams will only converge if your starting value of x0 is close enough to the root, and if the graph of f(x) isn't too steep. So the root is between –1.7 and –2 f(–1.8) = e(2 × –1.8) + (–1.8)2 – 3 = 0.26... P Parameter (hypothesis testing) A quantity that describes a characteristic of a population. If you're using the suvat equations in vector form, then acceleration due to gravity will be a = –gj (= –9.8j ms–2). ff Start by defining a random variable to represent the time taken. In each case, think about which way the ladder would slip — the frictional force will act in the opposite direction. x 2 a) Find the values of gf(1), fg(1) and f2(4). Indefinite integral An integral which contains a constant of integration that comes from integrating without limits. ff There are points of inflection (where the curve changes between concave and convex — see p.135-136) at x = m + s and x = m – s. So the graph of f(x) for x ≥ 0 is reflected in the y-axis for the negative x-values. Find: a) the length of time that the stone is at least 6 m above the ground, b) the distance by which the stone falls short of the target. The range is y ≥ 3 or y ≤ 1. 1 1 4 c) . ff If there's a fraction in the numerator or denominator (e.g. Learning Objectives: ff Be able to simplify rational expressions (i.e. algebraic fractions with linear or quadratic denominators) by factorising and cancelling. It has a constant acceleration of 0.1 ms–2 in the i direction, and a variable acceleration in the j direction, such that dn dn = (kn – 10) ms–2 and < 0 (k is a constant). b) gf(x) = g(x2) = 2x2 – 3, range: gf(x) ≥ –3. T 6m Given that the string holds the lamina in equilibrium at an angle of 20° to the horizontal, find the tension in the string. = 962 (to the nearest whole number) Exercise 3.1 MODELL ING Q1 The speeds of 100 randomly selected cars on a stretch of road are recorded by a speed camera. dt a a) Show that V = b – Ae–bt, where A is a positive constant. (see page 270). b) The graphs cross twice so the equation has 2 roots. 0 2 1 x 3 f(x) = x – x – 9 3 2 It has no breaks or jumps. Explanations Clear explanations for every topic. a) X ~ N(m, 62) and P(X < 23) = 0.9332 b) X ~ N(m, 82) and P(X < 57) = 0.9970 c) X ~ N(m, 1002) and P(X > 528) = 0.1292 d) X ~ N(m, 0.42) and P(X < 11.06) = 0.0322 e) X ~ N(m, 0.022) and P(X > 1.52) = 0.9938 Q2 X ~ N(m, 3.52). Magnitude is always a positive scalar, so the unit vector is always parallel to the vector, and has the same direction. -1 -1 ^8 + 2xh 3 = 1 `1 + x j 3 4 2 1 7x 3 x2 x = 2 (1 – 12 + 72 – 2592 + ...) 1 7x 3 x x2 = 2 – 24 + 144 – 5184 + ... 30º 30º 0.6g A particle of mass 600 g is propelled up the line of greatest slope of a smooth plane inclined at 30° to the horizontal. b) Find an expression for the rate of change of the volume of the crystal with respect to time. ff The centre of mass of a non-uniform beam could be at any point along the beam. b) P(12 ≤ X ≤ 15)? 32 Chapter 2 Algebra and Functions PROBLE M SOLVING 6. b) (1 + 4x)4 = 1 + 16x + 96x2 + ... Find the horizontal and vertical components of its initial velocity, giving your answer in metres per second. Often used to describe the initial state of a body. Work out what that is first. So there are two equations to solve: 1 x + 2 = 2x fi x = 2 2 2 x + 2 = –2x fi x = - 3 2 So the two solutions are x = 2 and x = - 3 . The area under the curve is 1, so 1 – 0.025 = 0.975. Assuming that the variance has remained unchanged, test at the 5% level whether there is evidence that the mean time has decreased. 24 Chapter 2 Algebra and Functions Graphs of inverse functions The inverse of a function is its reflection in the line y = x. JK - 1NO JK 2 NO JK - 3NO KK OO KK OO KK OO K O Q2 GH = OH – OG = K 4 O – KK 3OO = KK 7 OO KK OO KK OO KK OO 4 9 5 L P L P L P JK - 3NO JK 3 NO KK OO KK OO HG = – GH = – KK 7 OO = KK - 7OO KK OO KK - OO 5 5 L P L P Q3 OJ = 4i – 3k, OK = –i + 3j, and OL = 2i + 2j + 7k JK = OK – OJ = –i + 3j – (4i – 3k) = –5i + 3j + 3k KL = OL – OK = 2i + 2j + 7k – (–i + 3j) = 3i – j + 7k LJ = OJ – OL = 4i – 3k – (2i + 2j + 7k) = 2i – 2j – 10k Q4 a) Point B is twice the distance of A from the origin in each direction, so OB = 2 OA . 264 Chapter 11 Probability ff Draw the tree diagram and write on as much as you know so far: P(N|M) N 0.4 M 0.2 N' N M' P(N'|M') 0.7 N' ff You can find the probabilities of the remaining branches by doing 1 – the probability on the other branch: 0.2 M 0.8 M' 0.4 N 0.6 0.3 N' N 0.7 N' b) Find P(N). y = f(x) This is the root. b) Find the particle's velocity vector 8 seconds after it begins accelerating from rest. ff Calculating with 3D vectors is just the same as with 2D vectors, as the next example shows. ff You can use a calculator to find probabilities for any normal distribution. The toy is modelled as a particle moving with constant acceleration of (0.1i + 0.5j) ms–2. In cases like this, one variable will usually be t, time. dy dy b) 3x2 – y – x = 4y dx dx 2 dy 3x - y = dx 4y + x dy 3 (1 2) - (- 1) - 4 At (1, –1), = = 3 dx 4 (- 1) + 1 2 - a1 k 2 1 dy 3 (1) 5 At b1, 2 l, = = 6 1 dx 4a 2 k + 1 Answers 423 Q6 a) Putting x = 1 into the equation gives: y + y2 – y – 4 = 0 fi y2 – 4 = 0 so it cuts the curve at y = 2 and y = –2. 40 45 50 55 60 time, mins 65 70 75 x Tip: You could have used any of the normal conditions involving m and s here. dx Then use the product rule to find f'(x): du dv 1 u = x2, v = x + 3 fi = 2x, = dx dx 2 x+3 1 dy dv du 0) So (2, 0) is a minimum point. MODELL ING Chapter 12 The Normal Distribution 287 3. Example 3 If the random variable X ~ N(53, s2) and P(X > 56) = 0.2, find s. dr b) Hence find for a material with k = 1.5, when V = 4 cm3. + 1#2 1 Q5 a) 3 4 x = 1 + 5 + a 8 ka

25 x 2k + ... k^ x - 2h 2x + 3 fi y= Example 4 db Find the particular solution to the differential equation = 4 b, dt given that when t = 12, b = 900. ff If r = 0 (or more likely, pretty close to 0), that would mean the variables aren't correlated. 35 So the 1st term is 9. b) gf(3) = g(32) = g(9) = 2(9) + 1 = 19 c) f2(5) = f(52) = f(25) = 252 = 625 d) g2(2) = g(2(2) + 1) = g(5) = 2(5) + 1 = 11 Q2 fg(90°) = f(2(90°)) = f(180°) = sin 180° = 0 gf(90°) = g(sin 90°) = g(1) = 2(1) = 2 3 Q3 a) gf(1) = ga + k = g(1) = 2(1) = 2. Similarly, C contains two data clusters, each with negative correlation, despite r suggesting virtually no correlation. b) Differentiating: dy dy 1 x + 2y = 2xy + x2 dx dx 1 2xy - x2 dy 2x y - 1 = = dx 2y - x 2 2xy - x 3 2^1 2h^3h - 1 dy At (1, 3), = =1 dx 2^1 h^3h - ^1 h3 so the gradient of the normal is –1. Like differentiating with the chain rule, to integrate by substitution you have to write part of the function in terms of u, where u is some function of x. E.g. it says p ≠ a for a parameter p and constant a. ff Resolving parallel to the plane (): F – 0.1g sin 20° = 0 This is the component of the stone's weight acting down the plane. ff A test statistic is a statistic calculated from the sample data — here it's r. Find the magnitude and direction of the resultant force on the particle. The probability of any person ordering a vegetarian meal is 0.15. Tip: Remember, the critical value(s) depend on the significance level, and on whether it's a one-tailed or a two-tailed test. Example 2 Tip: Under H0, X ~ N(30, 32), 32 so X ~N(30, 20) . N Natural logarithm The inverse function of ex, written as ln x or loge x. You don't need to label all the lines, just the ones that help you answer the question: 0.71 C 0.62 0.29 C' 0.38 S' C' 454 So P(S Ç C) = 0.62 × 0.71 = 0.4402, and so: P(S È C) = 0.4402 P(S|C) = = 0.617 (3 s.f.) P(C) 0.539 P °C) h Answers 2. Q8 Given that P(X) = 0.44, P(Y) = 0.72, P(Z) = 0.61, P(X|Y) = 0.75, P(Z|X) = 0.25, P(Y Ç Z') = 0.2 and that P(X Ç Y Ç Z) = 0.1 , find: a) P(Y) b) P(Y|Z) c) P(X Ç Z') Using conditional probability to find independent events To show that A and B are independent, then P(B) is the same, whether A happens or not. du will be Then dx Replace with f′(x) and f′(u) will be replaced with (n + 1)[f(x)]n. ff A parameter describes a characteristic of a population. A lot of the probability in this chapter is stuff you've seen before, but using different notation. ff Dividing equation 1 by equation 2 : 10g - R Q sin 10° Q cos 10° = 0.7R 10g - R tan 10° = 0.7R 0.7R tan 10° = 10g - R R = 10g ^1 + 0.7 tan 10°h = 87.232... ff Write out the division in the form f(x) º q(x)d(x) + r(x): x3 + 5x2 – 18x – 18 º (Ax2 + Bx + C)(x – 3) + D ff Putting x = 3 into the identity gives D = 0. x3 + 5x2 – 18x – 18 º (Ax2 + Bx + C)(x – 3) ff Now, setting x = 0 gives the equation –18 = –3C, so C = 6. To make it a bit clearer, divide both the top and bottom by h, keeping f(x) and g(x) aside. Inverse Trig Functions 4. 3 + 20 = 0.15 2 + 20 = 0.1 Tip: It doesn't matter whether the table gives the numbers or the probabilities — you can always find the conditional probability by dividing the entry by the total. 15x - 27 A + B + C / (x + 1) (x - 2) (x - 5) fi 15x - 27 / (x - 5) fi 15x - 27 A(x - 2)(x - 5) + B(x + 1)(x - 5) + C(x + 1)(x - 2) ... Repeated factors 3x + A B + C A / (x + 5) 2 fi (x + 5) (x + 3) fi 3x + A(x + 5) + B Equating coefficients: x terms: 3 = A constants: 0 = 5A + B fi B = -15 3x 3 15 This gives: 2 / + - + 2 (x 5) (x 5) (x 5) x 2 A B C Q2 2 / + + (x 2) (x 3) (x 1) (x 1) x fi 5x + 2 º Ax(x + 1) + B(x + 1) + Cx2 Equating coefficients: constants: 2 = B x terms: 5=A+B fi A=3 x2 terms: 0 = A + C fi C = -3 5x + 2 3 2 3 This gives: 2 / + + 2 - x + 2 - + (x 1) (x 2) (x - 2) ... 2 - 4 ... 2 fi 2 - 7 º A(x – 3) + B Substitution: x = 3 fi -1 = B Equating coefficients of the x terms: 2 = A 2x - 7 2 1 This gives: - 2 / (x 3) (x 3) (x - 3) Q1 6x + 7 A B fi º A(2x + 3) + B Substitution: x = - 2 fi -2 = B Equating coefficients of the x terms: 6 = A fi A: = 3x + 7 3 2 This gives: / ^2x + 3h2 ^2x + 3h ^2x + 3h2 fi 6x + 7 º A(2x + 3) + B Substitution: x = -4 fi -7B 8 fi B = -3 A + 3 B = 49C fi C 8 fi 8 Equating coefficients of the x2 terms: 0 = A + C fi A = 7 This gives: 7x + 3 3 fi 7 ... Q Putting it all together: f′(x) = 3x2 ... 2 + B(x - 1) + C(x - 1)2 Substitution: x = 1 fi 10 = 25A fi A = 5 ... 3 1 fi 45 - 5x = A + 5B fi 35 = 5C fi C = 7 Equating coefficients of x2 terms: 0=A+B fi B = -5 5x + 10 2 7 ... Substitution: x = 2 fi 8 = 4C fi C = 2 1 fi x = -2 fi -16 = 16A fi A = -1 ... 4 2 Equating coefficients of the x terms: 1 = -2A + B fi B = -1 ... This gives: /4 (x + 2) (x - 2) (x + 2)2 (x - 2) ... Q3 So, we have proved by contradiction that there is no largest even number. ff Check your answer by differentiating it to make sure you end up with the right function. 4 ^3h 4 ^3h - 1 fi 4 x 3 ... Q6 x 3 Q5 Q2 Q3 Q4 Q5 Q6 Q7 ... Q4 Solving: f(x) = 2 sin (2x) ... 1 Stretch horizontally, scale factor 2, then reflect in the x-axis, then translate by vector. 4 up. Q7 At t = 0, the particle is moving north-west. N At t = 0 dx E.g. The model could treat the ball as a threedimensional shape with a significant diameter, instead of a particle, to better estimate the distance from the wall. dx 2 As you saw on page 149, the derivative of tan x is sec2 x. Q2 a) Action Comedy Total Under 20 16 10 26 20 and over 6 8 14 Total 22 18 40 Action Comedy Total Under 20 0.4 0.25 0.65 20 and over 0.15 0.2 0.35 Total 0.55 0.45 or in terms of the probabilities, 1 b) You're told that they saw a comedy, so look at the Comedy column — the total is 18 (or 0.45). 0 a 3 a ≤ s 45 45 s ≤ s 50 50 s ≤ s 55 55 s ≤ s 60 s ≥ 60 28 40 27 2 0 a) Explain why it would be reasonable to use a normal distribution to model the cars' speeds. Functions Some mappings take every number in the domain to exactly one number in the range. Using vectors ff For a body moving in two dimensions (i.e. in a plane), you can describe its position, velocity and acceleration using the unit vectors i and j. ff Be able to find Magnitude The magnitude of a force is measured in newtons and the distance is measured in metres, the moment of a non-uniform rod or beam. ff So, P(C|D) = 7 5 outcomes in C Ç D number of outcomes corresponding to D Find P(D|C'). Irrational number A number that can't be expressed as the quotient (division) of two integers. Drawing a diagram will help you to see whether it acts clockwise or anticlockwise. ff A two-tailed alternative hypothesis says that the parameter you're investigating is not equal to the value in H0. Chapter 7 Differentiation 177 b) Find the equations of the tangents to the curve at each of these points. 2 x is valid for x ≠ 0. p 1 +1 = 1 +1 2 sin x p dx : 2 (cos x) p xD 0 p = :- 1 cos x + 1 xD p 0 2 1 1 p 1 1 1 1 :- 2 cos x + p a p x p kD :- 2 cos 0 + p a p x 0 kD = :- 1 ^ - 1h + 1D :- 1 + 1D = 0 D 2 2 = :; 1 + 1 ... Multiply out and integrate each term separately: #12 ^cos x + sin x dx = 1 sin x + cos x C 2 186 Chapter 8 Integration You can integrate linear transformations of functions of the form sin(ax + b) and cos(ax + b). ff Since P(Z ≥ z) = P(Z < z) you can interchange the ≤ and < signs — i.e. f(z) = P(Z < z) as well. Q7 Given that f A 4 dx = 10 and A ≥ 1, find A in terms of e. So # (n + 1)f′(x)[f(x)]n dx = [f(x)]n + 1 + C ff 1 3 3 2 2 2 2 (2x – 4)(x – 4x + 5) MODELL ING Q4 Find the coordinates of the stationary point(s) ... 311 Q4 A golf ball is hit off the edge of a 40 m high cliff with velocity (ai + bj) ms–1, where a and b are positive constants. Show that points A, B and C are collinear. OA = 2i – j + k, OB = i + 2j, OB = i + 8j + 7k. The beam is supported by a smooth peg at C, where AC = 11 m, so that it makes an angle of 20° with the horizontal, as shown. f(a) 0 a b x ff It's not a very good approximation, but if you split the area up into more trapeziums of equal width, the approximation will get more and more accurate because the difference between the trapeziums and the curve will get smaller. Start by identifying 'u' and 'v': u = x3 and v = tan x ff Now differentiate these two separately, with respect to x: du dv = 3x2 and = sec2 x dx dx ff Very carefully put all the bits into the formula: Tip: See page 149 for how to differentiate tan x. So it's a good idea to resolve forces parallel and perpendicular to the beam, rather than horizontally and vertically. Here's a quick reminder of what each one does: y = f(x + a) y = f(x) x 0 ff f(x – a) f(x) is translated a right. equations without needing to convert to Cartesian form. dy dy dy b) 2xy + x2 + 2y = 2xy ... Q9 x = t3 + t2 ... Q Implicit differentiation Q1 a) Differentiating: dy dy x 2x + 2x + 2 - 2y = 0 fi dy = ... 1 5 dx dx Q16 Chapter 8 Integration fi 3x4 sin x3 ... 5 sin(4x) ... Exercise 5.1 Q1 Find the following integrals: Q3 Q6 # sec x tan x dx ... Q12 Integrating f′(x) × [f(x)]n Some products are made up of a function and its derivative. This bracket is the derivative... 2 Q8 f'(x) = 3x2 + 4x + 3, so at x = –3, 2 2 2 5 4 8 f′(x) = 3b - 3l + 4b - 3l + 3 = 3 - 3 + 3 = 3 f′(x) ≠ 0 at the point of inflection, so it is not a stationary point. y You are told to find area C — so you need to find the x-coordinate of the point of intersection. centre of mass × Example 5 A uniform lamina, ABCD, of weight 8 N, is pivoted at point A. A force of magnitude 16 N also acts vertically upwards on the particle. Fortunately, this CGP Student Book makes it all much easier to digest. = 12.490... = 6.24 N (3 s.f.) ff Resolving horizontally: F = N sin 20° = (10.06...) sin 20° = 3.435... ff Resolve parallel to the plane ( ) for A: T + F – 3g sin 30° = 3 × 0 Mg + 10.184... If the function touches the x-axis but doesn't cross it, then there won't be a change of sign. = 3.896... ff The magnitude of the frictional force can take a range of possible values. Q1 Differentiate with respect to x: a) y = sin (3x) b) y = cos (–2x) b) p cos (f) y = cos 2 (q) y = sin q ... Using Trigonometric Identities in Integration Learning Objective: ff Be able to use the double angle formulas — for sin, cos and tan and alongside the methods learnt throughout this chapter to simplify difficult trig integrals. Q7 f(x) = Q8 f(x) = x2 with domain x d R , and g(x) = 2x – 3 also with domain x d R. ff You're Reading a Free Preview Pages 187 to 203 are not shown in this preview. Putting this into the Newton-Raphson formula gives: 3xn g (xn) xn2 – 1 = xn - e 3x4 xn + 1 = xn - 1 g (xn) 3e n - 8xn c) Differentiating f(x) gives h′(x) = cos x + 3x2. So the largest continuous domain that makes a function would be either x > 2 or x < - 2. = 0.0478 (3 s.f.) ff Tip: Almost all the possible values of x are within 3 standard deviations of the mean (see page 270), so choosing -9999 or 9999 means that the area you're calculating will cover the whole region you want. Horizontally, vx = u = 17 ms–1 Using Pythagoras: Vertically, vy2 = u2 + 2as fi vy2 = 02 + 2 × 9.8 × 20 ff Using trigonometry: = 14.071... Tip: q is the Greek letter theta. b) Maximum force is when F = mR, so max force is 58.8 N Q4 T R F 5g mu = 3 4 3 k6 a12 - a2 × 9.8 × 5 4k a2 × 9.8 ... 3 5 3 Q5 Gn y 5 = 0.02 (2 d.p.) Q6 a) Resolving vertically: F = 6 cos 25° Friction is limiting, so, using F = mR: 6 cos 25° = 0.15R fi R = 24.168... to (a, b, c). d) By using the iterative formula part c) with starting value x0 = 1 find the value of x1 x2... Example 4 The flight path of two different aeroplanes taking off from a runway are modelled as position vectors a = 5i –2j + k, and b = –3i + 3j + k. Tip: Remember — if the coefficients of a polynomial sum to 0, then (x – 1) is a factor. + 1.6791... Q6 A toy submarine is moving underwater at a constant depth, with an initial velocity at time t = 0 of (–0.6i + 3j) ms–1, where the i direction is due east and the j direction due north. Exercise 5.2 — Solving modulus equations and inequalities Q1 a) y = |x – 2| y=|6 2 2 y=|1 1 The graph shows that there are two solutions to |x – 2| = 6. dx 2 When 1 < x ≤ 5, x + 1 is positive and x – 1 is d2 y positive, so is positive for 1 < x, x 2 - 4x) Show that an alternative iterative formula is xn+1 = n 5 n - 4. You can also use the iterative formula to find the value of the function at that point, as shown in the examples below. dx 1 ff cosec x sin x, so use the quotient rule: du dv u=1 fi = 0 and v = sin x fi = cos x dx dx so dx ^sin x h dx ... ^ 1 x cos xh dx cos x ... dx = dx sin x 2 sin x 2 cos x 1 1 ff Since cot x sin x , and cosec x = cos x 1 - sin x = ... Example 2 Prove by contradiction that there are infinitely many prime numbers. Dividing by ... In 2.1 - (2.1 - 3)2 = -0.068... So the equation of the tangent is y = 2x + 1. Q6 Q1 Hint: You're looking for the tension in the string when the toy is on the point of sliding. Q Resolving vertically: R(≠) = 11 sin 75° – 12 sin 80° = –1.192... ff Using a calculator, P(X < 780) = 0.960 (3 s.f.) 766 780 x b) The machine fills 2138 boxes of cereal in an hour. Q2 a) (1 - 6x)–3 = 1 + –(3)(–6x) + 1 + 6x = ^1 + 6xh2 1 a1 - k 2 2 1 1 = 1 + 2 (6x) + 1 # 2 (6x)2 1 a 1 - 1a 1 - k 2 2 1 2 2 + (6x)3 + ... 226 Chapter 8 Integration ln P0 = –k(0) + C fi ln P0 = C ff So the equation is: ln P = –kt + ln P0 fi P = e(–kt + ln P0) = e–kteln P0 fi P = P0e–kt c) Given that k = 0.1, find the time at which the population of rabbits will have halved, to the nearest day. The trapezium rule for approximating is the number of strips i.e. trapezium. The model could be adjusted to be more accurate for very low values of N. Stationary point A point on a curve where the gradient is 0. ff Replace x with f–1(x) and y with x — this is the inverse function. It looks like this: dy dy dx (= d x dt fi dy The chain rule lets you find dy/dx from parametric equations, differentiate each equation with respect to the parameter t, then put them into the formula. Chapter 13 Hypothesis Testing 2. ff Now find the value of your test statistic: x = 30.9 30 = 1.3416... ms–1 Horizontally: uH = ux = u = 19 cos 28° Using Pythagoras' theorem: speed = ^19 cos 28°h2 + ^6.353...h2 = 17.9 ms–1 (3 s.f.) Q13 a) Resolving vertically, taking up as positive: s = u = u sin q, v = 0, a = –g v2 = u2 + 2as 0 = U2 sin2 q – 2gs U 2 sin 2 q fi s = 2g ff So the maximum height is U2 sin2 q / 2g ... b) Resolving vertically, taking up as positive: s = U sin q, v = 0, a = –g ... t = 0 U sin q fi t = g s Q14 a) Resolving vertically, taking down as positive: u = 0, a = 9.8, t = 1 s = ut + 2 at2 0.2 = 4.9t2 fi t = 0.063... s = 2.357... b) This is a one-tailed test. ff Be able to calculate integrals using substitution. Example 4 An amount of... Integration by parts ... two different vectors in three dimensions using trigonometry. ff Be able to ... For some functions y = |f(x)| looks very different to y = f(x) — as you'll see in the example on the next page. Tip: Swapping the limits and putting a minus in front of the integral might seem more complicated, but it often cancels with another minus, making the whole integration easier. two equations on the same set of axes. 1 c) f(x) = + with domain x > –2 Has a range x ≥ 2 f(x) > 0. then X can be approximated by the normal random variable Y ~ N(np, np(1 – p)) 1 2 and n is quite large, then you don't need to use B(n, p) to work out probabilities for X. Q7 The maximum value of sin x is 1, and the minimum is –1: Transformed Function f(x) + 2 f(x – 90°) f(3x) 4f(x) New equation y = 2 sin x + 2 y = 2 sin(x – 90°) y = 2 sin 3x y = 8 sin x Max value Min value 3 1 1 4 ... –1 –1 –4 Q12 a) y = 2x2 – 4x + 6 = 2[x2 – 2x + 3] = 2[(x – 1)2 + 2] = 2(x - 1)2 + 4 c) y = 2(x – 1)2 + 4 This graph ... can be approximated by a Normal distribution ... 2 1 1 - 3 ... Find an equation for the tension ... Q2 When there's an ff Draw a diagram, and label all forces. Example 1 Find the equation of ... c) Find, to 2 d.p., Normal distribution — this graph ... gradient, i.e. N ... 4 d.p. ... so 1 x ... y = x = ... q Show that ... 4 ... and the normal ... x ... so the greatest distance ... Tip: Remember — don't ... Example 2 Show that the equation ... Modelling with Probability Exercise 2.1 — Modelling assumptions and problems There are many possible answers to these questions — here are some examples: Q1 – S he has assumed that the probability of eating each face is equally likely — given that his cube is a cube, this is probably not a very good assumption. b) Find gf(x) and write down its range. Integration by Parts Learning Objectives: ff Understand that integration by parts comes from the product rule. Learning Objectives: ff Understand the meaning of the modulus, including modulus notation. Including plenty of modelling and problem solving It's easy to differentiate CGP from other Maths books — we're the best by miles P G C www.cgpbooks.co.uk A-Level Year 2 Mathematics — The Complete Course for AQA Year 2 of A-Level Maths will really test your limits... So here we've got the whole lot of A-Level Maths. Give your answers to 3 significant figures. You can also work this out by taking logs. dt dN So μN fi = kN, for some k > 0. p Putting this in y = mx + c gives: 6 = 13 p = 0) + c fi c = – 13 p p 6 So the equation of the normal is y = x – 13. Since z = –2.598 < –2.575, the result lies in the critical region and is significant. The particle's horizontal displacement and sy is the particle's vertical displacement. ff Resolving vertically (≠) for the particle, the result... in the critical region and is significant. The angle of ... is 0.3783 so your critical region will be... 2 1, so again c = 2. ff The alternative hypothesis was r > 0, you would use a one-tailed test. r = 0.3783 so your critical region will be... ff Tip: If you use integration, remember. Since you want to work out the integral using integration by parts... dN = kN at time t, differentiate... Mappings and functions with a continuous domain (such as x d R , i.e. not a discrete set of values) can be drawn as graphs. dN dN μ N fi = kN, dt dt dN 1 fi = kN fi ∫ dN = kt + ln A Q3 a) b) = kN = ekt... Newtons ... 2 a) N Aekt N = Aekt N = 20ekt In N = 20ekt ... 1 N = 20e0.25t In 1.5 So N = 1000 fi 1000 = 20e0.25t In 1.5 fi 50 = 0.25t In 1.5 fi t = In 50 ÷ In 1.5 = 38.59 So the field will be over-run in 39 weeks. ff Resolve vertically for B (θ): 10g – T = 10a T = 98 – 10a — equation 2 ff Substitute equation 1 into equation 2 : 4a + 36.597... = 9.39 kg (3 s.f.) 3m B 90 N MB g Tip: It's a good idea to take moments about a point where an unknown force is acting, as the moment of that force about the point will be zero. ff State the conclusion of the test. ff Use the normal approximation would still be appropriate to estimate the probability of at least 25% surviving to be over 25%. Be able to find the equation and the set of numbers they become is called the range. dy b) From part a), m = 1, so the normal gradient is –1. Explain whether or not a normal approximation would still be appropriate to estimate the probability of ... vector ... 25% surviving to... There are other ways you could have used the log laws in this question — you should get the same answer. Chapter 7 Differentiation 173 ff Step 1: Identify two functions... so... = 5.77 kg (3 s.f.) Q5 a) F Resolving perpendicular to the plane (): R = 5g cos 35° = 40.138... ds s = # dt dt dy = 2y(1 + x)2 when x = –1 and y = 4. = 0.663 (3 s.f.) Tip: Remember to use a lower bound (e.g. –9999) for 'less than' probabilities and an upper bound (e.g. 9999) for 'greater than' probabilities. - 5.883 × 100 = 0.0010% (2 s.f.) 5.8829... b) Differentiate y = (x + 1)3(x – 1). ff Draw a diagram showing all the forces acting on the rock. Calculate the total area of the grey sections of the logo. Functions are written using the following notation: f(x) = 5x + 1 or f : x Æ 5x + 1 You've probably seen at least the first notation before, but you need to be able to understand and use both. The Normal Distribution 2. N) is greater than the downward force of the two weights (0.1g + 2 = 2.98 N), so the reaction will act downwards to balance this out. Example 1 A function f(x) = x + ^7 has domain x > 0 and range f(x) > 0 ... Using the general formula with a = –6 and b = 1 gives: # sin ^1 - 6xh dx = –16 × –cos(1 – 6x) + C 1 = 6 cos(1 – 6x) + C Tip: The term cosec x is ... differentiating cosx(1 – 6x) with the chain rule gives 6 sin(1 – 6x). ff Resolve parallel to the plane (): Resolve parallel to the plane ( ): F = mR: 5g sin 20° – 3 × 0 R = 25... integrate by parts... Integration by parts to... 3 × 3 ... 8 ... dx = 8 ... ... Q2 ... Q1 Integrate the following functions with respect to x: a) ... integrate... b) ... Chapter 8 Integration... 35 ... ... 2 ... product but you can't use any of the methods you've learnt so far, you might be able to use integration by parts. Modelling with Trig Functions Exercise 9.1 — Trigonometry in modelling Q1 For each sector of the circle, 2 for so 1 Area = 2 r2q = (200q) m2, and Perimeter = 2r + rq = (40 + 20q) m Convert each angle to radians and then find the area and perimeter: 2p 120 × 120° = 180 = 3 2p Area = 200 × 3 = 419 m2 (3 s.f.) 2p Perimeter = 40 + 20 × 3 = 81.9 m (3 s.f.) 4p 144 × 144° = 180 = 5 4p Area = 200 × 5 = 503 m2 (3 s.f.) 4p Perimeter = 40 + 20 × 5 = 90.3 m (3 s.f.) 96p × 96° = 180 = 15 8p Area = 200 × 15 = 335 m2 (3 s.f.) 8p Perimeter = 40 + 20 15 × 15 = 73.5 m (3 s.f.) Q2 a) cos q has a maximum of 1 and a minimum of –1. ff Using the formula for conditional probability: P(C|W') = P^C + Wlh P(W') = P(C + Wlh Tip: If either walks or runs to the bus stop, so P(runs) = P(W'). 338 Chapter 15 Dynamics Friction and inclined planes Inclined planes ff For objects on an inclined plane, resolve forces parallel and perpendicular to the plane, rather than horizontally and vertically. Cosec, Sec and Cot 5. Geometric Sequences and Series Exercise 4.1 — Geometric sequences second term = 3 Q1 Common ratio r = 2 = 1.5. first term Then: 4th term = ar3 = 1.5 × 1.53 = 5.0625 5th term = ar4 = 1.5 × 1.54 = 7.59375 6th term = ar5 = 1.5 × 1.55 = 11.390625 b) 6th term = 5th term × 1.5 = 10.125 × 1.5 = 15.1875 7th term = 6th term × 1.5 = 15.1875 × 1.5 = 22.78125. Since 1.556 < 2.326, the result does not lie in the critical region and is not significant. The data does not provide significant evidence to support the claim that the more a person likes romance movies, the less they like horror movies. f(z) ff Now, use your calculator to find a and b. Q3 T 0.6 m 20° A 1.5g 0.5g A non-uniform beam, AB, of mass 1.5 kg and length 2.4 m, is freely hinged to a vertical wall at A. c) When x = 1, y = 31 = 3. ff Most of the weights lie close to the mean weight — with similar numbers distributed symmetrically above and below. e) Find P(M|A Ç E). So x1 = 3.5 Q3 a) 14x2 + 6x + 11 º (x2 - 15x + 21)(x + 1) - 10. fi T = 3311.10... A health authority tests a sample of 1000 people to see if they carry the disease. The string passes over a smooth pulley fixed to the apex of a wedge. N Resolving vertically: 46.581... Two of the three players who can play in goal are over 6 feet tall. RQ = – OP + OQ JKANO JK2NO KK OO KK OO = – KK3 OO + KK5 OO = KK OO KK4OO L P L P JK2 - 4NO JK5 2NO KK OO K O KK5 - 3 OO = KK2 2 OO KK - OO KK OO K 0 P L P y 2 P(4, 3, 0) Tip: Don't get confused between a point and its position vector. c) Q6 Show that the integral of the function y = 5 cosb 6l between x = 2p and x = p is 15(1 – 3). ff Note: When you're resolving a force, you should use the cosine of the angle if you're resolving. Example 4 22 children are at a Halloween party. Example 1 Integrate the following functions with respect to x. ff You solved the equation |2x – 4| = 5 – x on p.30, so you know that the graphs cross at x = 3 and x = –1. Identities Involving Cosec, Sec and Cot Exercise 5.1 — Using trig identities Q1 cosec2 x + 2 cot2 x = cosec2 x + 2 (cosec2 x – 1) = 3 cosec2 x – 2 1 Q2 tan2 x – tan2 x – sec2 x cos 2 x = tan2 x – (1 + tan2 x) = –1 1 1 Q3 x = sec q + tan q y = sec q – tan q (sec q + tan q) 2 q + 2 sec q tan q + tan2 q sec2 q tan2 q x2 y2 = sec2 q = tan2 q. But since sec q = tan q + 1: x2 = 1 + 2 ... 1 sec q + 2 sec q tan q + 2 sec q tan q ... required. These are approximations of the actual area under the curve. dy using implicit differentiation as usual: dx dy dy cos x + y sin x = 2 ... x+ 3x2 ... x = y sin y sin x ... x + 2 x x = 2x fi ... Tip: You should know cos p = –1 and sin p = 0 — these are standard trig results that were covered on page 53. fi T = 1539.21... At this point, the friction is said to be limiting. PROBLE M SOLVIN When t = 1, the body has position vector r = 21 m, relative to O. 2 x 2 (x – 2) 2 2 (x + 2) 2 = ... 2x + 2x ... Q13 1 a2 + 1 k x x 1 = x2 + 1 ... 2 x + 1 ... k x 1 2x + 1 ... 1 1 + 2x + 1 = ... 1 x x + ... 2x = ... 4x + 2 ... 2x x + 2 x ... ^ + 1 ... 1 2 ... 3 1 ... -12 Q15 Q16 Q17 ... 3 Q18 ... Q19 Q20 ... Q15 Q16 Q17 Exercise 1.2 — Adding and subtracting algebraic fractions Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9 ... Q10 Q11 Q12 Q13 Q14 Q15 ... Q16 ... Q17 ... Q18 ... Q19 ... Q20 ... Answers 371 Exercise 1.3 — Multiplying and dividing algebraic fractions ... Q1 a) ... Q2 ... Q3 ... Q4 ... Q5 ... Q6 ... Q7 ... Q8 ... Q9 ... Q10 ... Q11 ... Q12 ... Q13 ... Q14 ... Q15 ... Answers 472 Exercise 1.4 — Algebraic division Q1 a) ... Q2 ... Q3 ... Exercise 1.5 ... h) lw| Arcsin The inverse of the sine function, also written as arcsine or sin–1. ff Replace A and B in the original identity: 12x + 6 2 + 4 º 2x + 3 2x – 3 2x – 9 ... ff So # 12x + 6 dx = # 4 2x - 9 # 2 + 4 dx = ... 2x + 3 ... Q3 a) This is a two-tailed test, so the hypotheses are H0: r = 0 and H1: r ≠ 0. ... the quadratic formula gives x = 2 You can ignore the negative solutions to the quadratic equation because you're solving for a distance x. Q Substituting into either of the original equations gives: –14 = A + B, so B = –15. = 3330 N (3 s.f.) 5g Answers 471 b) Resolving vertically (≠) for the truck: RT = 2000g The you're told you can assume that friction is at its maximum value, so using F = mR: F = 0.08 × 2000g = 1600g N Resolving horizontally (Æ) for the truck: Fnet = ma W – F – T = 2000 × 0.022... x = 0°, 71.6°, 108.4°, 180°, 251.6°, 288.4°, 360°. You can say that y is undefined for –1 > y > 1. ff Using Trigonometric Identities in Integration 200 7. Comment on your answer. But for more complex functions, you need to rearrange the equation for t, or if it's a function of x. ff Their (i) component (the body's horizontal motion, and the j component usually describes the body's vertical motion. p c) If x = arctan (3 then x = tan 3 ... If x = arctan 3, then x = p 3 … ff Using Trigonometric Identities... You can write in ex in (x + 3) Tip: Remember to replace k... a pair of coordinate axes, it would be moving along the line y = x (for positive x and y). Use this information to find C, and hence v: 0i + 10j = 5e0 i – (cos 0)j + C 0i + 10j = 5i – j + C fi C = –5i + 11j fi v = (5e0.2t – 5)i + (–cos t + 11)j Tip: You'll be expected to know how to differentiate and integrate any of the types of expressions that were covered in Chapters 7 and 8 — so any calculus that comes up elsewhere in the course can also come up in Mechanics. sin 2A ² 2 sin A cos A 35 35 1 fi sin 2x = 2c 6 × 6 m = 18 35 sin 2x c) tan 2x cos 2x = 17 1 2 7 Q3 a) cos 2x = 1 – 2 sin2 x = 1 – 2a - 4 k = 8 b) First find cos x: 1 2 15 cos2 x = 1 – sin2 x = 1 – 4 = 16 15 fi cos x = – 4 x is in the 3rd quadrant of the CAST diagram where cos x is negative, so take the negative square root. 242 Chapter 9 Numerical Methods Combining the methods You might see a question that combines all (or at least most) of the methods covered so far in this chapter. Chapter 2 Algebra and Functions 15 Turning mappings into functions Some mappings that aren't functions can be turned into functions by restricting their domain. x2 It's given in terms of x and y, so: 4 1 P(Zombie|Girl) = 12 = 3 b) Two girls dressed as ghosts go home. - 3g sin 30° = 0 M = 2.54 kg (to 3 s.f.) b) What would M be if A were on the point of sliding down the plane? Q13 The curve C is defined implicitly by cos y cos x = sin x . You may assume that the frictional force between the truck and the horizontal plane takes its maximum value. Find x#(3x + 2)3 dx. a) Find the equation of the tangent to the path of the particle when t = 2, giving your answer in a suitable form. + 2.4138...] = 2.691 (4 s.f.) The corresponding trapezium rule answers from Q1 lie inside the lower and upper bounds. 4y 3y 5x 3y x 3 4x 3 3x 2 ... Q2 a) 3 ÷ 5x b) 3 + 23 6 3x 34 c) 2x 3 3x 4 ... Q Q3 ... 2 × 2x - 2 ... 5X Q6 ... Q14 ... 2 + 1 ... Q12 ... 2 2 + 2 x + 2 ... Q4 ... Q15 ... x x 5 ... 3 ... Q4 ... Q16 Ç ... 2 + x + 2 ... 1 ... = x + 1 ... 4 ... 2 ... 3 ... ... + 2 - 2 ... Vectors in Three Dimensions Exercise 1.1 — Three-dimensional position vectors J N J N KK - 3OO KK 4 OO Q1 a) b) KK OO c) KK OO ... KK 5 O ... Q2 a) ... JK -3NO ... b) ... c) ... OR = 4i – 5j + k OS = –3i – k There's no j component in vector OS because its j component is zero (you don't write 0j). Tip: This doesn't work for n = –1 because you'd end up having to divide by n + 1 = 0. The block is held in equilibrium by a force of 147 N, acting at an angle of 60° to the plane. So the range is g(x) ≥ 0. Tip: Try to be able to think of other limitations and improvements for part d) — these aren't the only possible answers. y For example, consider the graph of the mapping 1 y = - for x d R : x 1 1 The mapping y = x 1 for x d R is not a function, because it's not defined at x = 1. ff You just input the upper bound x = 105, a lower bound (e.g. -9999), m = 100, and s = 16 = 4 into the normal cdf function on your calculator. The projectile moves freely under gravity and passes through the point (q, m). Learning Objectives: ff Be able to differentiate functions defined parametrically. d) Given that the i and j components of the force shown... b) Given that the system is in equilibrium, find m and n. Q10 a) 2 a2 + j 4b (i + 5i – 4b) 2i + 3j 25k and 3j – 2k ... KK OO KK OO KK 2 OO K4O KK 2O ... 3 OO ... K OO ... LO ... Q4 The flight path of a toy aeroplane is modelled by the vector 2m – n, JK- 5NO JK- 4NO KK OO KK OO - ... M ... The model can be improved by ... So the number of successful first-serves in a match, M, can be modelled with a binomial distribution — M ~ B(100, 0.65) Alternatively, n is large, and 0.65 is close to 0.5 so the normal approximation to the binomial distribution is suitable. Use the iterative formula to find the value of x_2... M could be approximated by a normal distribution Y, where m = 65 and s2 = 22.75 — Y ~ N(65, 22.75). –15.733... fi t = 0.896 s (3 s.f.) Q6 Resolving perpendicular to the plane (): R = 5000 cos 10° Resolving parallel to the plane (): F = ma T – F – 5000 sin 10° = (5000 ÷ 9.8) × 0.35 T – mR – 5000 sin 10° = 178.57... Show that triangle AOB is right-angled. ff So the result is significant. b) Given that V = 4bi when t = 0, find A in terms of a and b. There are a fixed number of trials (20 meals), with probability of success (i.e. vegetarian meal) of 0.15. Put together that both components will have a turning point at ... Q Recurrence relations. Chapter 1 Proof 1 Tip: You can use the same method to prove the irrationality of any surd, although you need to prove the statement "if n2 is a multiple of p then n is a multiple of p", which is a bit trickier than the proof in Example 1. Give reasons for your answers. = –13.237...° i.e. 13.237...° below the positive horizontal So direction = 360° – 13.237...° = 347° (3 s.f.) ff Resolving vertically: R(≠) = 6.5 cos 69° + 10 sin 67° – 8 sin 61° = 2.152... A force, Q, acts on the box at an angle of 10° to the horizontal, as shown. Sigma notation Used for the sum of series. For example: 4x5 - 7x2 + 3x - 9 + x2 - 5x + 8. a) Find the speed and direction of motion of the stone after 0.5 seconds. c) A shop's store room has 1303 bottles of vinegar. ff Look for common factors in the numerator and denominator — factorise top and bottom and see if there's anything you can cancel. ff Using a calculator: P(X ≥ 7.5) = 0.961572... sin 4 x 1 ff Write as a negative power. Tip: The reason why a fixed inverse... d) A projectile... uses the sample... 2 (2.179...) Tip: Using the rule (p.139) is used to differentiate a function of a function f(g(x)): if y = f(u) and u = g(x) then: dy dy du = dx du dx Tip: Don't forget that for indefinite integrals (integrals without limits) you need to add a constant of integration, C. k = 83.788...° (measured clockwise from the downward vertical) So direction = 270° – 83.788...° = 186° (3 s.f.) Q5 a) Taking moments about A: (10 cos 30° × 2) + (T sin 30° × 4) = (1.75g × 4) = (2.25g × 8) fi T = 54.474... 3 Sp So cos–1 = 2 – (x – 6) + 2 = 60 +2 d) a = 2 sin 4° × tan(x) = 0.267 m s-1 2 3 ff If you ... and find... (cumulative frequency) ... mg the sum of moments about a point. Root A value of x at which a function f(x) is equal to 0. ff The y-values go in steps of h, starting with x0 = a. Chapter 11 Probability 267 2. a) Find m, the coefficient of friction between A and the plane. a) Find the tension in the string, K. 5 5x = 1 – 6 + a – 72 k(25x2) + ... A golf ball is struck from a point A on a horizontal plane. The expansion of (1 + 4x)4 is valid for all x, since x is a positive integer. k = 24.265...° (measured clockwise from the downward vertical) So direction = 270° – 24.265...° = 246° (3 s.f.) Exercise 2.2 — Friction Q1 a) Taking moments about A: 12g × 8 = T sin 20° × 16 fi T = 940.8 × 5.472... Let u = 2 - x2 . b) Evaluate f –1(2). So f –1(x) has a domain x > 3 and range f –1(x) > 0. c) Equating coefficients — x2 terms: 6a2 + 12ab + 6b2 = 24 fi 6(a + b)2 = 24 fi (a + b)2 = 4 fi a + b = ±2 Equating coefficients — x terms: 3ab = 3b + 1fi + 3ab fi b ... 3 ... 3 a ... b 2 ... 3 f ... = 2 ... 3 ... 2 ... 2 ... a = 2 ... 2 - 2 ... 2 ... b ... -5, b = 7 and a = 7, b = –9 You could also have rearranged to get either a or b in terms of the other, and then substituted that into the other equation. You can sometimes use the trig identities that you learnt in Chapter 3 to manipulate nasty trig integrals to give functions you know how to integrate. ff Find the value of ... 2 dy ... 2 = 0. ff ... x = 2 ... Differentiating ... ff You could have substituted for y ... b) 10, 14 ... each term. variables — such as height, weight, length, etc. Q1 f(x) - 1 x 0 1 2 3 2 - 1 ... 2 ... Q2 a) ... b) ... In the graph above, f(x) goes from positive to negative or vice versa — f(x) changes sign as it passes through a root. ff So rewrite the integration: # sin x cos x dx = # 12 sin 2x dx = 12 # sin 2x dx = 12 a - 2 cos 2x k + C = - 14 cos 2x + C b) # 0 p 4 Tip: If you need to integrate a function of the form sin x cos x, use the double angle formula for sin 2x. To do this, get all the terms containing y on the left-hand side, and all the terms containing x on the right-hand side and dy split up the . y y = 2f(–1)2 + 2(-2 ... 4 ... 2) fi ... ff So the point of inflection of x3 is at (0, 0), so: Transformed Function New equation Coordinates of point of inflection f(x) + 1 f(x – 2) –f(x) + 3 ... y = (x – 2)3 ... y = –x3 + 3 ... (0, 1) (2, 0) (0, 3) ... Q4 a) 5x3 ... The minimum point is at (1, 4). Q10 Given that f(x) = 3x + f(2, 1) ... fi f(x) = 3(x - 1)2 ... Q Resolving ... The stone is launched with velocity of 8 ms–1, q q where q is a constant. Q2 T A 0 A 0.2 m ... 0 T 2 x 4 ... 0.5 ... 16 Moments Taking moments ... x 0 ... 1 ... The resultant force on a body is given by its ... The line is ... 2 = 5.8 ... 4 ... 2 ... Q1 x 2 - ... 2 4 ... = 1 fi 2x = ... Take the ... fi 5 x + 4 = ... x ... 25 ... 2x ... 10 ... 4 ... x = 5 ... ff Use the i and j components to draw a right-angled triangle: v q 15 ff Now use Pythagoras' theorem to find the particle's speed, and trigonometry to find its direction of motion. 314 Chapter 14 Kinematics speed = |v| = 152 + 52 = 15.8 ms–1 (3 s.f.) 5 q = tan–1 15 fi q = 18.4° (3 s.f.) from the i-direction Example 2 A particle is moving in a vertical plane so that at time t seconds it has velocity v ms–1, where v = (t3 – 6t)j. stretched parallel to the x-axis (i.e. horizontally) 1 –180° by a factor of a . ff Fy is the perpendicular distance of 0.2 m from O, so using the formula: Moment = 20 sin 60° × 0.2 = 3.46 Nm (3 s.f.) anticlockwise Example 3 5 O Tip: You could've considered the i and j components separately — you'd get the same answer. Equating coefficients of x terms: 0 = B + 2C fi ... Moment = 25 cos 40° × 5 Chapter 16 Moments 25 N ff You need the perpendicular distance, d, between the line of action of the force and the pivot point: s 4 0 40° A force of 25 N acts upon a light rod of length 5 m, pivoted at A. ff Resolve perpendicular to the plane (): R = 3g cos 30°. Chapter 16 Moments 3 5 O Tip: You could've found this using the cos rule... - 13 a - 13 5o 1.8 = - 1.8 , making a and c is correct to 3 s.f. Exercise 2.2 — Finding a function ... Q2 ... Q3 ... Using the formula ... 2 ... A 4 ... x ... The point ... ff The iteration ... Q10-11 ... 4 ... 2 ... 3.9... = 2.7 to 1 d.p. ... Q2 a) ... x = 2.7 ... Q ... Q20 Q21 ... Q22 ... Q23 ... ... The ladder is in limiting equilibrium ... Reaction Forces and Friction Learning Objectives: ff Be able to solve problems involving rigid bodies that are freely hinged to a surface. ff Okay, this one looks a bit scary, but it'll make more sense when you write down the probabilities you know: P(W) = 0.4, P(C) = 0.54 and P(C|W) = 0.3 ff Now you can draw the Venn diagram, you need to find the intersection C Ç W. Vector A quantity which has both a magnitude and a direction. (1 - 1.03) In 20 days she runs a total of 322 miles (to the nearest mile). Q10-11 Hint: You'll need the double angle formulas here — see page 75. Use the iteration formula xn+1 = Start with x0 = 2. 6 = ... 2187 fi 2187 = 6 × 3n Find the term 6 in 2187 = ar and n ... ff 3 = ... 9. Find the value of h. The bead is on the point of sliding along the wire. The probability of ... ff In a ... We show that a projectile ... n = 9 ... Tree diagrams You may not have seen tree diagrams ... Then you can look at two conditional probabilities about probability ... vector ... acts on ... rod ... the ... is ... show. Q9 A curve is given parametrically by x = q2 sin q, y = q ... 172 Chapter 7 Differentiation 10. These graphs give you an idea of what different values of r look like. A light strut, fixed to the wall, supports the rod at its midpoint, so the rod is horizontal. ff Take the positive square root to leave: x = 6 + 13 . 478 Glossary Factorial n factorial, written n!, is the product of all integers from 1 to n. -1 0 1 x -5 The range of this is f(|x|) ≥ -5 Exercise 5.1 Q1-2 Hint: Don't forget to label the key points such as where the graphs cross the axes. Use the differential equation for D: So dD - k = dt D2 dD k = - 2 when D = 40 fi - 2 = -2 dt 40 fi k = 2 × 402 = 3200 Chapter 8 Integration 227 c) Given that D = 60 cm at t = 0 s , find a particular solution to the differential equation for D, and hence calculate D when t = 2 s. Find the root of the equation x2 + 3x – 8 = 0 to 2 d.p. Q4 a) Show that the equation 2x – 1 = 4 can be written as x = 22x – 6 . ff As the total area under the curve is 1: P(X > 78) = 1 – P(X < 78) ... So you only need to be able to ... xt ... ff To use the ... 3 ... 2 ... 8 ... ff So the integral ... 2 ... Multiply top and bottom by ... You'll find that ... is directly proportional to V, dt dt dt D dV dVin so = kV for some constant k, k > 0 and out = 20. = 1.40 s (3 s.f.) b) Resolving vertically, taking right as positive: s = s, u = 8 cos 59°, a = 0, t = 1.399... ff Resolve perpendicular to the plane ( ) for A: R – 3g cos 30° = 3 × 0 R = 3g cos 30° ff The friction is limiting, so: F = mR = 0.4 × 3g cos 30° = 10.184... So the sum to infinity is a = 9 = 9 = 90 SY = - r 1 r 0.9 0.1 Q3 So a = 2a fi - 2 fi 1 - 2 = 2r 1 r 1 r 1 - 2 r 1 2 = 0.5 Q4 a) Sum to infinity is 13.5 = SY a 1-r a (1 - r) = 3 = r a (1 – r3) a Divide S3 by SY: 13 = 1 - r 13.5 1 - r (1 – r3) 27 = 1 - r 3 27 1 27 r = 27 fi r = 3 1 ... You can see that the ... Chapter 16 Moments ... 8 ... P(A Ç B) ... ff Resolving ... Q6 Find ... For vectors in three dimensions (three-dimensional ... 3 ... 1 Q17 ... ff The ... 4A – 3B Solving simultaneously gives: A = 3, B = 4 11x 3 + 4 This gives: / 2x 2 + 5x - 12 (2x - 3) (x + 4) Q7 a) x3 - 9x º x(x2 – 9) = x(x + 3)(x - 3) 12x2 + 18 A B C / + x + x + 3 (x - 3) fi 12x2 + 18 º A(x + 3)(x – 3) + Bx(x – 3) + Cx(x + 3) Substitution: x = 0 fi 18 = –9A fi A = –2 x = –3 fi 126 = 18B fi B = 7 ... This gives: 3 / - 8 ... Substitution: x = 6 fi ... B = 36 ... fi 8 = 3B ... Equating ... A(x + 6)(x - 6) + Bx(x - 6) + Cx(x + 6) ... When x = 0 ... –36A ... 72B ... 72C ... So ... 12 ... the ... 3 / + ... (x + 6) ... ff Now put the denominator ... factorise the denominator ... Example 2 Solve ... Tip: You can ... You can use the factor ... as y = ... This gives ... x ... Find the range ... Q1 This is your sample ... so ... 20 ... So ... 12.049... = 76.4 N (3 s.f.) Chapter 16 Moments ... Iterative Methods Learning Objectives: ff Be able to use iteration formulas to find a solution of an equation to a given level of accuracy. 176 Chapter 7 Differentiation Example 2 Curve A has the equation x2 + 2xy – y2 = 10x + 4y – 21. Q6 Find f′(x) = e3x cot (4x). If y = uv dy dv du = u + v dx dx dx Where u and v are functions of x, i.e. u(x) and v(x). In other words, you have simultaneous equations, which you can solve to find m and s. Arithmetic sequence/series A sequence or series where successive terms have a common difference. According to the model, will he achieve this? Q13 If X ~ N(40, 25), find: a) P(X ≥ 45) b) P(X ≤ 30) Q2 If X ~ N(120, 40), find: a) P(X ≥ 28) b) P(X > 25) Q3 If X ~ N(120, 40), find: a) P(X > 107) b) P(X > 115) Q4 If X ~ N(17, 32), find: b) P(X < 12) c) P(X > 15) Q5 If X ~ N(50, 52), find: a) P(52 < X < 63) 272 Chapter 12 The Normal Distribution b) P(0.51 < X < 0.58) Q7 If X ~ N(260, 152), find: a) P(240 < X < 280) b) P(232 < X < 288) The standard normal distribution, Z ... Q6 If X ~ N(0.6, 0.04), find: a) P(0.45 ≤ X ≤ 0.55) b) P(0.51 < X < 0.58) ... cos (3x) ... Differentiating, we get dy du dy du = -4 Answers ... = -sin (3x) ... Q1 (i) = (2x) ... x2 + 1 ... = - 2 cos ... ... x ... e) x = 6 ... Q2 ... Q5 ... f) So ... = x = 3 x sec2 x = 15 sec2 2x ... ... Q3 First part: = 3 sec2 x ... dy du ... Q5 ... Putting it all together: f′(x) = 3(sec2 x) ... dy ... Q4 f′(x) = ... Q6 ... Q5 ... a) f(x) = 6 sin x ... a) Differentiate ... b) Find ... The curve ... ff ... = 0.983 ... fi ... Q7 ... b) ... ff ... = -22 ... q ... cos q ... 5 ... ... Q6 a) Find the differential equation ... describe the number of people ordering a vegetarian meal and give the values of any parameters. d) y = 2 cosec 2x is undefined where x = 0°, 90°, 180°, 270° and 360°. b) Let a = the time to beat. y = f(x) y ff Be able to calculate When you find yourself with an expression that you can't integrate, you can approximate the area under the curve using a series of trapeziums. Q3 Hint: Model the snowball as a sphere (Surface area A = 4pr2). Only these function to integrate. ff ... Q ... Sketching Iterations 4. Using column vectors ... to ... 2 ... ... 2 ... a = 2 ... 2 ... ... Q2 ... Q3 ... ... 4 3 ... ... Q6 ... 40p dt Chapter 7 Differentiation 165 Often you'll see wordier questions involving related rates of change, like the one in the example below. Numerical Methods Exercise 2.2 — Iterative methods ... So ... 2.3 ... ... Hint: ... Q4 Find a root to the equation x2 ln x = 5 to 5 s.f., using x0 = 2. If the middle 70% of the distribution lies between 61 and 75, find the value of s. 0.8 > 1 so log 0.8 > 0 and dividing through by log 0.8 changes the direction of the inequality because log 0.8 is negative. ff Resolving vertically: R = 11g + mg = (11 + m)g ff Resolving horizontally: F=N ff The ladder is in limiting equilibrium, so F = mR: F = mR fi N = 0.45R = 0.45g(11 + m) mg 11g N Taking moments about the base of the ladder: N sin 58° × 4 = (11g cos 58° × 2) + (mg cos 58° × 3) ... 1.8g(11 + m) sin 58° = 22g cos 58° + m(3 cos 58°) ... Q ... so the common denominator will just be another multiple of x. For events A and B: P(A Ç B) = P(A|B)P(B) or P(A Ç B) = P(B|A)P(A) ff Taking moments about A and B both happen, you multiply the Tip: The whole diagram... to find P(C|W)... ... P(A) ... = 0.1; ... In fact, since ... Q ... The probability that ... = 0.7 ... dy ... 2 ... 1 ... 5 ... ff ... be converges or diverges. x = 11.2, so z = 2 20 s n ff All that's left is to test for significance. ff Now put these values for t into the parametric equations to find the Cartesian coordinates of the turning points. ... 2 ... a, ... Hint: ... Q ... So ... ff ... ... 2 ... When t = 1 x = (1)2 – 1 = 0 y = (1)3 – 3(1) + 4 = 2 So there's a turning point at (0, 2). When t = –1 x = (–1)2 – 1 = 0 y = (–1)3 – 3(–1) + 4 = 6 So there's another turning point at (0, 6). Exercise 9.1 Q1 For each curve C, defined by the parametric equations given below, dy find the value of dx ... Sequences An ordered list of numbers (referred to as terms) that follow a set pattern. The council are planning to ... PROBLE M SOLVING 312 Chapter 14 Kinematics 2. ff Tip: As always, if you're writing a linear transformation of any of these functions, or the form (ax + b), then ... you ... rearrange ... ff The particles are moving, so friction is limiting: Tip: Find RA and RB by resolving perpendicular to the ... Q8 Hint: The function 1 f(x) = 2x - x is not defined for x = 0, so there's a discontinuity there... 2. Find the ... Q3 Hint: ... 0 mg mg cos q Example 2 R F 20° 0.1g A stone of mass 0.1 kg is placed on a smooth plane inclined at 20° to the horizontal. Chapter 13 Hypothesis Testing 2. You've seen how to differentiate products with the product rule. a = 2.36 ms–2 (3 s.f.) b) Resolving vertically: Fnet = ma R + 10 sin 45° – 3g = 0 R = 3g – 10 sin 45° = 22.328... p b) The graph of y = 2 sin b x + 6 l is the graph of y = sin x translated left by 6 , then a vertical stretch by a factor of 2. ff For a function with a point of inflection at x = 1: f (–2.67...) ... = ... + 9.8 fi t = 0.314... PROBLEM SOLVING 313 1 2 Q6 Show that the vectors a = 4i + 3j – 2k and b = i + j – 3k are ... Q5 b) ... r = 0.7 × 32.860... ff The values ... ff Work out the acceleration of P. Resolving vertically: R = 0.3g ... mR = 0.3 × 0.3g ... ff Using ... gives ... ff So ... a and c ... 2 ... ff So it ... Find the value ... = –2 – 3(2) + c fi c = 4 So the equation of the tangent to the curve ... y = 4 – 3x. Don't forget to change the Ghost Ç Girls entry to 6 – 2 = 4 and update the Girl, Ghost and overall totals before dividing. + p = 2.84 (2 d.p.) and –0.296... The ladder weighs 100 N, and makes an angle of 75° with the rough horizontal floor, where m = 0.1. Given that the ladder remains at rest, find the range of possible values of the magnitude of the force K. = 0.22 (to 2 d.p.) 340 Chapter 15 Dynamics F m 0.44 15° 20 N Example 4 R T a = 0.6 ms–2 F 20° reach 3g A mass of 3 kg is being pulled up a plane inclined at 20° to the horizontal by a rope parallel to the surface. 9 2 1 2x 1 = 5x2 - 3 - 27 + 81 - ... 1.61 ms–2 (3 s.f.) b) Use a constant acceleration equation. Magnitude is a scalar quantity that tells you a vector's magnitude... ff First consider only the velocity in the vertical... You're then told that the ... 1 ... Exercise 2.2 — Integration ... of ... x2 – 2x ... Q12 ... Q13 ... Exercise 2.2 — Integrating ex and 1/x Q1 a) ... 2 2 ... Q3 ... 1 2 ... Q2 ... ... cos ... ... Q2 ... 4 ... ... 2 ... 8x + 4 + ... 2 ... 1 3 ... Q3 ... ff So you're ... x3 ... gives y = 2 ... ... Q2 You've been given the derivative of the curve, so integrate it to get the equation of the curve. # cosec x cot x dx = -cosec x + C # sec x tan x dx = sec x + C # cosec2 x dx = -cot x + C 2 As always, you can integrate linear transformations of these functions (functions of the form cosec(ax + b)cot(ax + b), sec(ax + b)tan(ax + b) and cosec2(ax + b)). The particle's initial speed is 3 ms-1 at an angle of 70° below the horizontal. j h x y from p # a b f(x) dx ff Each part of the function will act over a different domain. The ... is continuous in this interval so x = 0.0160 is correct to 4 d.p. Q5 a) f(x) = ln(2x) + 0.43 = -0.159... Chapter 7 Differentiation 151 Exercise 4.1 Q10 Hint: Don't worry, you don't need to use the chain rule to integrate this one... Each part of the function ... Q3 ... ... Q ... ... 2 ... cos2q ... Exercise 8.3 Q1 Find the exact value of p where - 2 < q < 2 . 640x 3 20x 160x 2 1 = 1 + 3 + 9 + 27 + ... You've already seen how to integrate functions of the form xn in Year 1. b) Find the equations of the tangents at these points. The tension in the string is 41 N, the length of the rod is 5.5 m and the centre of mass of the rod is 1.7 m from A. r cm h cm Q10 Hint: A hemisphere is half a sphere. Then imagine a third axis sticking straight through the page at right angles to it — this is the z-axis. Q3 a) Express Q4 Given that f(x) = 3x + 5 and g(x) = x5 e(x-10) and x ≥ 3. ff Integrate ... P(Y ≤ 0.5) = ... 1 ... y = ... 2 ... 3 ... The crocodile ... modelled by a normal distribution, N(m, s2). R mg sin q q The force is ... where ... 10 ... Chapter 4 Sequences and Series 2. ff ... 2 ... dx ... 2 ... 0.7 ... Q ... 12 P ... P(E) = ... 100 = 39 = 13 60 60 20 P(M) = 100 = 19 ... P(A|E') = ... P(Y Ç Z) = P(Y) – P(Y Ç Z') = 0.72 – 0.2 = 0.52 ... P(Y|Z') = P(Y Ç Z') / P(Z') = 0.2 ÷ 0.39 = 0.513 (3 s.f.) Then there are lots of ways to write an expression for P(X Ç Y Ç Z) — e.g. P(X Ç Y) = P(X|Y)P(Y) = 0.75 × 0.72 = 0.54 ... P(X Ç Y Ç Z) ... 0.54 ... ff Resolving vertically, taking right as positive: Tip: Remember that the horizontal component of velocity stays constant throughout the motion — so you always equals ux. ff Resolving ... Q ... 2 ... Q3 ... b) Show that the area is equal to ... The probability that a randomly selected battery lasts more than 160 hours, but less than 9 hours ... ff ... a = 6.25 ... Example ... Q4 ... Q3 ... So ... Exercise ... Q ... ff ... dy ... Starting with x0 = 1, (-1) 2 - 5 (-1) = –1.857142... The next step is to write it as partial fractions. Find: a) the magnitude of the tension in the wire. ff Resolving vertically: T + R = 30 + 40 R = 70 – T = 70 – 36 = 34 N 354 Chapter 16 Moments The point of tilting If a rod is 'about to tilt' about a particular point of support, then any normal reaction forces acting at any other supports along the rod will be zero. So if x2 is odd, then x must also be odd. Tip: For example, a force which acts horizontally has zero vertical component, and vice versa. 1 Q6 a) 4 ... 1 ... x = ... 4 ... 5 ... 3 ... a = ... 3 4 ... 1 ... 0.76923... With wordy problems, don't forget to check that the units should be and include them. Example 3 4N 10° 20° The diagram shows a particle P, which is at rest on a smooth horizontal plane. ff Differentiate the substitution and rearrange. Tip: You need to pick the right substitution before you can integrate. Using partial fractions ... Q13 ... Q12 ... ff Draw a diagram. ff Draw a diagram of the curve and the line. c) Let b = the number of days taken by Bella. 4 - 9 ... Q14 Find ... to ... ... Chapter 2 ... 4 ... at the graph of y = f(x). y=3 3 y d dy 2 ... Q14 ... Differentiating: ... b) ... 18x2 – 2y + 2x ... dy dx ... 27x2 – 3y = 0 fi 3y = 27x2 fi y = 9x2 ... dy dx ... fi 27x2 = 3y ... Substitute y = 9x2 back into the original equation: 1 22 3 2 4 3 3 3 (9x) = (9x – 6x) ... 9x2 ... 18x 27x ... 0 fi 3 ... ff So the stationary points ... with ... 2 ... the root ... 0 ... 2 ... 2.4723 ... 2.4715 ... 2.4725 ... f(2.4725) ... 2.4715 ... f(2.4715) ... 2.4715 ... 2.4725 ... There's a sign change, so the root ... Questions and examples that involve problem solving are marked with stamps. Composite Functions 4. ff So rewrite the integration: # sin 2 x dx = # 12 ^1 - cos 2xh dx = 12 # ^1 - cos 2xh dx = 12 a x - 1 sin 2x k + C = 1 x - 1 sin 2x + C 4 2 2 2 ff Rearranging the cos double angle formula: cos2 x º 2 cos2 x - 1 ... Tip: For a reminder of integrating sin x and cos x (and linear transformations of them) see pages 186-187. Second order derivative The result of differentiating a function twice. ff f(2.2) is positive, so the root is between 2 and 2.2. Now try again with x = 2.1, as it's halfway between the two and you'll see that it's closer to. It's worth making sure you're familiar with trig identities (see Chapter 3) so you can spot which ones to use and when to use them. Integration Using Partial Fractions ff By rewriting algebraic expressions as partial fractions, you can turn difficult-looking integrations into ones you're more familiar with. 2m Find the values of MA and MB. g. 3(x2 + 4) 3 (x) + 4x2 ... of the moments ... Chapter 8 Integration 2 Q2 ... ... Q3 Example ... ff Be able to ... The ... 0 Q4 ... The dimensions ... Chapter 16 Moments ... ... 5 ... So ... Q3 ... 2 ... 1 ... 2 ... Q4 ... 3 ... x 3 ... 3 ... x - 1 ... 10 ... -3 ... 1.91293... Q4 A uniform ladder of mass 10 kg and length 6 m rests with one end on a rough horizontal ground and the other end against a smooth, vertical wall. ff Be able to write the modulus of a number or function. Add it to the stone's initial position vector to find the stone's position vector: 0 76.01... So don't rush in. State the conclusion of the hypothesis test. Q11 A curve is defined by e2y + ex – e4 = 2xy + 1. Example 1 Find # 2x ex dx. Equating the coefficients of x3 gives 1 = A. Q2 f(0.9) = sin (1.8) – 0.9 = 0.0738... 1 (sec x – 2) 2 cot2 x + 3 cosec x ... = 0 ... 1 ... 3 ... 2 ... 2 ... So ... x = 63.4° ... It has a ... 2 ... the ... on 5 ... Assume the statement is not true, i.e. ... that ... x ... cos ... 3 ... The ... 2 ... the ... Exercise ... to ... 2 ... The ... normal ... probability of getting less than 300 heads d) the probability of getting exactly 499, 500 or 501 heads e) the probability of getting between 250 and 750 heads (inclusive) f) the probability of getting at least 100 heads but less than 900 heads MODELL ING Normal approximation to a binomial distribution Certain binomial distributions can be approximated by a normal distribution. = 0.0139 = 0.00139) = 27 122 signatures So no, the target will not be achieved. Try solving this on your own with the substitution u = ... Q8 ... Q2 ... ff ... Q3 ... The PMCC is always between +1 and –1. N Resolving horizontally taking right as positive: (i) ... 5 ... = 16.055... d) t = 92 fi y = 30 000 – 20 000e18.4 fi y = 30 000 – 20 000 × ... ff ... = 1.939 ... Tip: The plank is on the point of tilting about B, so the reaction at A, RA, is zero. b) Find the value of the tension in the string ... Q1 a) ... at the origin ... 2 ... a ... 2 ... x ... Q ... 0 ... 2 ... a ... ... 4 ... Tip: ... Q5 ... Tip: This is a ... If ... is the ... = 11.7 cm (3 s.f.) Q5 ff First find the value of t ... y ... 720° ... Q9 Water is dripping from a hole in the base of a cylinder of radius r cm, where the water height is h cm, at a rate of ... 0.3 cm3 s–1. Q5 f(1.6) = ... 1.6 ... ... 1.24... It'll help a lot if you draw a Venn diagram showing what you know. 4-1 3p 7p fi tan ... = ... 4 4 Q10 ff Use the iteration formula ... Q10 ... 81 314 – 300 = 1.556 (3 d.p.) ... 2 sin (q + 45°) = 3 cos q 1 1 sin q + cos q = 3 cos q 2 sin q ... cos q 2 ... tan q ... q = 63.43° (2 d.p.) 63.43° + 180° = 243.43° (2 d.p.) The thing that connects the two components ... This will be the same no matter what you choose ... So the ... Q ... sec2 ... ff ... 81 You're interested in the right-hand tail of the distribution, so use P(Z > z) = 0.01. ff Resolving perpendicular to the plane ... So ... 2 ... Suppose that there is an integer ... that is a multiple of 3 but x is not. ... 4 ... 2 ... Q ... P(Z ... 0 ... Exercise 9.1 ... with coordinates 2 2 (0, 0), (0, 1), (a - 3 , 1.13 k and a - 3 , - 0.13 k. 1 translate by e (i.e. 1 up). ff The direction that the rod turns in (i.e. clockwise or anticlockwise) is called the sense of rotation. Start with the product rule. Tip: Take a look back at p.154 for more about this. Find the tension in the string and the magnitude of the force exerted on the rod at A. You assume that there is one, then you can simply add it to get a bigger number of the same type, which contradicts your initial assumption. dt Use the figure to show that ... integrate functions of the form sec2(ax + b). ff Resolving horizontally ... Q3 ... cos 4x ... 2 ... sin 2x ... Example 2 Find # cos 4x – 2 sin 2x + sec2 ... 2x dx ... ff If you ... 1 ... Tip: If the magnitude of the normal reaction at C is 49 N, find a) the magnitude of the tension ... b) the mass of the rod. Q9 Find ... Q3 Show that ... 2 ... Q1 a) P(Z > 1.8) = ... Q2 ... Q5 ... Find the value of ... dx ... 0.02 ... = 0.004 ... P(Z > 2.1) ... 0.0179 ... 0.9426 Chapter 12 The Normal Distribution 1 ... ff Draw another sketch showing ... 0.22 ... 2 ... dt ... the ... Tip: ... 1 4x = 1 + a 2 k(4x) + ... 1#2 1 3 ... #1 = 1 ... 2 ... If x = 3k + 1, then x2 = (3k + 1)2 = 9k2 + 6k + 1 = 3(3k2 + 2k) + 1 So x2 is not a multiple of 3. ff The projectile is moving only under the influence of gravity (i.e. there'll be no horizontal forces acting on it). Example 1 Find ... Example 1 ... x ... ff Below show how to use the

table of critical values to test for significance. 0 1 –3 y P(3, 7) y = f(x – 1) + 3 –1 Q(0, 0) 2 3 x –3 Answers 381 b) 4 y Q(–0.5, 3) –1 –3 c) 2 x y = –f(2x) P(1, –4) y Q(–3, 3) 4 P(0, 4) y = |f(x + 2)| –1 x 2 –3 7. Then the rate of change of V with respect to t dV dV dVin dV is = in – out . cos2 x + sin2 x º 1 cos3 x = cos2 x cos x = (1 – sin2 x) cos x = cos x – cos x sin2 x = cos x – cos x (sin x)2 ff Now write out the integral: # cos Tip: See p.197 for more on how to do this type of integration. 3 3 x = 2 fi y = (2 × 2 – 3)7 = 0 3 So the coordinates of the point of inflection are b 2 , 0 l. ff Treat the two particles as one combined particle with mass 6 kg + 8 kg = 14 kg, and resolve perpendicular to the plane ( ): Fnet = ma R – 14g cos 20° = 0 R = 14g cos 20° N ff The particles are moving, so the total frictional force acting on the two particles is: F = mR = 0.6 × 14g cos 20° = 8.4g cos 20° Tip: The acceleration of A will be the same as the acceleration of the two particles combined. Q6 If y = arctan x, show that PROBLE M SOLVING dy 1 = . 6 2 You could also have used the 'equating coefficients' method. a) Find the standard deviation of the heights of the 17-year-old boys in this population. Resolving north: u = 3, v = v, a = 0.5, t = 6 v = u + at v = 3 + (0.5 × 6) = 6 So it is moving due north at t = 6 s. One lunch time, 20 people order a meal. Q5 a) 4 sin x + cos x º R sin (x + a) º R sin x cos a + R cos x sin a fi R cos a = 4, R sin a = 1 1 tan a = 4 fi a = 0.245 (3 s.f.) R = 4 2 + 1 2 = 17 fi 4 sin x + cos x º 17 sin (x + 0.245) b) Maximum value of 4 sin x + cos x = 17, so the 4 greatest value of (4 sin x + cos x)4 = ^ 17 h = 289. So 2x3 + 5x2 – 8x – 17 º (2x2 + 9x + 10)(x – 2) + 3. d Using the chain rule, (3x + 4)5 = 5(3x + 4)4 × 3 = 15(3x + 4)4. Write down the values of a, b and n and then substitute them into the formula. Tip: You will often see asymptotes on a graph when a mapping is undefined at a certain value. a) Calculate the x-coordinates of the points on the curve where y = 1 and hence find the equations of the normals at these points. There's no real need to use the y = mx + c formula here as we already know the place it crosses the y-axis is (0, 1) (it's given in the question), but it's good to be safe. The gradient at x = 1 is 3(3 ln 3) = 9 ln 3. Q3 a) Taking moments about A: (1.5g × 0.6) + (0.5g × 1.2) = T sin 20° × 2.4 fi T = 14.7 ÷ 0.820... Find the magnitude of force F and the normal reaction, R, of the plane on the stone. x a) Show that the positive root, a, lies in the interval 7 < x < 8. Find the angle between the two flight paths, in degrees, to 1 d.p. MODELL ff a and b form two sides of the triangle AOB. ff For example, for an object on an inclined plane, you might want to resolve the forces in directions parallel and perpendicular to the plane. Chapter 7 Differentiation 161 7. Example 3 Z ~ N(0, 1). Tip: As x gets further away from the mean, f(x) gets closer to 0 but never reaches it (the x-axis is an asymptote). Trust us to give you the best extras! It's easy to differentiate CGP from other Maths books — we're the best by miles Prolonged use may result in genius ISBN 978 1 78294 720 2 P G C 9 781782 947202 MAT61 £27.99 (Retail Price) www.cgpbooks.co.uk ☻ A-Level Year 2 Mathematics — The Complete Course for AQA Year 2 of A-Level Maths will really test your limits... It's important that you get things in the right order, so concentrate on what's going where: d u - dv v u dy (2x + 1) (cos x) - (sin x) (2) = d x 2 dx = dx v (2x + 1) 2 Tip: This is quite similar to the product rule from the last section — the first stage is to identify u and v and differentiate them separately. The mean of X (m) will correspond to the mean of the values. 1 du dv = 4x3, = x dx dx dv du : 4 1 D f'(x) = u +v = x × x + [ln x × 4x3] dx dx = x3 + 4x3 ln x = x3(1 + 4 ln x) 2 du dv d) u = 4x, v = ln x2 = 2ln x fi = 4, = x dx dx dv du : f'(x) = u +v = 4x × 2x D + [ln x2 × 4] = 8 + 4 ln x2 dx dx du dv e) u = 2x3, v = cos x fi = 6x2, = –sin x dx dx dv du f'(x) = u +v = (–2x3 × sin x) + (cos x × 6x2) dx dx = 2x2 (3 cos x – x sin x) du dv f) u = x2, v = cos (2x) fi = 2x, = –2sin (2x) dx dx dv du f'(x) = u +v dx dx = [x2 × –2 sin (2x)] + [cos (2x) × 2x] = 2x cos (2x) – 2x2 sin (2x) = 2x(cos (2x) – x sin (2x)) du dv Q4 a) u = (x + 1)2, v = x2 – 1 fi = 2(x + 1), = 2x dx dx dy dv du = u +v dx dx dx 2 = [(x + 1) × 2x] + [(x2 – 1) × 2(x + 1)] = 2x(x + 1)2 + 2(x2 – 1)(x + 1) = 2x3 + 4x2 + 2x + 2x3 + 2x2 – 2x – 2 = 4x3 + 6x2 – 2 du dv b) u = (x + 1)3, v = x – 1 fi = 3(x + 1)2, =1 dx dx dy dv du = u +v dx dx = [(x + 1)3 × 1] + [(x – 1) × 3(x + 1)2] = (x + 1)3 + 3(x – 1)(x + 1)2 = (x3 + 3x2 + 3x + 1) + (3x3 + 6x2 + 3x – 3x2 – 6x – 3) = 4x3 + 6x2 – 2 Use the binomial formula to expand (x + 1)3 — you'll get the coefficients 1, 3, 3, 1. f(1.3) = 1.33 + ln1.3 – 2 = 0.459... Chapter 8 Integration 211 Exercise 8.2 Q1 Find the exact values of the following using the given substitutions: #2 (3x – 2)4 dx, u = 3x – 2 a) 1 3 b) # 2x(x + 3)4 dx, 1 u = x + 3 -2 p 6 # 8sin3 x cos x dx, u = sin x d) # x x + 1 dx, u = c) 3 0 0 x+1 Q2 Use an appropriate substitution to find the exact value of each of the following: # x(3x – 4)3 dx c) # x(x2 – 3)4 dx b) # a) 5 10 2 1 2 2 x dx x -1 1 between x = 1 and x = 4, 3- x using a suitable substitution. The rod is supported by a light strut at an angle of 70° to the wall, as shown. g 2x2 + 3x - 1 x - 5 2x 3 - 7x 2 - 16x + 11 – (2x3 – 10x2) 3x2 – 16x – (3x2 – 15x) –x + 11 – (–x + 5) 6 ff After subtracting, this term (6) has a degree that's less than the degree of the divisor, (x – 5), so it can't be divided. b) State the largest possible domain that would make f a function. Chapter 13 Hypothesis Testing 297 Chapter 14 Kinematics 1. This data does not provide evidence at the 1% level to support the claim that the machine is dispensing a mean volume different from 30 ml. The Binomial Expansion Exercise 1.1 — Expansions where n is a positive integer 3^3 - 1h 2 3^3 - 1h^3 - 2h 3 1#2 x + 1#2#3 x = 1 + 3x + 3x2 + x3 7^7 - 1h 7^7 - 1h^7 - 2h 3 (1 + x)7 = 1 + 7x + 1 # 2 x2 + 1 # 2 # 3 x + ... The probability that MODELLING he walks is 0.4. The probability that he catches the bus is 0.54. MODELL ING Show that counters placed at coordinates (1, 0, 3), (3, 1, 2) and (7, 3, 0) lie on a straight line. There is evidence at the 5% level of significance to reject H0 and to suggest that the mean time has decreased. Sector A section of a circle formed by two radii and part of the circumference. Mappings can be drawn as mapping diagrams, like the one shown here for 'multiply by 5 and add 1' acting on the domain {–1, 0, 1, 2}: Domain Use the notation {1, 2, ...} for the domain and range if they are a discrete list of values. Q4 a = 300 000, d = –30 000 nth term = a + (n – 1)d = 300 000 – 30 000(n – 1) = 330 000 – 30 000n 330 000 – 30 000n < 50 000 n > 280 000 ÷ 30 000 = 9.33... B Absolute value Another name for the modulus. In each case, find the magnitude of the missing tension, T, and the mass of the particle, m. The chain between the truck and the car is modelled as a light, inextensible string passing over a fixed, smooth pulley. ff You need to find two different integrals to work out the area of B: y 2 B1 1 0 B2 x 4 2 5 4 3 2 p 5p ff The area B is the area under the red curve between 4 and 4 (B1) p 5p minus the area under the blue curve between 4 and 4 (B2). dy y= # dx = # 10e -5x - 1 dx = 10 # e -5x - 1 dx dx = 10 a 1 e -5x - 1 + c k = - 2e -5x - 1 + C -5 To find C, use the fact that the curve goes through the origin (0, 0). C is just a constant. ff So the root is –2.27902 to 5 d.p. Tip: Remember that differentiating an equation gives you the gradient which you can use to find the tangent at a given point. N Resolving parallel to the plane (): D + F – 5g sin 35° = 0 D = 28.105... Given that the coefficient of friction between the box and the plane is 0.7, find the magnitude of the normal reaction force from the plane and the magnitude of Q. i.e. Tip: The formula for weight (W = mg) is derived from Newton's second law, where g is acceleration due to gravity. Example 2 a) Find dy x if y = cot 2 . f(3.5765) = e3.5765 – (10 × 3.5765) = –0.016... f(1.7) = (3 × 1.7) – 1.74 + 3 = –0.25... Q3 A particle of mass 5 kg is acted on by two forces, (6i – 10j) N and (4i + 5j) N. At time t seconds, the particle's position relative to a fixed origin O is given by r = [(sin t)i + (cos t)j] m. So the surface area of the cube is: A = 6x2 fi dA = 12x dx A decreases at a constant rate of 0.027 m2 s–1. a (1 - r n) Using: Sn = , where a = 12, r = 1.03, n = 20, (1 - r) 20 12 (1 - 1.03) S20 = = 322.44... Differentiating them is simpler than you might expect — but you'll have to combine this with all the other differentiation you've done in this chapter so far. = g(x)f'(x) + f(x)g'(x) Example 1 Differentiate x3 tan x with respect to x. ff It's symmetrical about the mean — so f(x) values the same distance above and below the mean are equally likely. MODELL Q1 Hint: It might help to draw a suitable diagram, even if the question doesn't ask for it. Integrate ACCELERATION (a) ff To differentiate or integrate vectors written in i and j notation, just differentiate or integrate each component separately: Differentiating Vectors If r = xi + yj is the position vector of a body, then: the body's velocity, v = dx dr = i + dy j dt dt dt and the body's acceleration, a = dv d 2 r = 2 dt dt d y d2x i+ 2j dt dt 2 2 = Tip: The shorthand for dr is ro (the single dot dt means differentiate r once with respect to time). G dy dx The rate of change of the radius, = –2. range of f(x) for values of x ≥ 0. Then, P(T > b) = 0.063. Find the magnitude and direction of the resultant force on each particle. so x = 2.3935 (5 s.f.) Q5 Differentiating the function gives 6x2 – 30x. 3 So, the speed of projection of A is 8.66 ms–1 (3 s.f.), and the speed of projection of B is 17.3 ms–1 (3 s.f.) Exercise 1.3 — i and j vectors Q1 a) u = (12i + 16j), v = v, a = –9.8j, t = 2 v = u + at v = (12i + 16j) – 19.6j v = (12i – 3.6j) ms–1 b) When the particle reaches its maximum height, the vertical component of its velocity will be zero, and the horizontal component of its velocity will be the same as when it was projected (there is no horizontal acceleration), so its velocity will be 12i ms–1. and solve differential equations that model real-life situations and interpret the results in context. Doing calculations based on probabilities is all very well and good, but in real-life situations, the hardest part is often finding the probabilities in the first place. Find the equation of the normal to the curve at t = p. Your calculator still works for negative z-values. This is written |8| = |–8| = 8. So f –1(x) has domain {x : x ≥ 3} and range {f –1(x) : f –1(x) ≥ 2}. Q3 A curve C is defined by the parametric equations x = sin 2t, y = t cos t + 2 sin t. 16 x - 4 7 7 11 - 16 x 7 7 Q3 a) Putting y = 1 into the equation gives: x2 + 1 = x + 7 fi x2 – x – 6 = 0 fi (x – 3)(x + 2) = 0 fi x = 3 and x = –2. 376 Answers 3 (0) + 4 =4 1-0 2 3b- 5 l + 4 - + 2 = 6 20 = 14 = 2 c) f –1 a - 5 k = 7 2 5+2 1+ 5 Q6 a) f(x) has domain x > 3, and range f(x) d R . When x = p: 1 cos y cos p + cos y sin p = 2 , 2p 1 –cos y + 0 = 2 fi y = 3 You can use the CAST diagram or the graph of cos x to find this solution. b) the magnitude of the reaction force, R, acting on the rod at A. 2 218 Chapter 8 Integration # x2ex dx a) # x2 cos x dx b) # 4x2 sin 2x dx c) d) # 40x2(2x – 1)4 dx Q2 Find # 0 -1 x2(x + 1)4 dx using integration by parts. # 2 sec x dx = b) c) cot x 5 dx Use the result for cot x above. Differentiation with Parametric Equations Exercise 9.1 — Differentiating parametric equations dy dx = 2t, = 3t2 – 1. = 38.6 ms–1 (3 s.f.) b) Resolving vertically, taking up as positive: s = s, u = (38.604...) sin 40° = 24.814..., a = –9.8, t = 2 1 s = ut + 2 at2 1 s = (24.814... dx x ff This is a function (cot) of a function (2), so you need the chain rule. Tip: Don't worry if you draw the components acting in the wrong direction — the numbers in your calculations will just come out negative. dt ff The rate of change is proportional to the difference between L and R, dL so ? If P(X < 9) = 0.5596 and P(X > 14) = 0.0322, find m and s. d) Using the new iterative formula with x0 = 3 x1 = 2.00855..., x2 = 0.74525.., x3 = 0.21069..., x4 = 0.12345..., x5 = 0.11313..., x6 = 0.11197..., x7 = 0.11184..., x8 = 0.11183... The formula appears to converge to another root at x = 0.112 (3 d.p.) (-1) 2 - (3 × (-1)) x 2 - 3x Q7 a) x1 = 0 2 0 – 5 = – 5 = –3 2 x2 = 4, x3 = –3, x4 = 4 The sequence is alternating between –3 and 4. To do this, choose the inverse normal function on your calculator. a) f(x) b) –2 4 2 x Q1 Hint: Work out which type of function is shown in each of the diagrams. = 10.1 m (3 s.f.) above the ground. When x = 1, y = 0 so the stationary point is at (1, 0). N ff Resolving vertically: RV + 0.1g + 2 = 16 cos 70° fi RV = 2.492... Chapter 15 Dynamics 341 Q3 A box of mass 7 kg is accelerated across a rough horizontal plane by a horizontal force, P. c) Find P(M'|N'). Chapter 12 The Normal Distribution 273 Tip: You could split this up like on p.272: F(0.82) – F(0.12) = 0.79389... Example 2 Differentiate the following with respect to x: a) y = cos (2x) ff Rewrite the function in 'chain rule notation': y = cos (2x), so let y = cos u where u = 2x ff So dy = –sin u = –sin (2x) and du dy dy du = × = –2 sin (2x) d x du dx du =2 dx b) y = 4 sin (x2 + 1) ff As before, work out which part needs to be u for the chain rule: y = 4 sin (x2 + 1), so let y = 4 sin u where u = x2 + 1 dy du = 4 cos u = 4 cos (x2 + 1) and = 2x du dx dy dy du = × = 4 cos (x2 + 1) × 2x dx du dx = 8x cos (x2 + 1) ff So c) Find dy when x = tan (3y). So the hypotheses are: H0: r = 0 and H1: r < 0 Chapter 13 Hypothesis Testing 293 ff Find the critical value for a sample size of n = 20 and a one-tailed significance level of 5%. Coefficient of friction A number greater than or equal to zero which measures the effect of friction between an object and a surface. Then input the required x-value(s), mean and standard deviation. • 3D vectors could be used to describe the variation in the toy's depth underwater. ff Resolving in the i-direction (Æ): –4.8 cos 28° = –4.24 N (to 3 s.f.) ff Resolving in the j-direction (≠): 4.8 sin 28° = 2.25 N (to 3 s.f.) ff You can resolve forces in any two perpendicular directions, not just horizontally and vertically. ff You then put the new value into the iteration formula, and keep going until your answers are the same when rounded to the level of accuracy needed. Here's the proof of this formula. Tip: An exam question might ask you to draw the trapeziums on a given graph. This is where the line changes from concave to convex — i.e. at x = 65. If X represents the number of survivors, then X ~ B(200, 0.23). The sketch you made will also show roughly where the roots are (it's the same x-value for both), so locating them is a bit easier. ff As before, start with the differentiation from first principles. e) What is a likely limitation of this model and how could it be addressed? 164 Chapter 7 Differentiation 8. 4-0 = 1, 4 Chapter 9 Numerical Methods 247 Tip: Increasing the number of strips increases the accuracy because the gaps between curve and line are smaller: ff Calculate the corresponding y-values: 6x2 y =— 3 x +2 y 0 1 2 3 0 1 2 3 6x 2 x3 + 2 x0 = 0 y0 = 0 x1 = 1 y1 = 2 x2 = 2 y2 = 2.4 x3 = 3 y3 = 1.8621 (4 d.p.) x4 = 4 y4 = 1.4545 (4 d.p.) ff Putting these values into the formula gives: 4 x 6x2 y =— 3 x +2 y y= x # 0 4 h 6x 2 dx ≈ 2 [y0 + 2(y1 + y2 + y3) + y4] x3 + 2 1 = 2 [0 + 2(2 + 2.4 + 1.8621) + 1.4545] 1 = 2 [13.9787] = 6.989 (3 d.p.) 4 x Upper and lower bounds You can calculate an upper and lower bound for the area under a curve using a simplified version of the trapezium rule that uses rectangles: Tip: For an increasing function like the one shown here, calculating the bound using the 'left hand corner' will give the lower bound, and using the 'right hand corner' will give the upper bound. ff This gives a domain of x ≥ 4 — in set notation, this is {x : x ≥ 4} ff Again, this can be demonstrated by sketching a graph of y = f(x): y x=4 Tip: Remember, the : means "such that", so this is the set of x, such that x is greater than or equal to 4. a - 12 k # a - 3 -1 2 2k 6 2 1 6 6 a 5 xk Q4 a) a1 + 5 x k = 1 + a 2 ka 5 x k + 1#2 1 3 5 a- 2 k # a- 2 k # a- 2 k 3 a6 + 5 x k + ... Find the value of a. Chapter 2 Algebra and Functions 27 Example Draw the graphs of y = |f(x)| and y = f(|x|) for f(x) = 5x – 5. Partial fractions A way of writing an algebraic fraction with several linear factors in its denominator as a sum of fractions with linear denominators. Model A mathematical approximation of a real-life situation, in which certain assumptions are made about the situation. ff You're usually told what significance level to use, but the most common values are a = 0.1 (or 10%), a = 0.05 (or 5%) and a = 0.01 (or 1%). Tip: Speed, distance and time must always be positive. ff Taking moments about A: (mg cos 20° × 3) = (mg sin 20° × 1) + (T × 6) fi 6T = mg(3 cos 20° – sin 20°) fi T = 19.6(2.4770...) = 48.6 N (3 s.f.) T mg 1m A 20° 20° sin cos g m 20° mg 3m 20° Exercise 1.2 Q1 For each of the following diagrams, find the values of x and y, given that the light rod AB is in equilibrium: a) xN 1.5 m A 16 N 2.5 m 4m yN Q2 Q2 Hint: The rod is uniform, so its weight acts halfway along its length. You can see the range of f(x) more easily from the graph of y = f(x) — there is a horizontal asymptote at y = 0. Using Pythagoras' theorem, the hypotenuse will be 5: 5 q 3 3 4 So sin q = 5 and cos q = 5 . You should remember from Year 1 that when particles are connected by a taut, inextensible string or a rigid rod, it will exert an equal tension force or thrust force respectively on both particles. 3 3 5. 3 3 p b) If x = arcsin 2 then 2 = sin x so x = 3 . It is caught when it is on its descent, so it is in the air for 1.558... Chapter 7 Differentiation 159 Example 3 Determine the nature of the stationary point of the curve y = Tip: To determine the nature of the stationary point you're going to have to differentiate twice. 3x 3x 2 - 9x b) 5 ÷ 20 3x 20 4 = 5 × 3x ^ x - 3 h = 4 x-3 Turn the second fraction upside down, and then cancel. c) n is large (100) but p = 0.85 is not close to 0.5. It might still be OK if np and n(1 – p) are > 5. = 15.0 (to 3 s.f.) b) magnitude = (15.011...) 2 + 7 2 = 16.6 N (to 3 s.f.) 10 N Q4 x 7.1 N 4° a) 10 cos x = 7.1 7.1 x = cos–1 a 10 k = 44.8° (to 3 s.f.) b) 10 cos (x + 4) = 6.59 N (to 3 s.f.) R 8.2 N 39° 8.2 cos 39° 8.2 sin 39° –1 q = tan a + k 7 8.2 cos 39° –1 = tan (0.3858...) = 21.1° (3 s.f.) 7N 60° R = (7 + 8.2 cos 39°) + ^8.2 sin 39°h = 14.3 N (3 s.f.) 2 2.5 + 4 sin 45° b) 2 R 5 cos 30° – 4 cos 45° Resolving horizontally: R(Æ) = 5 cos 30° – 4 cos 45° Resolving vertically: R(≠) = 5 sin 30° + 4 sin 45° = 2.5 + 4 sin 45° 2.5 + 4 sin 45° k q = tan–1 a 5 cos 30° - 4 cos 45° = tan–1(3.548...) = 74.3° (3 s.f.) R = (2.5 + 4 sin 45°) 2 + ^5 cos 30° - 4 cos 45°h2 = 5.54 N (3 s.f.) c) Resolving horizontally: R(Æ) = 10 – 11 cos 75° – 12 cos 80° = 5.069... cot x 1 1 5 = 5 cot x so there is a constant of 5 — put that at the front. The maximum of sin x is 1, so the maximum value of f(x) is f(x) = 29. 1 So overall, the expansion of (1 + 4x)4^1 - 5xh6 1 is valid for the narrower of these ranges, i.e. |x| < 5 . dt dt dy dx dy 3t 2 - 6 Using the chain rule, = ÷ , so = 2t . Q5 r = Exercise 2.1 — Using vectors (ii) When t = 3, v = (3(3)2 – 3)i + 2(3)j = (24i + 6j) ms–1 Using Pythagoras' theorem: speed = 24 2 + 6 2 = 24.7 ms–1 (3 s.f.) Using trigonometry: 6 q = tan–1a 24 k = 14.0° (3 s.f.) from the positive i-direction. Q7 a) y = ex + 1 fi ln y = x + 1 fi (ln y) – 1 = x So f–1(x) = (ln x) – 1 f(x) has domain x d R and range f(x) > 0, so f–1(x) has domain x > 0 and range f–1(x) d R . d) Use the iteration formula xn+1 = to find a to 3 s.f. e) Use the Newton-Raphson method with x0 = –2 to find another root of g(x), b, to 4 s.f. f) Explain why the Newton-Raphson method for g(x) 2 3 fails when xn = 3 . N Using Pythagoras' theorem: R = (16.828...) 2 + 13.475 2 = 21.558... Speed s < 35 35 ≤ s s, mph < 40 Freq. f) r = 5 = 1, r = 1 = 1 (and 1 is not less than 1), 5 so the sequence does not converge. Mappings and Functions Learning Objectives: ff Understand the definitions of mappings and functions and be able to deduce whether a given mapping is a function. So the identity looks like this: x4 – 3x3 – 3x2 + 10x + 5 º (x3 – x2 – 5x)(x – 2) + 5 Tip: After you've done a few of these you'll get used to spotting what the coefficients are going to be in terms of A, B etc. Integration by parts A method for integrating a product of two functions. x dx 2 2 d y d2 y So at x = 1, = 0 and the sign of changes, dx 2 dx 2 so this is a point of inflection. ff The area under the curve is 1, so subtract 0.15 from 1 to get the area to the left of z: 1 – 0.15 = 0.85 ff Use a calculator to find the value of z. ff On the sketch, it looks like the root is much closer to 2 than 3, so try again using x = 2 and x = 2.2 232 ln 2 – (2 – 3)2 = –0.306... Here, S depends on L, so L depends on S. Differential Equations 221 Chapter 9 — Numerical Methods 1. Q6 A cylindrical polishing block of radius r cm and length H cm is worn down at one circular end at a rate of 0.5 mm h–1. The 8th term is: 3 × 1.257 = 14.3 cm (to 1 d.p.) Q3 a = 40, d = 5 nth term = a + (n – 1)d = 40 + (n – 1)5 = 5n + 35 80 = 5n + 35 fi n = (80 – 35) ÷ 5 = 9 So he'll sell 80 sandwiches on the 9th day. ff P + 1 is a prime number that is larger than pn, which contradicts the assumption that pn was the largest, so there must be infinitely many prime numbers. You can see this clearly on a sketch of the function: f(x) f(x) = 2x 6 x 3 Only 3 in the domain is mapped to 6 in the range. N Resolving vertically: F + T sin 20° = 12g fi F = 117.6 – (171.919...) sin 20° = 58.800... Vectors in Three Dimensions In Year 1 you learnt how to represent, calculate with, and model situations using vectors in two dimensions. 3 So - s = –0.84162... Here is an example with two curves: Example 2 The diagram below shows the curves y = sin x + 1 and y = cos x + 1. ff Draw a diagram to show the particle's motion. 386 Answers 180° x d) The range is y ≥ 5 or y ≤ –1. = 46.581... 326 108 y = - 5 x + 3 fi 15y = –324x + 1630 fi 324x + 15y – 1630 = 0 Answers 417 Q8 First use the chain rule: dy = f'(x)ef(x) where f(x) = x2 + 3 . fi 9 – m = 0.14995...s 1st equation in m and s ff Now do the same thing for the second probability for X. Example 1 Tip: The 8g force acting downwards is the body's weight. N So F = 114.521... 1 q c m sin a 2 k 26 q 1 b) tan a 2 k = = = 5 5 q cos a 2 k c m 26 8. d2 y When x = –4, 2 = 4(–4 + 4)(5(–4)2 + 4(–4) – 10) dx = 4(0)(80 – 16 – 10) = 0 2 d y Check either side of x = –4: dx 2 When x > –4, (x + 4) > 0 and (5x2 + 4x – 10) > 0 d2 y fi >0 dx 2 When x < –4, (x + 4) < 0 and (5x2 + 4x – 10) > 0 d2 y fi 0 fi >0 dx 2 2 d y When x < 2, (x – 2) < 0 fi 0. # x2 sin x dx = –x2cos x + # 2x cos x dx = –x2cos x + 2x sin x + 2cos x + C Chapter 8 Integration 217 Example 2 Use integration by parts to find ff Let u = x2 and let Tip: The formula for integrating (ax + b)n is used a few times in this example — go back to p.180 if you've forgotten how it works. 6 -3 3 x x dx = ln|x | + C. b) cos q = = - 61 fi sec q = 11 hyp a - 11 k 61 1 1 61 = c) cosec q = sin q = 60 . f(2.75) = 2.753 – (7 × 2.75) – 2 = –0.45... a c bc So xy = b y = d fi y = ad Since bc and ad are both integers, this means that y is a rational number, which contradicts the assumption that y is irrational. Q3 a) The range shown is {f(x) : 3 ≤ f(x) ≤ 11}, so the 3-1 11 - 1 domain is: 2 ≤ x ≤ 2 , which is 1 ≤ x ≤ 5. ff x0 = 2, so x1 = 3 x0 + 4 = 3 2 + 4 = 1.8171... So the curve passes through (1, 3) and (1, –2). ff Differential equations tend to involve a rate of change (giving a derivative term) and a proportion relation, where the rate of change will be directly or inversely proportional to some function of the variables. But you can also use integration to find the area between a curve and a line, or even two curves. ff Draw a diagram. b) Find the speed of A when it reaches the pulley. ff Finding u = sin (2x + 1) so let u = sin v where v = 2x + 1 ff Then go through the usual stages: Tip: Calling it v just means you don't end up with a load of u's floating around. ff Be able to find the Differentiating parametric equations equations of tangents and normals to curves given parametrically. Then the sum of all the rectangles between a and b – dx is: b - dx /R x=a = b - dx / f (x) dx x=a As dx gets smaller and smaller, then this sum of rectangles comes closer and closer to the actual area under the curve. Tip: This example uses proof by contradiction to prove that there is no 'largest number' of a certain type. ... Look back at the previous sections of this chapter to check how to set out the working if you need to. The model could include other factors, such as air resistance or the spin of the ball. a a) What is the magnitude of the block's acceleration? Example 2 a) State the range for the function f(x) = x2 – 5, x d R . Q10 a) y = f(x) y d) y=x y = |x| – 9 y –9 Range: f(x) ≥ –9 9x –9 e) x = –1 1 1 y = –1 5 –1 y = f (x) b) There is one point where the graphs intersect. b) No, because the map is not defined for elements 4 and 5 of the domain. 2 c) Evaluate f –1 a - 5 k . ms–1 ff Resolving vertically, taking up as positive: s = 10, u = (23.811...) sin 45° = 16.837..., v = vy, a = –9.8 Using v = u + 2as: 2 2 vy2 = (16.837...)2 + (2 × –9.8 × 10) = 87.5 m s 2 –2 ff Now you can find the speed using Pythagoras' theorem: speed = ^16.837...h2 + 87.5 = 19.3 ms–1 (3 s.f.) Tip: Don't bother square rooting to find vy — you'll need to use vy2 in the next step. Using the critical values table, the critical value is 0.7067, so the critical region is r ≥ 0.7067. So a = –3 and b = 3 Q7 The graph of h(x) = + x + 1 is shown below. Using the formula for conditional probability: P (A + B) 0.3 P(A|B) = P (B) = 0.5 = 0.6 b) Find P(A Ç B'). Q14 15 ms–1 PROBLE M SOLVING MODELL ING 2 cm A dart is thrown horizontally towards a dartboard with speed 15 ms–1. Differentiating (ax + b)n + 1 using the chain rule gives a(n + 1)(ax + b)n. Tip: On pages 284-287 you saw how a normal distribution can be used to approximate a binomial distribution. ff Sketch the graphs of y = |2x – 4| and y = 5 – x (if you're thinking that this looks familiar, you're dead right — this is the graph you drew on p.30). So, resolving forces vertically ('forces up' = 'forces down'): 145 + 90 = MA g + MB g fi MA + MB = 235 ÷ 9.8 = 23.979... The Trapezium Rule Exercise 5.1 — The trapezium rule Q1 a) h = (2 – 0) ÷ 2 = 1, so the x-values are 0, 1, 2. The truck and the car move with constant acceleration. c) When A hits the pulley, the string breaks and B falls freely. Conditional probability A probability is conditional if it depends on whether or not another event happens. Since x is even, it can be written as x = 2n for some integer n. B1 = # 5p 4 p 4 5p 5p p 5p p sin x + 1dx = [- cos x + x] p4 = b - cos b 4 l + 4 l - b - cos b 4 l 4 1 4 = c - c - 2 m + 5p m - c - 2 + p m = 4 4 2 2 B2 = # p 4 5p 4 2 +p 5p 5p p 5p p cos x + 1dx = [sin x + x] p4 = bsin b 4 l + 4 l - bsin b 4 l + 4 l 4 = c - 2 + 5p m - c 2 + p m = - 2 + p 4 4 2 2 Tip: In the table on p page 53, cos 4 is 1 given as — here 2 2 it's written as 2 but it's the same thing (you can rationalise the denominator to check). Kinematics The study of the motion of objects. dy - q 3 sin q - 3q 2 cos q Q9 a) Using the quotient rule, = dq q6 - q sin q - 3 cos q = q4 dx = q2 cos q + 2q sin q dq dy - q sin q - 3 cos q Hence = dx q6 cos q + 2q5 sin q This looks a bit complicated but leave it as it is — you'll find the cos and sin terms usually disappear when you substitute. You can calculate the resultant force by adding the corresponding components of the forces. PROBLE M SOLVING 342 Chapter 15 Dynamics Connected particles, pegs and pulleys You also need to be able to resolve forces and calculate friction in situations with connected particles. ff Be able to explain why a probability distribution is or isn't suitable for a given context. ff Be able to add and subtract threedimensional vectors and multiply them by scalars. Chapter 12 The Normal Distribution 285 Example 1 The random variable X ~ B(90, 0.47). x 1 3 y = ln x c) S how that there is a solution between 2 and 3, and find this solution to 1 decimal place. 6 A + B + C Q2 a) º ^1 xh^1 + xh^1 + 2xh ^1 - xh ^1 + xh ^1 + 2xh 6 º A(1 + x)(1 + 2x) + B(1 – x)(1 + 2x) + C(1 – x)(1 + x) Using the substitution method: When x = 1, 6 = 6A fi A = 1 When x = –1, 6 = –2B fi B = –3 1 3 When x = - 2 , 6 = 4 C fi C = 8 Putting these values back into the expression gives: 6 1 - 3 + 8 º ^1 xh^1 + xh^1 + 2xh ^1 - xh ^1 + xh ^1 + 2xh b) f(x) can also be expressed as: (1 – x)–1 – 3(1 + x)–1 + 8(1 + 2x)–1 Expanding these three parts separately: (1 – x)–1 = 1 + x + x2 + ... Glossary 477 D F Degree The highest power of x in a polynomial. g 2x2 + 3x x - 5 2x 3 - 7x 2 - 16x + 11 – (2x3 – 10x2) 3x2 – 16x ff Multiply (x — 5) by 3x to get 3x2 – 15x, then subtract again and bring down the +11 term. = 1 + 9x + 24x2 + ... Find: a) the ball's time of flight, b) the horizontal range of the ball. So you would reject H0 if r ≥ 0.5494 or if r ≤ –0.5494. There are two negative solutions in the interval 0 ≤ x ≤ 360° at (180° + 30°) and (360° – 30°), so x = 210° and 330°. Examples and questions that include modelling are also indicated with stamps. Glossary i 477 Exercises (with worked answers) Lots of practice for every topic, with fully worked answers at the back of the book. Q5 Hint: To find sin q given cos q, you can draw a rightangled triangle and use trigonometry and Pythagoras' theorem. Given that the rod is in limiting equilibrium, find the magnitude of the normal reaction, N, at the peg and the magnitude of the frictional force, F, between the rod and the ground. It is projected across the surface at a point (0.4i + 1.6j) m from the origin at time t = 0 with velocity (0.5i – 1.2j) ms–1. In the form ax + by + c = 0, 5x – 6y + 8 = 0. Exercise 3.2 — Differentiating ln x dy 1 = x dx f ' (x) You could use that the derivative of ln (f(x)) is f (x) — it'd give the same answer. a) Explain why the distribution of the sunflowers' heights might reasonably be modelled using a normal distribution. 0 0 1 1 Q2 For the following functions, explain whether or not an inverse f–1 exists: a) f(x) = sin x, x d R b) f(x) = x2 + 3, x d R 2 c) f(x) = (x – 4) for {x : x ≥ 4} Q3 Find the inverse of each of the following functions, stating the domain and range: a) f(x) = 3x + 4, x d R b) f(x) = 5(x – 2), x d R 1 c) f(x) = + , x > –2 d) f(x) = x2 + 3 for {x : x > 0} x 2 Q2-6 Hint: If in doubt, sketch a graph of the function to check domains and ranges. There are 4 other solutions for 5x in the interval 0 ≤ 5x ≤ 10p, at 3p, 5p, 7p, and 9p, so 9p p 3p 7p x = 5 , 5 , p, 5 and 5 . x3 3 = 9t + C Chapter 14 Kinematics 317 ff x = 0 when t = 0, so C = 0. a) Find the angle between the direction of the force and the incline of the plane. 1 1 ^4 + xh2 = 2 `1 + x j2 4 x x2 x3 = 2(1 + 8 – 128 + 1024 – ...) x2 x x3 = 2 + 4 – 64 + 512 – ... y 1 x –1 3 –90° 90° x –2 b) 11 Range: 2 ≤ f(x) ≤ 11 f(x) tan x is undefined at e.g. –90° and 90° (it is also undefined periodically either side). ff The iteration might also stop working, like if you have to take the square root of a negative number. This is one of the endpoints of the arcsin x graph. dv ff Choose u and . Here you can see an exam-style question worked from start to finish, just how they'd want you to do it in the real thing. = 4a a = –9.149... Remainder (algebraic division) The expression left over following an algebraic division that has a degree lower than the divisor. Vertical transformations do not affect the position of the asymptotes, so they are in the same position as for the graph of y = cosec x, i.e. at np or 180n°, where n is an integer. dq Q3 A snowball of radius r cm is melting. N ff Use trigonometry to find the required direction: 1.354... dx # 6x 5 6 6 e x dx = e x + C f(u) = eu f'(u) b) # esin x cos x dx ff Here, u = sin x. 1 ff So substitute m = – 2 , x = 0 and y = –2p into y = mx + c, to find c: y = mx + c 1 –2p = – 2 (0) + c fi c = –2p ff Putting c back into the equation gives: 1 y = – 2 x – 2p or x + 2y + 4p = 0 Chapter 7 Differentiation 171 Exercise 9.2 Q1 A curve is defined by the parametric equations x = t2, y = t3 – 6t. The wire makes an angle of 60° with the rod, as shown. Find the equation of the tangent to the curve at t = 3, giving your answer in the form ax + by + c = 0. So there cannot be a largest multiple of 3. PROBLE M SOLVING Chapter 12 The Normal Distribution 283 2. 1 s = ut + 2 at2 s = 15 × 0.063... Chapter 2 Algebra and Functions 5 Exercise 1.2 Simplify the following: Q1 2x + x 2 - 1 3 5 Q2 3x 5x Q3 3 +2 x+1 x+2 Q4 3 + 4 x2 x Q5 2x + x - 1 3x - 2x - 1 7x Q6 4 3 5x Q7 2 +3 3 2 Q8 + + + x 1 x 2 x -1 x Q9 4 - 1 6 6 Q10 + + x 2 x 2 x-3 x+4 Q11 3 - 5 3 x Q12 + + + x 2 x 1 x - 2 2x + 3 Q13 5x - 3 5 3 Q14 + + + x (x 3) x 2 (x + 1) 2 x + 1 Q15 Hint: Factorise the first denominator before you do anything else. ff Substitute what you've got so far back into the original expression: # 4x(x2 – 2)4 dx = # 4xu4 21x du = # 2u4 du The remaining x's cancel. 26 q q q is acute, so 2 is also acute, and cos a 2 k is +ve so we can ignore the negative root. Answers 393 12 q 1 1 1 (ii) sin2 a 2 k = 2 (1 – cos q) = 2 b1 - 13 1 l = 26 q 1 1 So sin a 2 k = 26 . ff P(X > 620) = 0.025480... ÷ 9.8 = 2.29 kg (3 s.f.) Q7 R F W=8N Resolving vertically: R + T sin a = 8 Resolving horizontally: T cos a = F T will be greatest when the object is in limiting equilibrium: F = mR a) a = 0 fi R = W and T = F = mR = 8m T = 0.61 × 8 = 4.88 N b) a = 35° Resolving vertically: R + T sin 35° = 8 R = 8 – T sin 35° 1 Resolving horizontally: T cos 35° = F F = mR fi T cos 35° = 0.61R 2 W Resolving perpendicular to the plane: R = W cos a 1 Resolving parallel to the plane: F = W sin a 2 F ≤ mR when the object is at rest on the plane. Also known as the limit of a series. KK OO 6 L P Show that M is the midpoint of CD. Significance level (a) Determines how unlikely the observed value of the test statistic needs to be (under H0) before rejecting the null hypothesis in a hypothesis test. 188 Chapter 8 Integration Integration of other trigonometric functions There are some other more complicated trig functions which are really easy to integrate. product should be u and which bit should be ff For example, if you have a product that has a single x as one part of it, du choose this to be u. Here, the line is a tangent to f(x) at xn. b) Given that the population of his town is 30 000, he initially has 10 000 signatures on his petition and it takes 5 days for him to reach 12 000 signatures, how long, to the nearest day, will it take for him to reach 25 000 signatures? a - 13 F(z) = 0.7580 for z = 0.70, so 1.8 = 0.70 ff Now, just solve for a: a - 13 1.8 = 0.7 fi a – 13 = 1.26 fi a = 14.26 Tip: It's neater to write the variance as 1.82, rather than 3.24. MODELL Q3 A ball is kicked from a point on a flat, horizontal field. Find h, the difference in height between the two targets. Example 1 The diagram below shows the position of point Q. dt dA dA So µ A fi = –k A, for some k > 0. 2x 2 - 5x + 2 6 b) Hence find # dx where x > 2. It leaves the ground at an angle of 40° to the horizontal. MODELL ING If 200 robin chicks are randomly sampled from across the country, use a suitable normal approximation to estimate the probability that at least 25% of them survive to adulthood. d ^ 1 ln x h = x . g is usually assumed to be 9.8 ms–2. A random sample of 25 plums gives a mean weight of 43.5 grams. In this section you'll see how to differentiate reciprocals of trig functions — sec x, cosec x and cot x. All you have to do is draw on the lines and label the values of x1, x2 and x3. dy f l(x) 5 c) = = + dx 1 5x f (x) dy fl(x) 8 4 16 d) = 4 × =4× = = dx f (x) 4x - 2 4x - 2 2x - 1 Don't forget to simplify your answers if you can. ff So, b = –1.6904 (5 s.f.) Exercise 4.2 Q1 Let f(x) = x4 + 2x3 – 4x2 – 7x + 2 and g(x) = x3 – 4x + 1 244 a) Show that f(x) = (x + 2)g(x). Q6 Find f'(x) if f(x) = 2 sin3 x. = 4.526... Q3 – H e has assumed that his estimate of the probability (using the relative frequency from his data) is valid for children from any other school — there are likely to be lots of reasons why this is not the case. ff Taking moments about A: Clockwise moments = Anticlockwise moments (30 × 2) + (40 × 3) = 5T fi T = 180 ÷ 5 = 36 N This is the weight of the beam, acting at its centre. The Double Angle Formulas Exercise 7.1 — Using the double angle formulas Q1 a) sin 2A º 2 sin A cos A fi 4 sin A cos A º 2 sin 2A p 1 p p fi 4 sin 12 cos 12 = 2 sin 6 = 2 × 2 = 1 b) cos 2A º 2 cos2 A – 1 2p p 1 2 1 fi cos 3 = 2 cos2 3 – 1 = 2a 2 k – 1 = - 2 c) sin 2A º 2 sin A cos A sin 2A fi 2 º sin A cos A 3 1 3 sin 120° fi = sin 60° cos 60° = 2 × 2 = 4 2 2 tan A 1 - tan 2 A tan A tan 2A fi º 4 2 2 tan 2 A 3 tan 15° 1 tan 30° fi = = 12 4 2 2 tan 2 15° 4 3 e) cos 2A º 1 – 2 sin2 A fi 2 sin2 A – 1 º –cos 2A 3 fi 2 sin2 15° – 1 = –cos 30° = - 2 Q2 a) cos 2A º 1 – 2 sin2 A 1 2 17 fi cos 2x = 1 – 2 sin2 x = 1 – 2a 6 k = 18 b) First find cos x: 35 1 cos2 x = 1 – sin2 x = 1 – a 6 k2 = 36 35 fi cos x = 6 d) tan 2A º x is acute so take the positive root for cos x. ff p isn't particularly close to 0.5, but n is large, so find np and n(1 – p): np = 200 × 0.23 = 46 and n(1 – p) = 200 × (1 – 0.23) = 154. An accurate sketch of the graph will help to avoid this. Connected Rates of Change 165 9. The initial velocity of the particle is U ms–1 at an angle q to the horizontal: U ms–1 q h y x a) Find the maximum height the particle reaches above the ground, h, in terms of y, U and q. (i) P(R|S) (ii) independent? Given that the package is accelerating up the slope at a rate of 1.3 ms–2, find m, the coefficient of friction between the package and the plane. f '(5) = 0, which means that the tangent line at x1 is horizontal, and it won't intercept the x-axis. So for y = x + 3 and y = x2, at the points of intersection you know that x + 3 = x2, which you can rearrange to get x2 – x – 3 = 0. 2 Q5 3 cos 2x + 5 sin 2x º R cos (2x – a) fi 3 cos 2x + 5 sin 2x º R cos 2x cos a + R sin 2x sin a 394 fi 1 R cos a = 3 and 2 R sin a = 1 p 1 2 ÷ 1 gives tan a = fi a= 6 3 2 2 1 + 2 gives: R2 cos2 a + R2 sin2 a = ^ 3 h + 12 = 4 fi R= 4 =2 p So 3 sin x + cos x º 2 sin b x + 6 l. Set x = 0: 11 = C + D, so C = 21. ff First, differentiate f(x) to find f'(x) = 3x2 – 3. 1 radian is the angle in a sector of a circle with radius r that has an arc of length r. 2 tan x – 4 sec 2x tan 2x + cosec 2x + 2x 5. See page 184 for a method of integrating x–1 and (ax + b)–1. ff Resolving forces horizontally: N = 50 N fi F = 30 N 16g R 70° ff Taking moments about the base of the ladder: (50 sin 70° × x) + (16g cos 70° × 3) + (F cos 70° ×6) = (N sin 70° × 6) (50 sin 70°)x + 470.4 cos 70° + 180 cos 70° = 300 sin 70° (50 sin 70°)x = 300 sin 70° – 180 cos 70° – 470.4 cos 70° x = 59.457... The ball reaches a maximum vertical height above the ground of 4.8 m. N ff Resolve vertically (Ø) for B: Mg – T = M × 0 T = Mg ff Resolve parallel to the plane ( ) for A: T – F – 3g sin 30° = 3 × 0 Mg – 10.184... P G C Contents Chapter 5 — Sequences and Series Introduction About this Book i Pure Mathematics Chapter 1 — Proof 1. So this answer seems reasonable. ^ 3 + 1h × 2 6+ 2 = = 4 ^2 2 h × 2 Q6 sin 75° = sin (30° + 45°) = sin 30° cos 45° + cos 30° sin 45° 3 1 1 1 n+e 2 × o = d 2 × 2 2 c) cosec (x + y) = ^1 + 3 h × 2 1+ 3 6+ 2 = = 4 ^2 2 h × 2 2 2 tan 45° + tan 30° Q7 tan 75° = tan (45° + 30°) = 1 tan 45° tan 30° 3+1 1 e o 1+ 3 3 3+1 = = = . Tip: You can take constant factors outside of integrations and put them at the front to make things easier. Parameter (parametric equations) The variable linking a set of parametric equations (usually t or q). Use integration by parts again: Let u1 = x and let dv1 du 1 = (x – 1)–3. ff Step 2 — rearrange the equation: ds = –6t2 dt ff Step 3 — integrate both sides: # 1 ds = # –6t2 dt fi s = –2t3 + C Steps 4 and 5 aren't needed here — the equation doesn't need rearranging, and you're only looking for the general solution, so you're done. (The components will also be equal if the particle is moving along a path 45° below –i.) From part a), vA = [(3t2 – 2t – 4)i + (3t2 – 4t + 3)j] ms–1. Set x = 0: –4 = –3D + E, so D = 3. ff Multiply the total number of sunflowers by this probability: 1000 × 0.961572... Tip: For projections at ground level (y = 0) the formulae for time of flight, t, and horizontal range, x, simplify as follows: 2U sin q t= g 2U 2 sin q cos q x= g Chapter 14 Kinematics 303 The particle is used to model a ball thrown at a velocity of 30 ms–1 at an angle of 25° above the horizontal, from a vertical height of 1.5 m. Assumption A simplification of a real-life situation used in a model. co 25 0° 5 sin 4 25 25 N 40° m m O m 40° d 25 N ff d = 5 cos 40°, so: Moment = 25 × 5 cos 40° = 95.8 Nm (3 s.f.) clockwise Finding the sum of moments about a point ff If there are two or more forces acting on a rod or beam, then you can find the sum of the moments about a particular point. ff First, resolve forces horizontally (Æ), to find the frictional force. F: Fnet = ma 60 – F = 12 × 1.5 F = 42 N Tip: Don't get confused between F and Fnet. ff In fact, even if p isn't all that close to 0.5, this approximation usually works well as long as np and n(1 – p) are both bigger than 5. 2 2 dy 3t 2 - 2t + 5 Q2 = dx 3t 2 - 4t When t = – 1: dy 10 = 7 , x = –3 and y = –7. = 17.908... ^1 + axh4 b) = (1 + ax)4(1 – bx)–3 ^1 bxh3 = (1 + 4ax + 6a2x2 + ...)(1 + 3bx + 6b2x2 + ...) = 1 + 3bx + 6b2x2 + 4ax + 12abx2 + 6a2x2 + ... ff Input z = 0.64, a lower bound, m = 0 and s = 1 into the normal cdf function. t2 = 9 ÷ 36 = 0.25 fi t = 0.5 seconds ff ...and the i-component of its velocity is non-zero: 4(0.5) – 9(0.5)2 + 8 = 7.75 ≠ 0 ff So the particle is moving parallel to i at time t = 0.5 seconds. Exercise 10.2 Q1 A curve is defined implicitly by x2 + 2x + 3y – y2 = 0. You will have used Pythagoras' theorem to find the length of a 2D vector, and it works in three dimensions too: The distance of point (a, b, c) from the origin is: a2 + b2 + c2 You can derive this formula from the two-dimensional Pythagoras' theorem: ff The orange line shows z Tip: Remember the position vector of the point (a, b, c) is ai + bj + ck, or as a column vector: KJa ON KK OO KKb OO Kc O L P C (a,b,c) y c a2 + b2 O B a b A x the distance of the point (a, b, c) from the origin. – The number of signatures is a discrete variable, and the model uses a continuous function. Modulus Q1 a) x y = f(x) – 32 y b) c) There is one solution as the graphs intersect once. dy if y = sec (2x2). Q3 A = 4pr2 so dV b) V = A × h = 5 3 x2, so = 10 3 x. Differentiating: dy dy dy dy y - ex ex + 2y –y–x = –3 fi = dx dx dx dx 2y - x + 3 dy 1 - e0 At (0, 1), = =0 dx 2 (1) - 0 + 3 so this is a stationary point. Q1 5x 4 Q2 2x + 10 x 2 + 2x Q3 6x 2 - 3x 4 3 Q4 3 2 3x x + 3x 2 Q5 3x + 6 x 2 + 3x Q6 + + x 3x 2 x2 + x - 6 Q7 2x - 6 5x 2 - 20x Q8 x2 - 9 2x 2 - 5x - 12 Q9 x 3 - 4x 2 - 19x - 14 3x 2 - 7x - 6 Q10 2x 2 - x - 15 x 2 - 6x - 7 Q11 1 1+ x x 3 - 2x 2 Q12 3 x+1 x - 4x 2 1 1 1 + 2x 3+ x Q13 1 Q14 1 2+ x 2+ x 1 1 1 2+ x Q15 3x2 - Q16 2 6x + 3x 3x x 3x 1 2+ + x+2 x 1 Q18 x + 1 1 3+ + x+2 x+2 x 1 1 2 4- 2 1- + x x 3 Q19 Q20 1 1 x+2 2- x - 2 x Q17 Q20 Hint: Multiplying each term by x2 will get rid of all the fractions. Arctan The inverse of the tangent function, also written as arctangent or tan−1. Q6 Forces of magnitude P N and 2P N act on a particle vertically upwards and horizontally, as shown in the diagram. Find the horizontal and vertical components of the ball's initial velocity. 1 Q5 Use integration by substitution to find the integral of the function x y= between the limits x = 1 and x = 5. dt dV dV 1 is inversely proportional to the square of V, so ? Sum to infinity The sum to infinity of a series is the value that the sum tends towards as more and more terms are added. = 10.0 N (3 s.f.) b) Resolving vertically: R + (10.046...) cos 20° = 1.6g fi R = 6.239... a) 250 piglets are born. ff Be able to perform calculations involving projectile motion, including the use of i-j vectors. Hypothesis Tests of the Mean of a Population 295 Statistical Tables 369 Answers370 Glossary477 Index482 Formula Sheet 484 About this Book In this book you'll find... Using a calculator: P(T > 49.3) = 0.791 (3 s.f.) So, 79.1% are slower than Gary. Find the magnitude of: a) the acceleration of the particle, b) the normal reaction with the plane. b) Write down the number of solutions of the equation x + 1 = 2x. = 0.894 (3 s.f.) b) If X ~ N(13, 5.76), find P(X > 17) to 3 s.f. ff X is a continuous random variable, so P(X > 17) is just the same as P(X ≥ 17). It differentiates to 1, which makes integrating v dx very easy. ff Resolve parallel to the plane (R –2 s 15°): Fnet = ma 20 20 sin 15° – F = g × 0.44 F = 4.278... So fi # (n + 1)f'(x)[f(x)]n dx = [f(x)]n + 1 + C # –3 cos x(sin x)–4 dx = (sin x)–3 + C fi # - 3 cos x 1 dx = +C sin 4 x sin 3 x ff Divide everything by –3 to match the original integral: # cos x 1 = +C sin 4 x 3 sin 3 x Exercise 5.2 Q1 Find the following indefinite integrals: # 6x(x2 + 5)2 dx b) # (2x + 7)(x2 + 7x)4 dx a) # (x3 + 2x)(x4 + 4x2)3 dx d) # ^ x 22-x1h3 dx c) 3x # ^e 36x e- 5h2 dx f) # sin x cos5 x dx e) # 2 sec2 x tan3 x dx h) # 3ex(ex + 4)2 dx g) # 32(2e4x – 3x)(e4x – 3x2)7 dx j) # i) cos x ^ 2 + sin xh4 Q1-2 Hint: You'll need the derivatives of cosec, sec and cot: d (cosec x) = dx –cosec x cot x d (sec x) = sec x tan x dx d (cot x) = –cosec2 x dx dx # 5 cosec x cot x cosec4 x dx l) # 2 cosec2 x cot3 x dx k) Q2 Find the following integrals: # 6 tan x sec6 x dx b) # cot x cosec3 x dx a) Q3 Integrate the following functions with respect to x: a) 4 cos x esin x (esin x – 5)3 b) (sin x ecos x – 4)(ecos x + 4x)6 Q4 Integrate: 2 x # sec # cot x cosec x cosec x dx a) dx b) tan 4 x Chapter 8 Integration 199 6. To do this, you need to use integration. Identities Involving Cosec, Sec and Cot 6. 322 Chapter 15 Dynamics Example 1 The diagram on the right shows a force of 15 N and a horizontal force of 20 N applied to a particle. Wire A thin body often modelled as being light. So the range of values of x for which the ladder is in equilibrium is 1.265... y y1 y2 h When you approximate the area under a curve using the trapezium rule, that value will lie between these two bounds. So the gradient is 0 when x = ±1 fi the curve has stationary points at x = ±1. c) Find the probability that at least 5 people order a vegetarian meal. Tip: To get this formula, rewrite the 'reverse chain rule' on p.195, replacing the function 'u' with 'f(x)' and the function 'f(u)' with '[f(x)]n + 1 '. G g Acceleration due to gravity. Find the range of possible values for d which will cause PQ to rotate anticlockwise about the point O. Find: a) the magnitude of the normal reaction of the peg on the beam, b) the magnitude of the normal reaction of the ground on the beam, c) the magnitude of the frictional force between the ground and the beam, d) the coefficient of friction between the ground and the beam. Then = 2x and v = –cos x. Tip: Always mark on any key points such as the places that the graph touches or crosses the axes. The coefficient of friction between the block and the plane is 0.4. Given that the block is being held in equilibrium by a force D acting parallel to the plane, find the range of possible values of the magnitude of D. Find when x = 2.5. dt dt ff This is trickier because you're not given the expression for y. You've seen Newton's three laws before, so there's nothing new here. The rod is held horizontally in equilibrium by a light, inextensible string attached at C, 0.4 m from B, which makes an angle of 45° with the rod, as shown. Inverse Functions Exercise 4.1 — Inverse functions and their graphs Q1 a) Yes, as the graph shows a one-to-one function. Chain rule A method for differentiating a function of a function. Given that the particle is in equilibrium, find the magnitudes of the missing forces P and Q. Exercise 1.2 — Using conditional probability Q1 P(A') = 1 – P(A) = 1 – 0.75 = 0.25 P(A' Ç B) = P(A') P(B|A') = 0.25 × 0.4 = 0.1 P(A Ç B) = P(A) – P(A Ç B')= 0.75 – 0.45 = 0.3 P(B) = P(A Ç B) + P(A' Ç B)= 0.3 + 0.1 = 0.4 P(A)P(B) = 0.75 × 0.4 = 0.3 = P(A Ç B), so A and B are independent. dx 2 1 So x = 2 is a point of inflection. y r=1 × × × × × × × × y × × y × × x r = –1 × × × × × × × × × × × × × × Tip: The closer the value of r to +1 or –1, the stronger the correlation between the two variables (values close to +1 mean a strong positive correlation, and values close to –1 mean a strong negative correlation). Concave curve A curve with a negative second derivative — i.e. f''(x) < 0 for all x. Integration of (ax + b)n180 1 2. Q5 Show that # p 3 p 2 sin ^ - xh + cos ^ - xh dx = 1- 3 2 . Given that the block is on the point of moving, find the coefficient of friction between the block and the plane. b) OB = 2 OA = 2(3i + j + 2k) = 6i + 2j + 4k Q5 The position vector of the midpoint of CD is at JK - 1NO J N J N J N J N KK OO 1 KK 4 OO KKK - 1OOO KK 2 OO KKK 1 OOO 1 OC + 2 CD = KK 3 OO + 2 KKK - 4OOO = KK 3 OO + KKK - 2OOO = KK 1 OO = OM KK OO KK OO KK - OO KK OO KK - 2OO -5 6 5 3 L P L P L P L P L P So M is the midpoint of CD. Tip: The strut supports the rod at its midpoint — so the vertical component of the thrust acts 3 m from A. However, this method can be useful when the mean and variance are unknown (see p.277-279). For questions 4-6, you could also have used the formula method — you should get the same quotient and remainder. Example 4 Find the domain of fg(x), where f(x) = x , x ≥ 0, and g(x) = x + 5, x d R The range of g(x) is g(x) d R . Moments in equilibrium ff A rigid body which is in static equilibrium will not move. b) Using your graph from part a) write down the number of solutions of the equation 2x3 – x2 – 7x = 0 in this interval. Q7 For the curve y = Q8 Differentiate y = e x 2 x+3 Q7 Hint: To use the product rule here you'll need to rewrite the function at the bottom as a negative power. The next page has an example of subtracting integrals and then adding them. Find the horizontal distance travelled by the particle in the first 1.2 seconds of its motion. + yn - 1h + Example 1 Use the trapezium rule with 3 strips to find an 1.5 approximate value for # x2 + 2x dx . x (x - 2) 2 as partial fractions. ff If you know the rate of change of one of these linked variables, and the equations that connect the variables, you can use the chain rule to help you find the rate of change of the other variables. c) Find the horizontal distance between the wall and the point on the ground where the ball lands. dt because A is decreasing. c) Find the length of time the ball is at least 5 m above the ground. Chapter 15 Dynamics 331 ff The frictional force, F, acting on an object is given by: Tip: An object which is on the point of moving is said to be in 'limiting equilibrium'. So 68% of observations are within ±s of the mean, 95% of observations are within ±2s of the mean and 99.7% of observations are within ±3s of the mean. Q2 Write 3x4 – 8x3 – 6x – 4 in the form (Ax3 + Bx2 + Cx + D)(x – 3) + E, and hence state the result when 3x4 – 8x3 – 6x – 4 is divided by x – 3. Q4 Rearrange the equation to get x 2 ln x – 5 = 0. Integration of Trigonometric Functions 186 f l (x) 4. Small angle approximations Functions that approximate sin x, cos x and tan x for small values of x in radians. ff So the range is f(x) ≥ –5. × 1.2 = 4.35 m (3 s.f.) opp Q9 tan q = , so draw a right-angled triangle with 3 as adj the 'opposite' side and 4 as the 'adjacent' side. ff You can write the square root as a fractional power: # (x – 2) x 2 - 4x + 5 dx = # (x – 2)(x2 – 4x + 5). – 0.559... So the nth term un = ar n – 1 = a(0.85)n – 1 The price when new is the first term in the series and the value after 10 years is the 11th term. x3 terms fi –8 = –3A + B, so B = 1. = 1 – 25x + 250x2 – ... You're Reading a Free Preview Pages 122 to 130 are not shown in this preview. b) f(x) = 5(x – 2) with domain x d R has a range f(x) d R . dx 2 dy ff Start by finding — identify u and v and use the product rule: dx u = x3 and v = ln x du dv 1 fi = 3x2 and = x dx dx dy dv du b 3 1 l So =u +v = x × x + (ln x × 3x2) dx dx dx = x2 + 3x2 ln x Tip: You're told that there are two points of inflection, so these two x-coordinates must be the answers. Chapter 8 Integration 219 Example 2 Find the exact value of ff Start by writing # 3 4 2 dx, writing it as a single logarithm. dt dt Q4 The curve C is defined by the parametric equations x = t3, y = t2 cos t. × 2) + (2 × –9.8 × 22) s = 30.0 m (3 s.f.) Q12 a) Resolving vertically, taking up as positive: s = 2.5 – 0.5 = 2, u = 19 sin 28°, a = –9.8, t = t 1 s = ut + 2 at2 2 = (19 sin 28°)t – 4.9t2 4.9t2 – (19 sin 28°)t + 2 = 0 Using the quadratic formula: 2 19 sin 28° ! ^- 19 sin 28°h - ^4 # 4.9 # 2h t= 9.8 8.9199... 6x - 5 Chapter 8 Integration 185 3. = 0.7186... ff The critical region is the set of all values of the test statistic that would cause you to reject H0. ÷ cos 10° = 62.0 N (3 s.f.) Chapter 15 Dynamics 333 Example 4 A block of mass 3.2 kg lies at rest on a rough plane, inclined at an angle of q to the horizontal. Tip: There's a 'hidden' derivative here — the rate of decrease in area. Upper bound The upper limit of the values that a number could take and still be rounded down to the correct answer. through the point d 2 3 2 6 p 1 y = mx + c fi 3 = - 3 × +c 2 p+2 3 p 2 3 c= 3 + 3 = 3 2 6 p+2 3 So the equation of the tangent is y = - 3 x + 3 Q11 a) Differentiating: dy dy dy 2y - e x 2e2y + ex = 2x + 2y fi = dx dx dx 2e 2y - 2x When y = 0: 1 + ex – e4 = 1 fi ex = e4 fi x = 4 dy 2 (0) - e 4 e4 = = 6 (gradient of the tangent) dx 2e0 - 2 (4) e4 2e 4 0 = 6 (4) + c fi c = - 3 . Even so, the method below is quicker, so unless you're asked to find the term after one you've already got, you're better off doing this: Or: First term a = 2, common ratio r = 1.5 nth term = arn – 1 = 2 × (1.5)n – 1 7th term = 2 × (1.5)6 = 2 × 11.390625 = 22.78125 7 th term = 6561 Q2 Common ratio r = th = 3. = 1.35 N (to 3 s.f.) 12 N 9N 35° 50° P ff Resolving in the x-direction (Æ): 12 cos 50° – 9 cos 35° = 0.3410... ff The null hypothesis will be: H0: m = a for some constant a. N Resolving vertically: RV + T sin 60° = 0.6g fi RV = 0.6g – (4.526...) sin 60° = 1.96 N 474 Answers q 1.96 N R Using Pythagoras' theorem: R = (2.263...) 2 + 1.96 2 = 2.993... One string is horizontal, and the other string makes an angle of 30° with the vertical. So if the function was '+ 1', the inverse would be '− 1', if the function was '× 2', the inverse would be '÷ 2' etc. dx If the sample expands at a constant rate, given by = 0.05 cm min–1, dt dV find an expression in terms of x for , where V is the volume of the prism. ff Write out the division in the form f(x) º q(x)d(x) + r(x), replacing q(x) and r(x) with general polynomials of degree 3 and 0: x4 – 3x3 – 3x2 + 10x + 5 º (Ax3 + Bx2 + Cx + D)(x – 2) + E ff Substitute x = 2 into the identity to make the q(x)d(x) bit disappear: 16 – 3(8) – 3(4) + 10(2) + 5 = 0 + E 16 – 24 – 12 + 20 + 5 = E fi E = 5 So now the identity looks like this: x4 – 3x3 – 3x2 + 10x + 5 º (Ax3 + Bx2 + Cx + D)(x – 2) + 5 Tip: You know from the Remainder Theorem that the remainder when f(x) is divided by (x – a) is equal to f(a). dy b) Hence find . a) Find the tension in each rope. Of the people with a smartphone, 29% have a contract costing £25 a month or less. ff Next, use one of the constant acceleration equations: v2 = u2 + 2as 0 = 32 + 2(–4.9)s s = 0.918 m (3 s.f.) b) the magnitude of the normal reaction from the plane. But for a decreasing function, it's the other way around. ff So if you have a rigid body resting on a rough surface, you will have a normal reaction force acting perpendicular to the surface, and a frictional force acting parallel to it, in the opposite direction to any potential motion. And since the total probability is 1, the total area under the curve must also be 1. Q4 A light, inextensible string attaches a block P of mass 60 kg to a block Q of mass 20 kg over + a fixed, smooth pulley. Q1 a) One-to-one: 5 374 Restricting the domain to e.g. –90° < x < 90° would make this a function. Transformations of Graphs You should be familiar with the basic graph transformations. E.g. the function f(x) = 3x2 − x3 − 2 (below) has 3 roots, since it crosses the x-axis three times (i.e. there are 3 solutions to the equation 3x2 − x3 − 2 = 0). 3 x The domain of f –1(x) is the range of f(x): x < 3. 362 Chapter 16 Moments ff This means that the frictional force at A acts upwards. ff So (2x3 − 7x2 − 16x + 11) ÷ (x − 5) = 2x2 + 3x − 1 remainder 6. Given that m = 0.75, find the value of q. Find the mass of the rod, m, and the coefficient of friction, µ, between the wall and the rod. i unit vector The standard horizontal unit vector (i.e. along the x-axis). More Differentiation Learning Objectives: ff Be able to differentiate cosec, sec and cot. a) Calculate the size of the angle x. So the critical value is z such that P(Z < z) = 0.05. There isn't enough data to conclude that one variable necessarily causes the change — there might be another factor that has an effect on both variables, such as age. dx dx ff Putting these into the formula gives: ff So # 2x cos x dx = 2x sin x – # 2 sin x dx = 2x sin x + 2cos x + C Tip: Calling the different parts u1 and v1 just means you don't get confused with the u and v used in the first integration. The integrations you need to do should be obvious if you draw a picture. ff You might be given a starting condition — e.g. the initial population. Tip: dx = - 0.00225 = - 0.00225 = –0.00015 m s–1 x 15 dt Exercise 8.1 MODELL ING Q1 A cube with sides x cm is cooling and the sides are shrinking dV by 0.1 cm min–1. < –1.645, the observed value of the test statistic lies in the critical region. From the calculator, F(z) = 0.9678 for z = 1.8494... dx Putting this into y = mx + c gives: p p 1 2 = - 4 (0) + c fi c = 2 So the equation of the tangent is y = p – 1 x. ff Rearranging Tip: Rearranging the equation first, then finding C would give you a quadratic in C to solve, so it's easier to leave the rearrangement till last here. Taking moments about D: 20g × 1.63 = (80g × y) + (2.5g × 0.8) fi 32.6 = 80y + 2 fi y = 30.6 ÷ 80 = 0.3825 So the distance from B is: 1 – 0.3825 = 0.6175 m Q6 Taking clockwise as the positive direction: a) Sum of moments = (6 × 5) – (10 × 10) = 30 – 100 = –70 Nm (i.e. 70 Nm anticlockwise) b) Sum of moments = (6 × 0.5) + (10 × 5) = 3 + 50 = 53 Nm clockwise Answers 473 c) Sum of moments = (6 × 2) + (10 × 4) = 12 + 40 = 52 Nm clockwise d) Sum of moments = (6 × 3.5) – (10 × 6) = 21 – 60 = –39 Nm (i.e. 39 Nm anticlockwise) For parts a)-d), F acts either vertically or horizontally, so you can just use x or y respectively as the perpendicular distance when taking moments. Given that the coefficient of friction between the wire and the bead is 0.15, find: a) the magnitude of R, the normal reaction of the wire on the bead, b) the value of m. Tip: Equating coefficients means comparing the coefficients of each power of x on the left hand side and the right hand side of the identity. So A, B and C must all lie on the same line — they are collinear. Chapter 14 Kinematics 305 Exercise 1.2 Hint: Drawing a diagram might help you in some questions if you're struggling. Chapter 16 Moments 365 Example 5 B 2.4 m C 25 N 10 N 28° A A uniform rod, AB, of length 3.3 m and weight 10 N, rests with A on rough horizontal ground. Exercise 4.2 — Geometric series a (1 - r) Q1 The sum of the first n terms is Sn = (1– r) , a = 8 and r = 1.2, so the sum of the first 15 terms is: a (1 - r15) 8 (1 - (1.2) 15) = 576.28 to 2 d.p. = S15 = (1 - r) (1 - 1.2) n Q2 For a geometric series with first term a and n –1 a^1 - r nh common ratio r: / ar k = , 1-r k=0 9 9 25^1]0.7g10h so: / ar k = / 25]0.7gk = = 80.98 (2 d.p.) 1 - 0.7 k=0 k=0 402 Answers Q3 a = 3 and r = 2, the sum of the first n terms is: 3 (1 - 2 n) = 3 (1 - 2 n) (1 - 2) 196 605 = Sn = –3(1 – 2n) fi –65 535 = 1 – 2n fi 65 536 = 2n fi log 65 536 = log 2n log 65 536 fi log 65 536 = nlog 2 fi n = = 16 log 2 Q4 The first term is a = 4, second term = 5 = the common ratio is r = 4 1.25 first term The sum of the first x terms is a (1 - r x) 4 (1 - (1.25) x) = Sx = = 16(1 – (1.25)x) (1 - r) (1 - 1.25) So: 103.2 = –16(1 – (1.25)x) fi –6.45 = 1 – (1.25)x fi 7.45 = 1.25x fi log 7.45 = xlog 1.25 log 7.45 fi x= = 9.00 to 2 d.p. log 1.25 Q5 a) 3rd term = ar 2 = 6, 8th term = ar7 = 192. Now test at the 1% level: Again, you're interested in the higher end of the distribution, so find the critical value, z, such that P(Z > z) = 0.01. Exercise 11.2 — Solving differential equations dy Q1 a) = 8x3 fi dy = 8x3 dx fi # 1 dy = # 8x3 dx fi y = 2x4 + C dy 1 b) = 5y fi y dy = 5 dx dx 1 fi # y dy = # 5 dx fi ln |y| = 5x + ln k fi y = e5x + ln k = ke5x dy 1 c) = 6x2y fi y dy = 6x2dx dx 1 fi # y dy = # 6x2 dx fi ln|y| = 2x3 + ln k 3 3 fi y = e 2x + ln k = ke 2x y dy 1 1 d) = x fi y dy = x dx dx 1 1 fi # y dy = # x dx fi ln|y| = ln|x| + ln k = ln|kx| fi y = kx dy 1 e) = (y + 1)cos x fi + dy = cos x dx dx y 1 1 fi # + dy = # cos x dx y 1 fi ln|y + 1| = sin x + ln k fi y + 1 = esin x + ln k fi y = kesin x – 1 ^3x - 6h y dy 3xy - 6y f) = = dx ^ x - 4h^2x - 5h ^ x - 4h^2x - 5h ^3x - 6h 1 fi y dy = dx ^ x - 4h^2x - 5h ^ h x 3 6 1 fi # y dy = # dx ^ x - 4h^2x - 5h The integration on the right hand side needs to be split into partial fractions before you can integrate. The component of the force in the direction parallel to the plane is 7.1 N. dy x+5 = ×y dx ^ x - 2h^ 2x + 3h 1 x+5 y dy = ^ x - 2h^ 2x + 3h dx ff First, separate the variables: ff To make the right-hand side easier to integrate, write it as partial fractions (see p.219): x+5 A + B x - 2 2x + 3 fi x + 5 º A(2x + 3) + B(x – 2) ^ x - 2h^ 2x + 3h Tip: Since all the other terms are ln (something), it makes sense to use ln k as the constant of integration here, then you can use the log laws to simplify. The set of integers has the notation Z. Rod A long, thin, straight, rigid body. A force of magnitude 3.5 N is applied to the rod at B, at an angle of 80° to the rod, as shown. Reaction forces If a rigid body is fixed to a surface by a hinge or pivot, you can deal with the reaction force on the body from the hinge by splitting it up into two components, parallel and perpendicular to the surface. The triangle has side lengths |a|, |b| and |b – a|. Putting this into y = mx + c gives: 9 7 1 = - 5 (–2) + c fi c = - 5 and so the equation of the normal is y = - 7 x - 9 5 5 or 7x + 5y + 9 = 0. f(0.24) = ln(0.24 + 3) – (5 × 0.24) = –0.024... Otherwise you won't be able to see the correct domain and range for the graph of the inverse when you do the reflection. When you're using this method, you might have to use simultaneous equations to work out some of the coefficients (have a look back at your Year 1 notes for a reminder of how to do this if you need to). dx dx dy dy dy fi 2x – 2y –4 = 10 – 2x – 2y dx dx dx dy fi (2x – 2y – 4) = 10 – 2x – 2y dx dy 10 - 2x - 2y 5 - x - y = = fi dx x-y-2 2x - 2y - 4 So when 5-x-y dy = 0: - - = 0 fi 5 – x – y = 0 fi y = 5 – x as required dx x y 2 b) Find the coordinates of the stationary points of A. a) Find the value of a. x A 0.2 m 2.5g 1m 1m C D RC 100g 0.2 m RD B 2.5g The reaction at C is 4 times the reaction at D, i.e. RC = 4RD Resolving vertically: 2.5g + 100g + 2.5g = RC + RD 105g = 4RD + RD 105g = 5RD fi RD = 21g N fi RC = 84g N Taking moments about A: (2.5g × 0.2) + (100g × x) + (2.5g × 3.8) = RC + 3RD 10g + 100gx = 84g + 63g 10g + 100gx = 147g x = (147 – 10) ÷ (100) = 1.37 m b) The distance between the plank's COM and the point D is 4 – 1 – 1.37 = 1.63 m y 1.63 m A B 0.2 m C D RC 20g RD 80g 2.5g When the plank is about to tilt about D, RC = 0. 222 Chapter 8 Integration in terms of the two variables, without a derivative term. Tip: Be careful with questions like this — the question tells you the probability of S conditional on L (and L'). Example 3 Show that the derivative of x2(2x – 1)3 is 2x(2x – 1)2(5x – 1). Integration by Substitution Learning Objectives: ff Understand that integration by substitution is the reverse of differentiating using the chain rule. dx ff Start by differentiating the two parametric equations with respect to t: dx = 2t dt dy y = t3 – 3t + 4 fi = 3t2 – 3 = 3(t2 – 1) dt x = t2 – 1 fi ff Now use the chain rule to combine them: dy d y d x 3 (t 2 - 1) = = ' 2t dx dt dt b) Find the gradient of C when t = –2. Modelling Problems 96 101 103 108 117 Chapter 6 — The Binomial Expansion 1. Integration of ex and x 182 3. Just go through it step by step. f(2.5) = e2.5 – 2 – 2.5 = 0.067... It has initial speed 8 ms–1 and leaves the ground at an angle of 59°. d ^ h= 1 For these functions you need to consider the fact that ln x x, dx which you should remember from Chapter 7. Q3 A particle of weight 10 N rests on a smooth inclined plane. N Q7 a) F A Answers F B C 75° WN 30° Usually, the coefficient of friction between a body and a surface is between 0 and 1, but it can actually be any number greater than 0 — as in this case. Write your answer in the form ax + by + c = 0, where a, b and c are integers. dH Find when r = 12.3. Give your answer to 2 d.p. dr Chapter 7 Differentiation 167 Q6 Hint: Treat r as a constant as the radius will not change. Then multiply top by top and bottom by bottom. a) Find fg and gf and give the range of each in terms of b. = 0.958... u q ff Be able to derive general equations that describe the path of a projectile. Series An ordered list of numbers, just like a sequence, but where the terms are being added together (to find a sum). Tip: An irrational number is a real number that can't be a written as a fraction b (where a and b are both integers and b ≠ 0). N q R 69.681... You're Reading a Free Preview Pages 323 to 378 are not shown in this preview. 3 xn + 4 to solve x3 − 4 − x = 0, to 2 d.p. ff The notation xn just means the approximation of x at the nth iteration. Putting s = 0.198675... ff Next, transform F to Z. 27 13.5 27 b) 13.5 = S¥ = a fi a = 13.5(1 – r) 1-r 2 = 13.5 × 3 = 9. x+1 So the range is 1 < f(x) ≤ 2. Example 1 Find the general solution of the differential equation ds = –6t2. The rate of decrease is directly proportional to the current number of rabbits (P). You should just know this result, but if not you can take logs (p ln 2 = ln 32, so p = ln 32 ÷ ln 2 = 5). Q10 Use the chain rule to find f'(x): y = arccos u, u = x2 dy dy du 1 2x = × =× 2x = dx du dx 1 - u2 1 - x4 1 p 1 When x = , y = arccos x2 = arccos 2 = 3 2 This is the only solution for y in the given interval. f –1(x) x –8 The inverse function is f−1(x) = x + 8 . Using the Binomial Expansion as an Approximation 129 3. Q5 2N ° 35° 1N 10° 3N a) Resolving horizontally (Æ): 1 cos 10° + 2 cos 35° + 3 cos q = 0 - cos 10° - 2 cos 35° k = cos–1(–0.874...) q = cos–1 a 3 = 150.970...° = 151° (to 3 s.f.) q must lie in the range 180° ≤ q ≤ 360° cos q = cos (360° – q), therefore q = 360° – 150.970...° = 209.029...° = 209° (to 3 s.f.) b) R = 1 sin 10° + 2 sin 35° + 3 sin (209.029...°) = –0.135 N (to 3 s.f.) So, R has magnitude 0.135 N (to 3 s.f.) (and acts in the negative j-direction) Q6 Resolving horizontally: 2P = 20 sin q 1 Resolving vertically: P = 20 cos q 2 2P Divide 1 by 2 : tan q = P = 2, –1 q = tan (2) = 63.43...° = 63.4° (to 3 s.f.) Substituting into 2 gives P = 20 cos q = 20 cos (63.43...°) = 8.94 N (to 3 s.f) Answers 467 Exercise 1.3 — Harder equilibrium problems Q1 a) Resolving horizontally: T cos 50° = 14 N 14 T = cos 50° = 21.78... Integration of 191 f (x) du l 5. Tip: The stone is thrown horizontally, so the vertical component of its initial velocity is zero. d) By rearranging the equation x + 1 = 2x, use the quadratic formula to find the solution of the equation from part c) to 3 s.f. Q5 a) Sketch the graphs of y = e2x and y = 3 – x2 on the same axes. To write this in the form ax + by + c = 0 where a, b and c are integers, multiply by 15 and rearrange. The domain is all real numbers except those for which cosec x is undefined (i.e. at the asymptotes). Q2 The results for a random sample of 13 students in a maths test and a science test were recorded. For example, using the symmetry of the graph, you know that m is exactly in the middle of 6.45 and 20.17, so you can simply find the average of these 2 values. Find the magnitude and direction of the resultant of the three forces. ff Be able to carry out hypothesis tests on the product moment correlation coefficient. ff The results are getting further and further apart with each iteration. x 4 tan 2 2 dx 1 - tan 2 2 ff Rearrange, then use the double angle formula for tan: x x 4 tan 2 2 tan 2 = 2f p = 2 `tan `2 × 2x jj = 2 tan x 2 x 2 x 1 tan 2 1 tan 2 ff Rewrite the integration: x 4 tan 2 = # x dx 1 - tan 2 2 # 2 tan x dx = - 2 ln cos x + C Tip: Remember the integral of tan x is –ln|cos x| + C (which is the same as ln|sec x| + C using the laws of logs). ff Resolving parallel to the plane ( ): 70 – F – 100 sin 35° = 0 fi F = 70 – 100 sin 35° = 12.6 N (to 3 s.f.) Example 4 A rock of mass 9 kg rests on a smooth plane inclined at 35° to the horizontal. ff First, find the speed of A when B hits the ground: v = u + at v = 0 + (4.385... The plane is inclined at an angle of q to the horizontal. N 86 N 10° B R 5.4g Mg Resolving horizontally: R = 86 cos 10° = 84.693...N Using F = mR: F = 0.55 × 84.693... trapezium rule to approximate the value of definite integrals. At time t seconds, the particle's position relative to a fixed origin O is given by rA = [(t3 – t2 – 4t + 3)i + (t3 – 2t2 + 3t – 7)j] m. Give a suitable distribution to model the number of successful first-serves in each of the situations below. Putting this into the Newton-Raphson formula: 2x 3 - 15x 2 + 109 xn + 1 = xn - n 2 - n 6xn 30xn Starting with x0 = 1: 2 (1) 3 - 15 (1) 2 + 109 = 1 - 96 = 1 + 4 = 5 x1 = 1 - 24 6 (1) 2 - 30 (1) 3 - 15 (5) 2 + 109 2 (5) = 5 - 16 x2 = 5 0 6 (5) 2 - 30 (5) Since this involves dividing by 0, the method fails, so x0 = 1 cannot be used to find a root. 23 - m f(z) P(X < 23) = P a Z < 2 k = 0.9015 Tip: s = 4 = 2 ff Draw a sketch to show the information. Quotient: 3x2 – x – 5 Remainder: 0 Q2 3x4 – 8x3 – 6x – 4 º (Ax3 + Bx2 + Cx + D)(x – 3) + E Set x = 3: 243 – 216 – 18 – 4 = E, so E = 5. b) Hence find the equation of the tangent to C when t = p. 12x + 6 dx where x > 2. dx Q9 Show that if y = dy x sin x x - sin x then = + . = 1 + 3bx + 6b2x2 + ... Infinite sequence A sequence that has no 'last term', so it goes on forever. ff The derivative term is the rate of change of L with respect to t, or dL . For a given sample size and significance level (for either a one-tailed or a two-tailed test) you can find the critical value — the point at which your test statistic r would be significant. In real life, the forces acting on an object change as a result of its motion, so its acceleration will vary. This formula uses the same ideas to integrate, although you don't need to use it for the integration you came across in Chapter 8. Q3 a) 2x3 + 5x2 – 3x = x(2x – 1)(x + 3) 5x - 6 5x - 6 b) 3 + 2 º 2x 5x x^2x - 1h^ x + 3h A B C º x + ^2x - 1h ^ x + 3h 5x - 6 º A(2x – 1)(x + 3) + Bx(x + 3) + Cx(2x – 1) Using the substitution method: When x = 0, –6 = –3A fi A = 2 1 5 7 When x = 2 , 2 - 6 = 4 B fi B = –2 When x = –3, –15 – 6 = 21C fi C = –1 5x - 6 2 2 1 So 3 + 2 º x ^2x - 1h ^ x + 3h 2x 5x 3x c) (2x – 1)–1 = –(1 – 2x)–1 = –(1 + 2x + 4x2 + ...) 2 1 -1 1 1 (x + 3)–1 = 3 b1 + 3 x l = 3 b1 - 3x + x9 + ... Therefore: 12 / a (- 2) k = –5735.1 a^1 - r h a^1 - ^ - 2h h = = 2731a ^1 - r h ^1 - ^ - 2hh fi –5735.1 = 2731a fi a = –2.1 k=0 12 / a (- 2) k = 13 13 k=0 Exercise 4.3 — Convergent geometric series 1.1 Q1 a) r = 1 = 1.1, |r | = | 1.1| = 1.1 > 1, so the sequence does not converge. Example 4 The diagram shows the light rod AB. 4 60° o i.e. 60° right, c) Translate by e 0 then stretch vertically, scale factor 2. x2 + 2x x0 = 0 y0 = 0 x1 = 0.5 y1 = 1.25 = 1.118... Chapter 9 Numerical Methods 231 Q10 Hint: Roots of f(x) are given when f(x) = 0, so you might need to rearrange the equation before doing any calculations. Tip: You're testing to see if the mean time has decreased. You'll often need to use trigonometry to resolve the forces into their horizontal and vertical components. P(N) = P(M Ç N) + P(M' Ç N) = P(M)P(N|M) + P(M')P(N|M') = 0.2 × 0.4 + 0.8 × 0.3 = 0.32 Tip: There are 2 'paths' giving N, so find the probability of each, then add them together. JK4NO KK OO OP = 4i + 3j + 0k = KK3 OO KK0 OO L P OQ = 2i + 5j + 4k = and z Q(2, 5, 4) PQ JK2NO KK OO KKK5 OOO K4 O L P 5 3 O b) Hence find PQ as a column vector. Integrate each term separately. 1 b) The expansion is valid for |x| < 4 . T= = 0.9400... dx If 3x2 – 3 = 0 fi 3(x2 – 1) = 0 fi x2 = 1 fi x = ±1 3 and if e x - 3x - 5 = 0, there are no solutions. Q7 An object is placed on a rough plane inclined at an angle a to the horizontal. Example 4 Tip: You can usually assume that supports and strings are fixed, and won't move with the rod. x-2 4 4-2 3-2 = :ln x D3 = ln 4 ln 3 a log a – log b = log b b l 3 1 1 = ln a 2 k - ln a 3 k = ln b 2 l Exercise 10.1 Q1 Integrate the following functions by writing them as partial fractions: x - 82 # 249 ^-x4-x12 h dx b) # ^ x - 221 a) dx h^ x - 3h^ x - 4h 1 x Q2 Find # dx by expressing as partial fractions. = 171.919... 1 c) Evaluate f –1a 2 k . Solve this like a normal equation: 2x – 3 = 12 fi 2x = 15 fi x = 7.5 12 Chapter 2 Algebra and Functions Functions can also be given in several parts (known as 'piecewise' functions). The claim is that the leeks' height increases by 15% every 2 days. dx dx ff Putting these into the formula gives: # x2 sin x dx = –x2cos x – # –2x cos x dx = –x2cos x + # 2x cos x dx ff 2xcos x isn't very easy to integrate, but you can integrate by parts again: Let u1 = 2x and let dv1 du = cos x. A function is many-to-one if some values in the range correspond to more than one (many) values in the domain. f(2.75) = 2.753 – (2 × 2.752) – 5 = 0.67... Find the values of: a) fg(3) b) gf(3) c) f2(5) d) g2(2) Q2 f(x) = sin x for {x : x d R } and g(x) = 2x for {x : x d R }. (y + 2)2 – 4 – 8 = 0 fi y + 2 = ± 12 = ±2 3 so y = –2 ± 2 3 c) At (–2, –2 + 2 3): - 16 + 4 3 dy ^ - 4 + 4 3 h - 3 (- 2) 2 = = dx ^ - 4 + 4 3 h - 2 (- 2) 4 3 3-4 = 3 3-4 3 Rationalise the denominator: = 3 4 =1- 3 3 Q5 a) Putting x = 1 into the equation gives: 1 – y = 2y2 fi 2y2 + y – 1 = 0 fi (2y – 1)(y + 1) = 0 1 So y = –1 (given as the other point) and y = 2 . At t = 0, the particle is moving north-west with a speed of 5 ms–1. Natural number A positive integer, not including 0. Tip: s2 = 4, so s = 2. Tip: You might have to do a bit of rearranging to get u in terms of x. Chapter 8 Integration 181 2. dt db db ? 4 x Tip: Just add along each row of the column as you do with 2D column vectors — don't forget that you've multiplied OP by the scalar '–1' so all its entries will change signs. Set x = 0: –5 = C + D, so C = –5. = 3a T = 3a + 23.402... In Year 1 you saw how to find the area between a curve and the x-axis using integration. So a = 5 and r = 1.15. Q5 Hint: Be careful — make sure the angle you calculate is in the range given for q. If F represents the force needed in N, then F ~ N(13, 1.82). M Magnitude The size of a vector. ff Take all the individual 'bits' from the bottom lines and multiply them together. = 1 + 18x + 135x2 + ... Resolving vertically, taking up as positive: s = 2.8 – 0.6 = 2.2, u = 7.5 sin a, v = 0, a = –9.8 v2 = u2 + 2as 0 = (7.5 sin a)2 + (2 × –9.8 × 2.2) sin2 a = 43.12 ÷ 56.25 sin a = 0.875... ÷ 17.64 = 4.768... b) By using the formula in part a) and a starting value of x0 = 10, find an approximation to a root of the equation f(x) = 0. Example 1 42 N 1.7 m 22° A B mg A non-uniform rod, AB, rests against a rough vertical wall at A. The equation of a straight line is y = mx + c 23 1 fi 4 = (1 × 6) + c fi c = - 4 23 So the equation of the normal is y = x - 4 Q7 y = (7x2 – 3)–4, so let y = u–4 where u = 7x2 – 3 dy du fi = –4u–5 = –4(7x2 – 3)–5, = 14x du dx dy dy du = × = –4(7x2 – 3)–5 × 14x = –56x(7x2 – 3)–5 dx du dx dy 7 When x = 1, = –56(1)(7(1)2 – 3)–5 = –56(4–5) = – 128 dx 1 7 Q8 y = 3 , so let y = 7u - 3 where u = 3 - 2x 3 2x dy 7 4 du 7 fi = – 3

u- 3 = – 3 = –2 4 , du 3^ 3 - 2x h dx dy du 7 14 f ’(x) = × =– 3 4 × –2 = 4 du dx 3^ 3 - 2x h 3^3 3 - 2x h 4 14 f ’(x) could also be written as 3 ^3 - 2x h- 3 . c) m = 15 gives the first three terms 25, 15, 9. dt So the water level falls at a constant rate 1 1 of 120p cm s–1 and 2p = 0.159 cm min–1 (3 s.f.) You don’t actually need to use h in the calculation. x 3 20 Chapter 2 Algebra and Functions For a composite function fg(x), the domain can also be found by putting the range of g(x) into f(x). x PROBLE M SOLVING Q5 A company has designed a logo based on multiples of the sine curve, shown below. So the counters must lie in a straight line. MODELL ING ff n = 250, and p (the probability that the piglet is male) is 0.5. So if X represents the number of male piglets born, then X ~ B(250, 0.5). A force of 15 N acts on A at an angle of 25° to a rough horizontal plane, as shown. N) is greater than the upwards vertical component of the tension in the string (42 sin 22° = 15.733... If the surface is smooth, no frictional force will act on the object. = 38.2° (1 d.p.) cos q = b) |a| = 14 as before JK0ON KK OO b = KK1 OO , |b| = 1 KK OO 0 L P KJK - 1ONO K O b – a = KK - 2OO , |b – a| = KK OO -2 L P Using the cosine rule, ^- 1h2 + ^- 2h2 + ^- 2h2 = 3 2 ^ 14 h + 1 2 - 3 2 6 3 = = 2 # 14 # 1 2 14 14 3 n = 36.6992... 1 ff And as cosec x º sin x , then: dy -1 = = −cosec2 x dx sin 2 x 162 Chapter 7 Differentiation d cosec x = –cosec x cot x. You can work out the probability of B given A, using this formula: P^ A + Bh P^B|Ah = P ^ Ah Tip: The probability of A given B is: P^ A + Bh P(A|B) = P^Bh Here’s an explanation of where this formula comes from... Test statistic A statistic calculated from sample data which is used to decide whether or not to reject the null hypothesis in a hypothesis test. c) U se the iteration formula xn+1 = 6xn + 13 and x0 = 7 to find a to 1 d.p. Using xn+1 = 6xn + 13 with x0 = 7, gives x1 = 6 × 7 + 13 = 7.4161... ff Proof by deduction: using known facts and logic to show that the statement must be true. a cos x y b) Rearranging, cosec y – cot 2 = –2 = –cot y 1 fi cot y = 2 fi tan y = 2 . They are the derivatives of the functions cosec x, sec x and cot x. Example Use the Newton-Raphson method with a starting point of x0 = –2 to find a root of f(x) = x3 – 3x + 5 correct to 5 decimal places. Example 1 1 Let f(x) = x , x > 0. If more than 75 people test positive they will offer a vaccination to the whole population. But if you did use the formula, you’d work out: 5 P(C Ç D) = 20 and 12 P(D) = 20 And so, P(C|D) = P ^C + D h = P^Dh 5 20 12 20 = 5 12 The Venn diagram represents two events, C and D. A third force of 20 N acts at an angle q to the downward vertical. Q10 a) Find any stationary points of the curve y = (x – 2)2(x + 4)3. Earlier, you saw how to use the chain rule in reverse to integrate a function of du the form f’(u). 1 So the full expansion is valid for |x| < 2 , x ≠ 0. Calculating with Vectors Mechanics 269 284 288 Chapter 13 — Hypothesis Testing 1. ff Step 2: For negative x-values, reflect the part of the line where x is positive in the y-axis. So, to avoid adding on an extra rectangle beyond the point b, you need to sum only up to the point b – dx. Friction If a body is in contact with a rough surface, and there is a force acting that could cause the body to move, then friction will act between the body and the surface to try and prevent motion. ff First, resolve parallel to the plane ( ) to find a: Fnet = ma –0.6g sin 30° = 0.6a a = –4.9 ms–2 Tip: The particle is slowing down, so its acceleration is negative (i.e. it’s decelerating). f(–1) = e(2 × –1) + (–1)2 – 3 = –1.86... dV - k = for some constant k, k > 0. For example, if your final answer has to be to 3 d.p., find the y-values to at least 4 d.p. = 6.255 (3 d.p.) b) n = 4 ff For 4 strips, the width of each strip is h = so the x-values are 0, 1, 2, 3 and 4. Area = 0.9015 z 0 23 – 2 From the calculator, F(z) = 0.9015 for z = 1.2901... The rod is held horizontally and in equilibrium by a light wire attached to the rod 0.6 m from A. The model could describe the ball’s motion in three dimensions rather than two, as it’s unlikely to rebound perpendicular to the wall. ff Step 2: Use the chain rule to differentiate terms in y only: dy d d f(y) = f(y) dx dy dx In practice, this means ‘differentiate with respect to y, and stick a ff Step 3: dy on the end’. 286 b) Use your approximation to estimate the probability that there will be more than 130 male piglets born. Here you’re told that the child is dressed as a werewolf, so look in the Werewolf column — the Boys entry is 0.2, and the total is 0.3, so: 0.2 2 P(Boy|Werewolf) = 0.3 = 3 Exercise 1.2 Q1 Events A and B have the following probabilities: P(A) = 0.75, P(B|A’) = 0.4, P(A Ç B’) = 0.45 Are A and B independent? Sketch y = f(x) and y = f –1(x) on the same set of axes, marking the points where the functions cross the axes. Q5 M is a point on the line CD, where C has coordinates (–1, 3, –5), JK 4 NO K O M has coordinates (1, 1, –2) and CD = KKK- 4OOO . 2x + 5 Using the general rule, a = 2 and b = 5 so the integral is: 1 1 dx = 2 ln|2x + 5| + C 2x + 5 # Exercise 2.2 Q1 Find the following: # 19x dx b) a) # 1 7x dx c) # 1 dx 7x + 2 d) # 4 dx 1 - 3x 1 20 Q2 Integrate y = 8x - x with respect to x. a) Find the critical region for a hypothesis test, at the 2.5% significance level, of the claim that the PMCC for the population, r, is greater than zero. Q2 a) Resolving vertically, taking up as positive: s = s, u = 10, v = 0, a = –9.8 v2 = u2 + 2as 0 = 100 – 19.6s s = 5.102... Q8-9 Hint: Sketching the graphs of the composite functions will help you find the ranges and domains. Normal distribution A ‘bell-shaped’ continuous probability distribution where the further from the mean a value is, the less likely it is to occur. So the first non-zero value of t for which the object is moving east is t = 2p seconds. cos–1 (–0.76923...) = 140.3° (1 d.p.) There are two negative solutions in the interval 0 ≤ x ≤ 360° at 140.3° and (360° – 140.3°), so x = 140.3° and 219.7° (1 d.p.) e) cot x = –2.4 fi tan x = –0.41666... There is significant evidence at the 1% level to reject H0 in favour of the alternative hypothesis that Bree’s mean winnings have changed. Q3 A rocket is fired vertically upwards with speed 45 ms–1. Mean weight ff Be able to apply the ff A variable like this can often be modelled by a normal distribution. Q1 For each of the following light rods, find the moment of the force about A. f(0.02) = e0.02 – 2 – 0.02 = –0.0033... a) (x3 – 14x2 + 6x + 11) ÷ (x + 1) b) (x3 + 10x2 + 15x – 13) ÷ (x + 3) c) (2x3 + 5x2 – 8x – 17) ÷ (x – 2) d) (3x3 – 78x + 9) ÷ (x + 5) e) (x4 – 1) ÷ (x – 1) f) (8x3 – 6x2 + x + 10) ÷ (2x – 3) Q2 Hint: This is just another way of asking you to use the formula. ff You can go straight to identifying u to use in the chain rule: dy x x du = 1 y = cot u fi = – cosec2 u = – cosec2 2 and u = 2 fi du dx 2 dy dy du 1 x So = × = - 2 cosec 2 2 dx du dx b) Find Tip: Look back at p.139 for a reminder of the chain rule. Find the distance of the centre of mass of the rod from A, and the magnitude and direction of the reaction force on the rod at A. Newton’s laws of motion Here’s a quick recap of Newton’s three laws of motion: ff Newton’s first law: A body will stay at rest or maintain a constant velocity unless a resultant force acts on the body. The ball’s initial speed is 19 ms–1, at an angle of 28° above the horizontal. a) f(x) b) f(x) x x Q2 For each of the following functions, sketch the graph of the function for the given domain, marking relevant points on the axes, and state the range. x Example 2 Find # x3 ln x dx. dt 1 k dx dx So μ 2 fi = 2 , for some k > 0. Q4 c) y b) 8 4 a) 6 y = |5x + 4| –2 –0.8 1 x You can see from the graph that the minimum possible value of |5x + 4| is 0, and the maximum is when x = 1, i.e. |5x + 4| = |5 + 4| = |9| = 9. – 15 cos 25° = 4a T = 4a + 36.597... 30° 10° a) Calculate the magnitude of the tension in the string. 1#2#3 - 4 # -5 2 - 4 # -5 # -6 3 = 1 + (–4)x + x + x + x + ... The coefficient of friction, m, between the box and the plane is 0.2 and the box accelerates at a rate of 0.8 ms–2. The system P Q is held in limiting equilibrium with P F on the point of sliding down the plane by a second light, inextensible string 20g K 60g attached to Q at one end and 50° the horizontal ground at the other. If X is the number of people ordering a vegetarian meal, then X ~ B(20, 0.15). Q9 f(x) = x2 + 3, x > 0. 304 Chapter 14 Kinematics ux = u cos a, a = 0, t = t Rearrange to make t the subject: x t = u cos a — call this equation 1 ff Resolving vertically, taking up as positive: s = y, uy = u sin a, a = –g, t=t 1 Using s = ut + 2 at2: 1 y = (u sin a × t) – 2 gt2 — call this equation 2 PROBLE M SOLVIN G y x ff t is the same horizontally and vertically, so you can substitute equation 1 into equation 2 and eliminate t: 2 1 x x y = `u sin a # u cos a j – 2 g` u cos a j Tip: t doesn’t appear in the formula you’re given in the question, so you know that’s what you need to be trying to eliminate. x3 + 1 ff In general terms, this is written as: the denominator, e.g. # f l(x) dx = ln f (x) + C f (x) ff This rule won’t surprise you if you remember differentiating ln (f(x)) using the f l(x) . Example 3 Each piglet born on a farm is equally likely to be male or female. x0 x1 x2 x Starting at x0, the next iterations x1 and x2 are getting further away from the root. Tip: Make sure you always link the numbers back to the situation. fi vy = 6.353... Tip: Subscripts x and y are often used to denote the horizontal and vertical components of a vector respectively. chain rule (see p.146) — the derivative with respect to x of ln (f(x)) is Tip: You sometimes see this rule written without the modulus signs, but if it’s possible for f(x) to take a negative value, you need the modulus so you don’t end up trying to take ln of a negative number. ff The graph of X has points of inflection at x = m – s and x = m + s. f(x) 13 17 ff So P(X > 17) = P(X ≥ 17) = 0.047790... Example 2 The volume of interdimensional space jelly, V, in a container is decreasing over time, t, at a rate inversely proportional to the square of its volume. P F 12g Resolving vertically (≠): R = 12g So, using F ≤ mR: F ≤ 0.4 × 12g F ≤ 47.04 N This is just R – 12g = 0 rearranged. Find the value of b. If there were an one number of roots, the sign would appear not to change and you would miss those roots — e.g. on the graph above, f(–1) and f(2) are both positive, but there are two roots between them. These are the kind of questions that usually come up in exams. b) f –1(0) = b) f(x) has domain of 1 ≤ x ≤ 7, which will give a range of (4(1) – 2) ≤ f(x) ≤ (4(7) – 2), 2 ≤ f(x) ≤ 26. ff First you need to find the value of a: a = sin–1 (0.25) = 14.4775...° ff Now resolving perpendicular to the plane ( ): R – 147 sin 60° – 30g cos a = 0 R = 147 sin 60° + 30g cos(14.4775...°) = 412 N (to 3 s.f.) Exercise 1.3 Q1 Each of the following diagrams shows a particle held in equilibrium by two light, inextensible strings. Q9 A ball is hit by a bat from a point on flat, horizontal ground. Another way of finding roots of an equation is with iteration formulas. Let u = 2x2, then y = eu. Differential Equations Exercise 11.1 — Differential equations dN Q1 The rate of change of N with respect to t is . Chapter 10 Vectors 253 Formula Sheet These are the formulas you’ll be given in the exam, but make sure you know exactly when you need them and how to use them. b) One 17-year-old boy is selected from the population at random. ff Taking moments about A (with clockwise being the positive direction), and finding their sum: Sum of moments = (5 × 2) – (4 sin 30° × 5) = 10 – 10 This term is negative because the moment is anticlockwise. ff Remember that weight always acts vertically downwards, so you need to find the components of the weight in these directions. Integration of sin x and cos x In Chapter 7 you learnt how to differentiate sin x and cos x. The modulus of a function, f(x), makes every value of f(x) positive by removing any minus signs. Using a starting value of x0 = 5 find the values of x1, x2, x3 and x4, giving your answers to 4 significant figures. a) Find the critical region for a test of Mary’s claim at the 5% level. Q2 Suppose that there is some even number x for which x2 is odd. Q6 dr 1.5 = = 0.155 cm °C–1 (3 s.f.) dq 2p^1.240...h2 9. 236 ff Now just take the square root of both sides: x = ff This makes the iteration formula: xn+1 = x 3 - 9 as required x n3 - 9 ff This is the iteration formula used in the example on page 235, so if you tried to use it to find a root you’d end up with a diverging sequence and it wouldn’t find a root. T + 10 cos 35° – 9g sin 35° = 0 T = 9g sin 35° – 10 cos 35° Chapter 15 Dynamics 10 N = 78.0 N (to 3 s.f.) ff Resolving parallel to the plane (328 T R – 10 sin 35° – 9g cos 35° = 0 R = 10 sin 35° + 9g cos 35° = 42.4 N (to 3 s.f.) R): 35° 35° 9g Example 5 147 N 60° The diagram shows a block of mass 30 kg at rest on a smooth plane inclined at an angle a to the horizontal. Q5 Use the quotient rule to find the coordinates of x any points of inflection of the graph y = e x . u ms –1 –1 10 m 45° 45 m ff First of all, you need to find the ball’s initial speed, u. Q10 The diagram on the right shows the graph of y = f(x). z f(z) –0.42 0 z Tip: You could use symmetry here — i.e. P(Z > –0.42) = P(Z < 0.42) = F(0.42). 2nd term = 8.1 = Q2 Find the common ratio: r = st 0.9. 9 1 term The first term is a = 9. t = = 0.959 s (to 3 s.f.) 9.149... 4-0 = 2, 2 ff Calculate the corresponding y-values: y= x 6x 2 x3 + 2 x0 = 0 y0 = 0 x1 = 2 y1 = 2.4 x2 = 4 y2 = 1.4545 (4 d.p.) ff Putting these values into the formula gives: # 6x 2 dx ≈ x3 + 2 0 = 4 h 2 [y0 + 2y1 + y2] 2 2 [0 + 2(2.4) + 1.4545] Tip: If you have to round the y-values, make sure you leave enough decimal places. × tan 5°) + 0.02 4.9 × (0.958...)2 = (U cos 5°)2 × ((0.958...)tan 5° + 0.02) 4.9 × (0.958...) 2 U2 = = 43.666... Q2 Prove that if x2 is odd, then x must be odd. – 15 cos 25° = 4a –36.597... Tip: You might be asked to use different values for g (e.g. 10 ms–2 or 9.81 ms–2), so read the question carefully. N), so potential movement is downwards at A. Domain –1 x2 + 1 1 0 2 1 Domain Range ! x Range 5 2 –2 1 –1 4 1 2 The mapping shown here is not a function, because a value of x in the domain can map to more than one value in the range. fi s = 3.56 (3 s.f.) When you’re asked to find the mean and the standard deviation, the method is a little bit more complicated. The set of natural numbers has the notation N. x3 - 9 ff Start by isolating the x2 term: x2 = x3 – 9 Tip: This shows why you’ll never just be given an equation and told to find a root by first making an iteration formula. Starting with a, F(z) = 0.1 for z = –1.281... ff If x is odd, then you can write x as 2k + 1, where k is an integer. (1 + x)–1 = 1 – x + x2 + ... b) Find the velocity when t = 2.5: u = (–0.6i + 3j), v = v, a = (0.1i + 0.5j), t = 2.5 v = u + at v = (–0.6i + 3j) + 2.5(0.1i + 0.5j) v = –0.6i + 3j + 0.25i + 1.25j = –0.35i + 4.25j 464 Answers N b 45° b The initial velocity, u, can be written as a vector –bi + bj, with a magnitude of 5 ms–1. 1 c) Rearranging, f(x) = x – x – 2, and the roots are at f(x) = 0 1 f(2.4) = 2.4 – 2.4 – 2 = –0.016... 1 d = (0 × 2) + (2 (4.385... x 5 b) Use the iterative formula xn+1 = 2 + 2 with starting value x0 = 2 xn to find x5 to 1 decimal place. Given that x ≥ 0 and y ≥ 0, 1 1 show that the velocity vector, v, at time t, is given by v = 3t 3 i + 2t 2 j. Functions inside brackets, denominators of fractions and square roots are often good places to start. Resolving Forces ff Be able to resolve You’ve seen problems with more than one force acting on an object before, but things get more complicated when these forces are acting at different angles. Find the angle between the 10 N force and the slope of the plane using alternate angles. ff Be able to solve problems involving connected particles, friction forces and inclined planes. 1 Q4 a) = (1 – 4x)–2 ^1 - 4xh2 - 2^ - 2 - 1h 2 = 1 + (–2)(–4x) + 1 # 2 (–4x) - 2^ - 2 - 1h^ - 2 - 2h + (–4x)3 + ... The rate of increase of the number of signatures (y) he’s gathered dy can be represented by the differential equation = k(p – y), dt where p is the population of his town and t is the time in days since the new campaign was launched. Finding Area using Integration 204 8. = 0.958 m (3 s.f.) b) Resolving horizontally, taking right as positive: s = 0.958..., u = U cos 5°, a = 0, t = t 1 s = ut + 2 at2 0.958... Integration by substitution is a more general version of this dx technique. Exercise 1.1 Q1 f(x) = x3 – 5x + 1 Show that there is a root of f(x) = 0 in the interval 2 < x < 3. C Arccos The inverse of the cosine function, also written as arccosine or cos−1. ms–1 vy is negative because the projectile is travelling downwards when it hits the ground. ff It also means that the sum of the moments on the body about any point is zero. They’re especially useful for things like cos2 x, sin2 x and sin x cos x. So, when t = –2: dy 3^^ - 2h2 - 1h 3 (3) = = =-9 -4 4 dx 2^ - 2h Chapter 7 Differentiation 169 c) Find the coordinates of the turning points. T2 cos 30° – 8g = 0 T2 cos 30° = 8g T2 = 8g ÷ cos 30° = 90.5 N (to 3 s.f.) ff Resolving horizontally (Æ): T1 – T2 sin 30° = 0 T1 = T2 sin 30° = 90.5 sin 30° = 45.264... 1 f(2.5) = 2.5 – 2.5 – 2 = 0.1 There is a sign change (and the function is continuous in this interval) so there is a root in this interval. Find the speed and direction of projection of the ball. c) Suggest how the model used to calculate the bullet’s motion in air should be refined to calculate its motion in the water. The integral you found will be the same but without the constant of integration — it’ll have limits instead. Does the function allow negative values that don’t make sense? Chapter 9 Numerical Methods 3 4xn - 1 with x0 = 2 5. 4 (4 + 3x)–1 is valid if |x| < 3 . Implicit Differentiation For equations that you can’t write in the form y = f(x), you need to use implicit differentiation. ff It has height (b – a) and parallel sides of length f(a) and f(b). ff Resolving in the x-direction (Æ): 5 – Q cos 70° – 4 cos 60° = 0 Q cos 70° = 3 3 fi Q = cos 70° = 8.77 N (to 3 s.f.) ff Resolving in the y-direction (≠): P + 4 sin 60° – Q sin 70° = 0 3 P = cos 70° sin 70° – 4 sin 60° fi P = 4.78 N (to 3 s.f.) Example 4 y F 17 N 20° 12 N P 60° 30° 6N x 18° The diagram shows the forces acting on particle P. 3 1 1 p p c) arcsin 2 = 6 , so cos barcsin 2 l = cos 6 = 2 . Q5 A particle with a mass of 5 kg rests on a smooth plane inclined at 10° to the horizontal, as shown. F 3m B ff Both the lamina’s weight and F act vertically, so the perpendicular distance from each force to the pivot is just the horizontal distance. du dv and into the integration by parts formula: dx dx u = 2x fi du dv =2 = ex fi v = ex dx dx Tip: You don’t always need to work out both possible versions. Example 9 a) Show that x3 − x2 − 9 = 0 can be rearranged into x = . So: 1 # cos x sin 2 x dx = 3 sin 3 x + c ff So the whole integral is: # cos 3 x dx = # cos x dx - # cos x^sin xh 2 dx = sin x - 1 sin 3 x + C 3 p 3 1 6 sin 3x cos 3x + tan 2 2 x + 1dx. Given that a = 45°, find the speed of the ball as it passes over the tree. Integrating f (u) 195 dx 6. ff Be able to find the unit vector in the direction of any vector in three dimensions. Q3 Integrate the function y = Q4 Find # 2ex(1 + ex)3 dx, using the substitution u = 1 + ex. If x can take any real value, it’s usually written as x d R . ff Start with x1 = 2.5 3 - 9 = 2.5739... T ff First resolve forces vertically (forces ‘up’ = forces ‘down’): 4m d M T = 30 + 45 = 75 N 30 N 45 N Tip: The line of action of T passes through M, so the moment of T about M is zero. Q13 Hint: You won’t need your calculator for the trig here but you will need to remember your common angles. Chapter 14 Kinematics 299 The constant acceleration equations To solve problems involving projectile motion, you will need to make certain modelling assumptions: Tip: An external force is any force acting on a body other than the body’s weight — e.g. air resistance, friction, tension etc. Don’t forget, you’ll often need to relate your answer back to the question when you’ve finished calculating. ff You saw in the previous example that if a2 is even, then a must be even as well. E e An irrational number for which the gradient of y = ex is equal to ex. You can see from the units (m2 s–1) that it’s a change in area (m2) with respect to time (s). + yn - 1 + yn @ This formula sums the areas of the rectangles which meet f(x) with their right hand corner. dx The gradient of the normal is 2. Since for fg(x) the input into f is g(x) (i.e. x + 5): x + 5 ≥ 0 fi x ≥ –5. 3 dx 2 x u = 1 − 2ln x fi du dv 2 = − x and v = x3 fi = 3x2. Tip: The length of time the projectile is in the air is usually referred to as the ‘time of flight’. Find: 10 a) the particle’s maximum height above the ground, b) the speed of the particle as it hits the ground, c) the direction of motion of the particle when it hits the ground. 2 - x 2 looks like the most awkward bit to integrate, so that might be a good choice for the substitution. dx b) Show that the acceleration of the particle at this time is 384i – 4j. or t = 1.558... dt dy dy So μ (y – l) fi = –k(y – l), for some k > 0. But this expands to 1 1 3 ln|x| + 3 ln 3 + C 1 and the 3 ln 3 becomes part of the constant of integration. R ff Resolving parallel to the plane: F = 3.2g sin q ff Resolving perpendicular to the plane: F 3.2g R = 3.2g cos q ff The block is on the point of moving, so, using F = mR: 3.2g sin q = 0.75 × 3.2g cos q 3.2g sin q 3.2g cos q = 0.75 tan q = 0.75 q = 36.9° (to 3 s.f.) Exercise 2.1 Q1 A particle of weight W rests on a rough horizontal surface with coefficient of friction m between them. (18g × 1.2) + 0 = Mg × 0.8 fi M = (18 × 9.8 × 1.2) ÷ (9.8 × 0.8) = 27 kg ff Resolving vertically: RA + RB = Mg + 18g 0 + RB = 27g + 18g fi RB = 441 N Laminas A lamina is a flat 2D object (its thickness can be ignored). Using u = x , x = 9 fi u = 9 = 3 and x = 16 fi u = 16 = 4 16 4 4 4 So # dx = # 2u dx 9 3 u^9u 2 - 4h x ^9x - 4h 2 = # 3 4 8 ^9u 2 - 4h dx This function still needs some work before it can be integrated. Example 1 Functions f(x) Each value of x is mapped to a single value of f(x). Tip: You need to check that the i component of velocity is non-zero for the value of t you’ve found. ff Integrate each bit in turn: Tip: These three integrals should really all have a constant of integration on the end, but we’ll just add a combined constant of integration when we do the final integration. ff Use the diagram to find the point of inflection. 5% 2.5% 1% 0.5% two-tailed 20% 10% 5% 2% 1% n 4 0.8000 0.9000 0.9500 0.9800 0.9900 ... ff Be able to identify the domain and range of a given function. PROBLE M SOLVIN G y 2 y = sin x + 1 B 1 y = cos x + 1 0 x 3 2 Find out where the graphs meet by solving sin x + 1 = cos x + 1 sin x fi sin x = cos x fi cos x = 1 fi tan x = 1 5p p p 5p fi x = tan–1 (1) = 4 , 4 . The maximum points are at (–p, 1) and (p, 1). dq 9. More Differentiation 162 8. ff Understand and use the relationship F ≤ mR. ff When a moving object is acted on by a frictional force, friction is limiting, and the frictional force (F) is at its maximum value: F = mR. Exercise 2.1 So the critical region is Z > 2.576 or Z < –2.576. Tip: Any constant can be expressed as a logarithm because the range of the ln function is f(x) ! R . Step 1: Draw f(x). The range of f –1(x) is the domain of f(x): f –1(x) > 3. du : dx du x x = = -u dx 2 - x2 u fi u du = –x dx fi - x du = dx ff Now differentiate to find ff Now substitute what you’ve got into the original integral: # 3x 2 - x 2 dx = # 3x × u × - ux du # –3u2 du = = –u3 + C = –^ 2 - x 2 h + C 3 Tip: The chain rule is used here to differentiate. or t = –0.245... You’ll always be told what n is (it could be 4, 5 or even 6). ff Each pair of branches will show either M and its complement M′, or N and its complement N′. 318 Chapter 14 Kinematics ING Q3 A body is moving in a plane. There’s not too much to learn here — it’s mostly just applying what you already know. x-1 x+4 a) Find the equation of the tangent to the curve when x = 5 in the form ax + by + c = 0 where a, b and c are integers. State any parameters. 27x 2 135x 3 3x = 1 + 2 + 8 + 16 + ... ff Next, consider the graph of the composite function over the domain and range of the original functions: fg(x) As g(x) is restricted to x > –3, so the domain of fg(x) is also restricted to x > –3. – 20g = 216.63... ff Be able to integrate linear transformations 1 of ex and x , i.e. functions of the 1 form eax + b and . Q4 Hint: Start by rewriting everything as powers of 2 or x if you’re struggling. dx PROBLE M SOLVIN G ff Rearrange the equation to get rid of the arcsin by taking sin of both sides: Tip: Sin and arcsin are inverses of each other, so they cancel out. a) Draw a Venn diagram to show the event that it is sunny (S) and the event that she goes to the park (P). Draw an updated two-way table showing the probabilities of selecting a child from each category. Putting this into y = mx + c gives: 8 1 –2 = 3 (2) + c fi c = - 3 So the equation of the normal is 3y = x – 8. Q3 a) # 6 - 3 dx = # 3 dx = 3 ln x + C x x x = ln x 3 + C = ln x 3 + C 5 b) #4 6x - 3x dx = 6ln x3 @ 54 = 6ln - 6ln = 3 ln 5 - 3 ln 4 = 3 ^ln 5 - ln 4h = 3 ln a 5 k 4 125 You could also have written your answer as ln 64 . You always start by defining a normally-distributed random variable to represent the information you’re given. More Differentiation Exercise 7.1 — Differentiating cosec, sec and cot dy = – 2cosec (2x) cot (2x) dx dy dy du b) = × = (2cosec x)(–cosec x cot x) dx du dx = –2cosec2 x cot x dy dy du c) = × = –7cosec2 (7x) dx du dx dy dy du d) = × = (7cot6 x)(–cosec2 x) dx du dx = –7cot6 x cosec2 x dy dv du e) = u +v = [x4 × (–cosec2 x)] + [cot x × 4x3] dx dx dx = 4x3cot x – x4cosec2 x = x3(4cot x – xcosec2 x) dy dy du f) = × = 2(x + sec x)(1 + sec x tan x) dx du dx dy dy du g) = × = [–cosec (x2 + 5) cot (x2 + 5)] × 2x dx du dx = –2x cosec (x2 + 5) cot (x2 + 5) dy dv du h) = u +v = [e3x × sec x tan x] + [sec x × 3e3x] dx du dx = e3x sec x (tan x + 3) dy dy du i) = × = 3(2x + cot x)2(2 – cosec2 x) dx du dx du - dv v u (x + 3) sec x tan x - sec x Q2 a) f’(x) = dx 2 dx = v (x + 3) 2 dy du 1 lb - 1 1 b) f’(x) = × = bsec 1 x tan x x2 du dx 1 1 sec x tan x = x2 dy du 1 1 n c) f’(x) = × = ^sec x tan x hd 2 × du dx x sec x tan x tan x = = 2 x cos x 2 x dy du Q3 f’(x) = × du dx = 2(sec x + cosec x)(sec x tan x – cosec x cot x) 1 tan x Q4 f(x) = x cot x = x du - dv v u x sec 2 x - tan x f’(x) = dx 2 dx = v x2 dv du Q5 f’(x) = u +v = [ex × (–cosec x cot x)] + [cosec x × ex] dx dx = excosec x (1 – cot x) dv du Q6 f’(x) = u +v = [e3x × (–4 cosec2 4x)] + [cot 4x × 3e3x] dx dx = e3x(3 cot 4x – 4 cosec2 4x) Q1 a) The chain rule was used here to differentiate cot 4x — use y = cot u and u = 4x. It hits a target 0.5 seconds after being fired. Q4 Let the variable t represent time. They wouldn’t ask you if there’s a root if the function wasn’t continuous in this interval, but it’s worth saying anyway just to keep your answer ‘strictly true’. Step 3: Reflect f(x) in y = x to get f–1(x). P(X ≤ 78) = 0.022750... The R Addition Formulas 9. 1 So the question tells you that S¥ – S10 < 100 S3 99 99 fi 100 S3 – S10 < 0 fi 100 S3 < S10 a (1 - r10) Then: 0.99(S¥) = 0.99a < = S10 1 r (1 - r) You can cancel and keep the inequality sign because the series is convergent so |r| < 1 fi 1 – r > 0, and you know a > 0 from the question: fi 0.99 < 1 – r10 fi r10 < 0.01 fi |r| < 10 0.01 fi |r| < 0.631 (to 3 s.f.) Q9 Using the formulas for S¥ and S4 4 9 a (1 - r) a - = 8 # 1 r 1-r Cancelling (1 – r) and a gives 8 = - 4 4 =1 & r= 4 1 9 1 r & r 9 9 3 So, given that r is positive and real, r = 1 = . b) The total distance she runs in 20 days is the sum of the first 20 terms of the sequence. b) Hence show that the velocity of the particle can expressed as v = 70.1ti + 5^e -2t - 1h j A ms–1. For the first bit: fi y = 2u2 where u = cos x ff For the second bit: y = sin u where u = 2x fi dy du = 4u = 4 cos x, = –sin x du dx dy du = cos u = cos 2x, =2 du dx ff Putting it all back into the chain rule formula: dy = [(4 cos x) × (–sin x)] + [(cos (2x)) × 2] dx = 2 cos (2x) – 4 sin x cos x ff From the target answer in the question it looks like you need a sin (2x) from somewhere, so use the double angle formula for sin (page 75): sin (2x) º 2 sin x cos x fi 4 sin x cos x º 2 sin (2x) fi dy = 2 cos (2x) – 2 sin (2x) dx = 2(cos (2x) – sin (2x)) as required. Chapter 16 Moments 349 Example 2 20 N 60° O Tip: ‘Find the turning effect’ means the same as ‘find the moment’. Q5 A block of mass 1 kg is sliding down a rough plane of length 2 m, which is inclined at an angle of 50° to the horizontal. ff Rewrite the original integral in terms of u and du. seconds after being thrown: 0.314... a b) Suppose that 3 is rational, so 3 = b for some non-zero integers a and b that share no common factors. Q5 The heights of a population of 17-year-old boys are assumed to follow a normal distribution with a mean of 175 cm. So the ball is 2.5 m above the ground 0.261... In set notation, this is {x : 1 ≤ x ≤ 5}. E.g. 2, 6, 10, ... Being a bit clever about it saves you a lot of effort though, so always try to use the simplest common denominator possible (the LCM). g (x + h) - g (x) f (x + h) - f (x) g (x) f (x) d c f (x) m = h h lim g (x + h) g (x) dx g (x) h"0 ff The green bits on top are the definition of f’(x) and g’(x). Find p and q, in simplified surd form. Contributors: Andrew Ballard, Jean Blencowe, Katharine Brown, Jane Chow, Michael Coe, Claire Creasor, Margaret Darlington, Anna Gainey, Stephen Green, Dave Harding, Barbara Mascetti, James Nicholson, Charlotte O’Brien, Andy Pierson, Rosemary Rogers, Mike Smith, Janet West. ^1 + 6xh4 -1 So = (1 + 6x)4^9 + 4xh 2 + 9 4x 1 2x 2x 2 ≈ (1 + 24x + 216x2)a 3 - 27 + 81 k 1 1 2x 2 2x 1 2x ≈ 3 – 27 + 81 + 24xa 3 - 27 k + 216x2a 3 k 2 2 16x 1 2x 2x = 3 – 27 + 81 + 8x – 9 + 72x2 1 5690x 2 214x = 3 + 27 + 81 1 2. represents the total probability of the random variable taking one of its possible values. R 30 N F 4g ff Resolving horizontally: F = 30 N ff Resolving vertically: R = 4g ff The block is on the point of moving, so friction is limiting: F = μR 30 = μ × 4g Tip: m is usually given correct to 2 decimal places. Then, P(T < a) = 0.2. P(T < a) = 0.2 for a = 49.2 s (3 s.f.) Q3 a) Let V = volume of vinegar in ml. Q9 sec2 x = 3 + tan x fi (1 + tan2 x) = 3 + tan x fi tan2 x – tan x – 2 = 0 fi (tan x – 2)(tan x + 1) = 0 So tan x = 2 or tan x = –1. 12 ms–1 q v 5 ms–1 The vertical component of velocity is negative, so this component is acting downwards. In the following questions you can choose which method to use. du This result’s quite difficult to grasp — but after a few examples it should make complete sense. Example 1 a) If y = 3e5x and dy dx = 2, work out when x = –1. |f(x)| = −f(x) when f(x) < 0. º You know x > 2, so x – 2 is positive and the modulus can be removed. ff It’s a product of two functions, so start by identifying ‘u’ and ‘v’: u = e2x and v = 2x - 3 ff Each of these needs the chain rule to differentiate: du dv = 2e2x and = dx dx 1 2x - 3 ff Put it all into the product rule formula: dy dv du de 2x × =u +v = dx dx dx 1 n + (2x - 3 × 2e2x) 2x - 3 Tip: The chain rule bit has been done all in one go here to save time, but if you need to, do it in steps just to make sure nothing goes wrong. Implicit Differentiation 173 Chapter 8 — Integration 1. MODELL ING Q6 20 ms–1 45° 4m 306 Chapter 14 Kinematics ING An object is fired from a point 4 m directly above flat, horizontal ground. 5 + 3b Q5 The graph of the curve y = f(x) passes through the point (1, 2). mg R P F mg R Here, a force, P, is acting horizontally on the object, which could potentially cause the object to move. ff Step 4: Rearrange your answer to get it in a nice form — you might be asked to find it in the form y = h(x). v = 13.5 ms–1 (3 s.f.) Q5 a) 60 N F 40° A 8g 20 cm T b) Using 1 from part a): a = 5T – 0.7g cos 40° – g sin 40° = (5 × 2.363...) – 0.7g cos 40° – g sin 40° = 0.2648... Since 0.994 > 0.7067 the result is significant. To write this in the form ax + by + c = 0 where a, b and c are integers, multiply by 12 and rearrange. ff Differentiate f(x): f’(x) = 6 – 2x. Q2 Give GH and HG as column vectors, where JK 2 NO KKJ- 1NOO KK OO K O OG = KK- 3OO and OH = KK 4 OO . Tip: You first saw the normal distribution on page 269. The question gives each component of acceleration as a differential equation. String A thin body, usually modelled as being light and inextensible. Resolving perpendicular to the plane: R + 5 sin 20° = W cos 50° R = 4F, so: 4F + 5 sin 20° = W cos 50° 1 Resolving parallel to the plane: F + W sin 50° = 5 cos 20° F = 5 cos 20° – W sin 50° 2 Substituting 2 into 1 , 20 cos 20° – 4W sin 50° + 5 sin 20° = W cos 50°, 20 cos 20° + 5 sin 20° W= = 5.53 N (to 3 s.f.) 4 sin 50° + cos 50° 2. Given that the coefficient of friction between each particle and the surface is 0.2, find the magnitude of the acceleration of A. 1 So the full expansion is valid if |x| < 6 . = 7.509... The coefficient of friction between the particle and the surface is 0.5. a) Will the 50 N force cause the particle to move? hyp adj 1 11 = - 61 . ff Be able to formulate ff Before you even think about solving them, you have to be able to set up ff Be able to find MODELL ING ff The question tells you that you need to write a differential equation, so you know there’ll be a derivative term. dy cos p + sin p = = –1 2 (1) + cos p dx 1 = –p + c fi c = 1 + p So the equation of the tangent at (p, 1) is y = 1 + p – x. Find the following probabilities: a) P(X Ç Y) b) P(Y′ Ç Z′) c) P(Y|Z) d) P(Z′|Y′) e) P(Y|X′) Q5 Amena sometimes goes to the park, depending on the weather. Q2 Joe records the ages of people going to a cinema on a Sunday evening. Q2 |x| = 5 fi x = 5 or x = –5 If x = 5, |3x + 2| = |15 + 2| = 17 If x = –5, |3x + 2| = |–15 + 2| = |–13| = 13 Q3 a) y y = |2x + 3| 3 –1 y=x–1 1 x b) You can see from the graph that the lines do not intersect, so there are no solutions to |f(x)| = g(x). 4e 8 + 144 c) Show that these two lines intersect when x = . ff

the x-axis. H1: m < a or H1: m > a (one-tailed test) or H1: m ≠ a (two-tailed test) ﬀ State the significance level, a — you'll usually be given this. Q1 Using the Newton-Raphson method, give an iteration formula for finding the roots of each of these functions: a) f(x) = 5x2 – 6 b) g(x) = e3x – 4x2 – 1 c) h(x) = sin x + x3 – 1 Q2 Use the Newton-Raphson method to find a root of x4 – 2x3 – 5 = 0 to 5 s.f., starting with x0 = 2.5. Q3 Use the Newton-Raphson method to find the negative root of f(x) = x2 – 5x – 12 to 5 s.f. Use x0 = –1 as your start value. So for Example 1 on the previous page: (x4 – 3x3 – 3x2 + 10x + 5) ÷ (x – 2) = x3 – x2 – 5x remainder 5. Example 4 Prove that the derivative of tan x with respect to x is sec2 x. ﬀ Be able to solve problems involving rigid bodies under the influence of a frictional force. b) Given that in 4 weeks the number of mice in a particular field has risen from 20 to 30, find the length of time, to the nearest week, before the field is over-run with 1000 mice. du dv b) u = x3, v = (5x + 2)4 fi = 3x2, = 20(5x + 2)3 dx dx dy dv du = u +v = [x3 × 20(5x + 2)3] dx dx dx + [(5x + 2)4 × 3x2] 3 3 = 20x (5x + 2) + 3x2(5x + 2)4 = x2(5x + 2)3[20x + 3(5x + 2)] = x2(5x + 2)3(35x + 6) du dv c) u = x3, v = ex fi = 3x2, = ex dx dx dy dv du = u +v = x3ex + ex 3x2 = x2ex(x + 3) dx dx dx du dv d) u = x, v = e4x fi = 1, = 4e4x dx dx dy dv du = u +v = 4xe4x + e4x = e4x(4x + 1) dx dx dx 2 2 du dv e) u = x, v = e x fi = 1, = 2xe x dx dx 2 2 2 dy dv du = u +v = x × 2xe x + e x = e x (2x2 + 1) dx dx dx du dv f) u = e2x, v = sin x fi = 2e2x, = cos x dx dx dy dv du = u +v = e2x × cos x + sin x × 2e2x dx dx dx = e2x(cos x + 2 sin x) 1 Q3 a) u = x3, v = (x + 3) 2 du dv 1 + - 12 fi = 3x2, = 2 (x 3) dx dx du :x 3 × 1 (x + 3) - 12 D 7 + 12 +v = + (x 3) × 3x 2 A 2 dx dx 3 1 x 2 2 = 1 + 3x (x + 3) 2 (x + 3) 2 1 du dv - 1 - - 32 b) u = x2, v = (x - 7) - 2 fi = 2x, = 2 (x 7) dx dx dv du f'(x) = u +v dx dx 3 1 1 2 = ;x × b - 2 l × (x - 7) - 2 E + 7(x - 7) - 2 × 2x A x2 2x = 3 + 2^ x - 7 h x-7 f'(x) = u c) y = (x + 1)2(x2 – 1) = (x + 1)2(x + 1)(x – 1) = (x + 1)3(x – 1) x2 – 1 is the difference of two squares. Examples a) Simplify: 2 - 3 x - 1 3x + 2 ﬀ Multiply the denominators to get the common denominator: (x – 1)(3x + 2) ﬀ Multiply the top and bottom lines of each fraction by whichever factor changes the denominator into the common denominator: 2 × (3x + 2) 3 × (x - 1) (x - 1) × (3x + 2) (3x + 2) × (x - 1) ﬀ All the denominators are the same — so you can just subtract the numerators: 2 (3x + 2) - 3 (x - 1) 6x + 4 - 3x + 3 3x + 7 = = (3x + 2) (x - 1) (3x + 2) (x - 1) (3x + 2) (x - 1) b) Simplify: 2y 1 - x + x^ x + 3h y 2 ^ x + 3h y ﬀ The common denominator is: xy (x + 3) 2 The individual 'bits' here are x, (x + 3) and y, but you need to use y2 because there's a y2 in the denominator of the second fraction. ﬀ Again, turn the question into a probability for X. – 16.055... Q1 For each graph below, draw a diagram to show the convergence or divergence of the iterative sequence for the given value of x0, and say whether it is a convergent or divergent staircase or cobweb diagram. Example 1 Find the following: a) Tip: Use one of the cos double angle formulas when you've got a cos2 x or a sin2 x to integrate. x2 dv = 2x dx (x 2) b 1x l - (ln x) (2x) dy x - 2x ln x 1 - 2 ln x = = = So dx x4 x4 x3 dy ﬀ The stationary point occurs where = 0 (i.e. zero gradient), dx so this is when: 1 1 - 2 ln x 1 = 0 fi ln x = 2 fi x = e 2 x3 ﬀ To find out whether it's a maximum or minimum, d2y 1 - 2 ln x differentiate using the quotient rule to get . The estimate is more than the real area. = 1.62 s (3 s.f.) Q7 First find the ball's vertical displacement from its starting point when it passes through the first target. That's because the area under a graph at a single point is zero. gets you closer to the root, as shown: y = f(x) y y = f(x) y The orange line shows the tangent to f(x) at x0. ﬀ From your calculator, P(Y > 130) = 0.264 (3 s.f.) Chapter 12 The Normal Distribution Example 4 a) Only 23% of robin chicks survive to adulthood. dx Sometimes integration by parts leaves you with a function for v Example 1 Find # x2 sin x dx. pn – 1 pn + 1) ÷ p1 = p2 p3... ﬀ Magnitude is always a positive, scalar quantity. – 0.634... Q5 y = arccos x fi cos y = x Differentiating: dy dy 1 –sin y = 1 fi = - sin y dx dx 1 = sin 2 y 1 = 1 - cos 2 y dy 1 Using cos y = x: =dx 1 - x2 Q6 y = arctan x fi tan y = x Differentiating: dy dy 1 sec2 y = 1 fi = dx dx sec 2 y Using the identity sec2 y º 1 + tan2 y: dy 1 = dx 1 + tan 2 y dy 1 Using tan y = x: = as required. 2 7y0 + 2 ^ y1 + y2 + y3 + y4 + y5h + y6 A 2 bpl 6 3 3 = E ; 2 0 + 2 (0.5 + 2 + 1 + 2 + 0.5) + 0 = p 60 + 2 ^2 + 3 h + 12 2 ^2 + 3 h m = pc 12 p ^2 + 3 h = as required 6 p p b) It's an under-estimate because between - 2 and 2 the curve of the graph y = cos x is concave, and so the top of each trapezium lies below the curve. Q3 The rate of depreciation of the amount (£A) a car is worth is directly proportional to the square root of A. 3 Q8 3 sin q – 2 cos q º R sin (q – a) º R sin q cos a – R cos q sin a 3 fi R cos a = 3, R sin a = 2 1 tan a = 2 fi a = 0.464 (3 s.f.) 3 5 3 2 45 R = 32 + b 2 l = 4 = 2 3 5 3 fi 3 sin q – 2 cos q º 2 sin (q – 0.464) 3 5 So solve 2 sin (q – 0.464) = 3 in the interval –0.464 ≤ (q – 0.464) ≤ 5.819. b) Find the gradient of C when t = p. dx dx with, you guessed it, the chain rule: dy dx du x = tan u, u = 3y fi = sec2 u = sec2 (3y) and =3 du dy dx dx du = × = 3 sec2 (3y) dy du dy ﬀ First find Tip: See p.142 1 for more on . Q3 A stone is thrown with velocity (6i + 9j) ms–1 from a window 2.5 m vertically above flat, horizontal ground. ﬀ Divisor — this is the thing you're dividing by. Trapezium rule A way of estimating the area under a curve by dividing it up into trapeziumshaped strips. Show this as a differential equation. –a y = f (a x) y 1 x 0 –1 Tip: It might be easier to think of a stretch by a scale factor 0 < a < 1 as a 'squash' — but make sure you use the word stretch in the exam. A y r=1 × × B y r = –0.596 × ××× ××× x C y ×× ×× × x ×× × r = 0.023 ×× × × ×× × x ﬀ In A, there are only two data points, so r would always be +1 or –1 (you can always draw a straight line between two points). Example 1 T 45° A 0.6 m A uniform rod, AB, of mass 2 kg and length 1.2 m, is freely hinged on a vertical wall. For example, when flipping a coin, you assume that each side is equally likely to come up, so the probability is 0.5 for both. 2 Chapter 1 Proof b) Hence prove that 3 is irrational. Constant of integration A constant term coming from an indefinite integration representing any number. Resolving vertically: R = W, so using F = mW: 12 N a) F = 0.25 × 5 = 1.25 N 2g b) F = 0.3 × 8 = 2.4 N c) F = 0.75 × 15 = 11.25 N Q2 Resolving parallel to the plane: 12 = F + 2g sin q F = 12 – 2g sin q Resolving perpendicular to the plane: R = 2g cos q The object is on the point of moving, so F = mR. r = –0.8925 is inside the critical region so there is sufficient evidence to reject H0 and to support the alternative hypothesis that kidney function and weight are correlated. Since the leeks only reach a height of 25 cm, the claim on the compost is not justified. So (1 + 4x)4^1 - 5xh6 1 5x 125x 2 = (1 + 16x + 96x2 + ...)(1 – 6 – 72 + ...) 125x 2 5x = 1(1 + 16x + 96x2) – 6 (1 + 16x) – 72 (1) + ... ﬀ Resolving horizontally, taking right as positive: s = s, u = 38.6, a = 0, t = 4 1 Using s = ut + 2 at2: s = 38.6 × 4 = 154.4 m c) Find the position vector of the stone relative to O when it is at its highest point. dx dx dx 3y 2 - 2 3y 2 y 1e0 At (0, 1) the gradient is =1 3 3 (1) 2 d y d y dy 2y - 3x 2 Q4 a) 3x2 + 2y – 2y – 2x =0 fi = dx dx dx 2y - 2x b) Putting x = –2 into the equation gives: –8 + y2 + 4y = 0 Complete the square to solve... So there are two equations to solve: 1 4x – 1 = 2x + 3 fi x = 2 1 2 –(4x – 1) = 2x + 3 fi x = - 3 1 So the two solutions are x = 2 and x = - 3 . ﬀ Differentiate y and u: y = u3 fi dy = 3u2 = 3 sin2 x du and du = cos x dx ﬀ Then put it all into the chain rule formula: dy dy du = × = 3 sin2 x cos x dx du dx When you're differentiating trig functions, you'll sometimes be asked to rearrange your answer to show it's equal to a different trig function. = 3.70 (2 d.p.) So m = 3.7 and s = 0.2 20 - m c) P(X < 20) = 0.7881 fi P a Z < s k = 0.7881 F(z) = 0.7881 for z = 0.80 (2 d.p.) 20 - m fi = 0.8 fi 20 – m = 0.8s 1 s 14 - m P(X < 14) = 0.0548 fi P a Z < s k = 0.0548 f(x) 0.025 0.025 Area = 0.95 6.45 20.17 x So, P(X < 20.17) = 0.95 + 0.025 = 0.975 20.17 - m k = 0.975 fi PaZ < 3.5 F(z) = 0.975 for z = 1.96 (2 d.p.) 20.17 - m fi = 1.96 fi m = 20.17 – 1.96 × 3.5 = 13.31 3.5 There are different ways you could do this question. a) Draw a diagram to show this information, given that gravity acts in the negative y-direction. Tip: The point O is the pivot point. Don't be put off by the differentiated term — use it the same way as any other iteration formula. dx Given the derivative of ln x, integration is the opposite of differentiation, so: Integrate f(x) with respect to x, given that ln x Differentiation ln x + C So Tip: You'll be working with logs all the time when integrating 1 functions of the form x — it'll help to remember the log laws: log (ab) = log a + log b a log ` b j = log a - log b log ^a bh = b log a # Integration 1 x 1 x 1 x dx = ln x + C 1 So now we have a general result for integrating x : # 1 x dx = ln |x| + C Notice that this result uses |x| instead of just x. ﬀ Using v = u + at: 0 = 0.5i – 1.2j + 4a fi a = (–0.125i + 0.3j) ms–2 b) Find the position vector, r, at the point that the puck comes to rest. Either a binomial model or a normal approximation is OK in this case, since n is sufficiently large. Method 1 — using the formula There's a handy formula you can use to do algebraic division: A polynomial f(x) can be written in the form: f(x) º q(x)d(x) + r(x) where: q(x) is the quotient, d(x) is the divisor, and r(x) is the remainder. Q9 Hint: You should be able to write down the values of sin q and cos q without a calculator. Q6 a) Find the range of the function h(x) = sin x, 0° ≤ x ≤ 180°. So resolving east: u = –0.6, v = 0, a = 0.1, t = t v = u + at 0 = –0.6 + 0.1t fi t = 6 s Check that the velocity in the j-direction is positive at this time (otherwise the toy could be moving due south). ﬀ This is a one-tailed test and values more likely to occur under H1 are at the lower end of the distribution. A force, F, also acts on the particle, down the plane, parallel to the slope. ms–1 Horizontally, vx = ux = 3 cos 7° ms–1 3 cos 7° q 17.712... Differentiating: dy dy y2 + 2yx + 2y + 2x - 9x2 = 2x dx dx 2x - y 2 - 2y + 9x 2 dy = 2yx + 2x dx dy 4 - 25 + 10 + 36 - 25 At (2, –5), = = 16 - 20 + 4 dx This is the gradient of the tangent at (2, –5), so: 15 25 –5 = - 16 (2) + c fi c = - 8 So the equation of the tangent at (2, –5) is: y = - 25 x - 15 or 16y = –25x – 30 8 16 dy 25 4 - 9 - 6 + 36 At (2, 3), = = 16 dx 12 + 4 This is the gradient of the tangent at (2, 3), so: 1 25 3 = 16 (2) + c fi c = - 8 So the equation of the tangent at (2, 3) is: y = 25 x - 1 or 16y = 25x – 2 8 16 The two tangents intersect when: 14 –25x – 30 = 25x – 2 fi 50x = –28 fi x = - 25 14 And 16y = 25a - 25 k – 2 fi 16y = –16 fi y = –1. Objects hanging from strings and objects on inclined planes are some common examples of where you might see forces in equilibrium. dx 2 The gradient of the normal is 3 . Chapter 15 Dynamics 339 Example 2 A block of mass 12 kg is being accelerated across R a rough, horizontal plane by a force of magnitude 1.5 ms–2 60 N, as shown. Explain your answer. ﬀ Using the tree diagram: P(L Ç S) = P(L)P(S|L) = 0.4 × 0.7 = 0.28 P(S) = P(L Ç S) + P(L' Ç S) = 0.28 + P(L')(S|L') = 0.28 + 0.6 × 0.2 = 0.4 ﬀ So P(L|S) = P^L + Sh 0.28 = 0.4 = 0.7 P^ S h ﬀ This means P(Horace was late, given he is shouted at) = 0.7 Example 3 For events M and N: P(M) = 0.2, P(N|M) = 0.4 and P(N'|M') = 0.7. a) Draw a tree diagram representing events M and N. You should remember that sin x differentiates to cos x, and cos x differentiates to –sin x. dx dy 1 You could also use the fact that dx = 2 (x2 – 1)2 to show that the gradient is always ≥ 0, which means that any stationary points must be points of inflection. A rests on a rough plane (mA = 0.7) inclined at an angle of 40° to the horizontal and B rests on a different rough plane (mB = 0.05) inclined at an angle of 20° to the horizontal, as shown above. Find the weight of the particle, W, W and the angle x. y h) y = 2|x – 4| y=x 3 1 –3 2 1 4 4 x The graph shows that there are two solutions, one where both 2x + 3 and 4x – 1 are positive, and one where 2x + 3 is positive but 4x – 1 is negative. Q2 a) Differentiating: dy dx 3x2 + 2x + = 2y dx dx dy 3x 2 + 2x fi = . with in-depth answers at the back • 100% matched to the new AQA course... — If X ~ N(m, s2), then X + N bm, sn l fi Z = 2 X-m ~ N(0, 1) s n — Then the value of your test statistic will be: z = x-m s n ﬀ Use a calculator to test for significance, either by: — finding the probability of your test statistic taking a value at least as extreme as your observed value (this probability is called the p-value) and comparing it to the significance level a. ﬀ If x = 2.472 is a root rounded to 3 decimal places, the exact root must lie between the upper and lower bounds of this value — 2.4715 and 2.4725. Kyle believes that the mean amount of time he spends studying each week has increased. dt dt x x Q3 Let the variable t represent time. 1 2 1 a = b3 × 4 l i + b 3 × 1 l j + b3 × - 3 l k 1 a = 3b in the i and k directions, but 3 b in the j direction, so they are not parallel. Find the functions: a) fg(x) b) gf(x) Q5 f(x) = 2x – 1, x d R and g(x) = 2x, x d R . ﬀ Quotient — the bit that you get when you divide by the divisor (not including the remainder — see below). 1 : 1 + 4 cos x p a) Find the equation of the tangent to the curve when x = 2 . Examples Plenty of step-by-step worked examples. At time t seconds, the side length is 1 increasing at a rate of 2 + cm s–1. fi x = 4.507... ﬀ 25% of 200 = 50, which means you need to find P(X ≥ 50) ≈ P(Y ≥ 50). N Using Pythagoras' theorem: R = (90.432...) 2 + (9.842...) 2 = 90.966... Its velocity at time t seconds is given by the expression: 1 v = [(3 t3 + 2t2 – 12t)i + 14j] ms–1, where 0 ≤ t ≤ 4. ﬀ Sometimes a rearrangement of the equation leads to a divergent iteration when you come to working out the steps. Integration by substitution A method for integrating a function of a function. This is how we define an integral — as the limit of a sum of areas. Transforming to Z, you get: 14 - m P(X > 14) = P a Z > s k = 0.0322 278 Chapter 12 The Normal Distribution z f(z) ﬀ Draw a sketch to show the information. dy Then the rate of change of y with respect to t is . Could the probabilities be biased in some way? Alternative hypothesis The statement that you will accept instead if you decide to reject the null hypothesis in a hypothesis test. Example 3 The forces needed to snap lengths of a certain type of elastic are normally distributed with m = 13 N and s = 1.8 N. Exercise 2.1 Q1 Draw a mapping diagram for the map "multiply by 6" acting on the domain {1, 2, 3, 4}. The particles will also have the same magnitude of acceleration. b) Find fg(x) and write down its range and largest possible domain. Chapter 15 Dynamics 335 3. ﬀ Almost all the data is within 3 standard deviations of the mean: 9.8 – (3 × 1.3) = 5.9 and 9.8 + (3 × 1.3) = 13.7 ﬀ So the random variable X ~ N(9.8, 1.3²) seems a reasonable model. a) X ~ B(350, 0.45) b) X ~ B(250, 0.35) c) X ~ B(70, 0.501) Q3 It is estimated that 5% of people are carriers of a certain disease. Putting x0 in the formula for xn gives you x1 — the first iteration. m = 25.3 m (3 s.f.) Tip: Distance must be positive, so ignore the negative square root. Its height above the water after x seconds is modelled by the equation f(x) = 2x² – 8x + 7, where f(x) is the height in metres. A force acting up the plane, parallel to the slope, holds the particle in equilibrium. It has no horizontal component as acceleration due to gravity always acts vertically downwards. Exercise 1.1 Simplify the following: Q14 Hint: You need to multiply the top and bottom by the same term to get rid of the fractions. a-b a a So 1 + 2 = b fi 2 = b – 1 fi 2 = b Since a and b are integers, (a – b) is also an integer, which means that 2 is rational, which is not true. Q4 Find the area of the region bounded by p p y = cosec2 (3x), the x-axis and the lines x = 12 and x = 6 . Common things that you should think about are: Tip: Continuous and discrete variables were covered back in Year 1 of the course — discrete variables only take certain values (e.g. integers, names etc.) while continuous variables can take infinitely many values (e.g. real numbers between 0 and 1). - ^t + 3 h A + B / ^3t + 2h^t + 1h ^3t + 2h ^t + 1h fi –(t + 3) º A(t + 1) + B(3t + 2) Substituting t = –1 gives: –2 = –B fi B = 2 Equating coefficients of t gives A + 3B = –1 fi A + 6 = –1 fi A = –7 - ^t + 3 h 7 2 So / ^3t + 2h^t + 1h ^t + 1h ^3t + 2h The integral can be expressed: 2 2 #0 3 ^3t -+^2t +h^t3+h 1h dt = #0 3 ^t +2 1h - ^3t 7+ 2h dt 3 = :2 ln t + 1 - 7 ln 3t + 2 D 3 0 = :2 ln 5 - 7 ln 4 D - :2 ln 1 - 7 ln 2 D 3 3 3 = :2 ln a 5 k - 7 ln ^4hD - :0 - 7 ln ^2hD 3 3 3 = 2 ln a 5 k - 7 ln ^4h + 7 ln ^2h 3 3 5 7 = 2 ln a k - ^ln ^4h - ln ^2hh 3 3 5 7 = 2 ln a k - ln a 4 k = 2 ln 5 - 7 ln 2 3 3 2 3 3 du 1 Q6 u = x fi = fi dx = 2 x du = 2u du dx 2 x and u = x fi x = u2. Q4 B O 21 N 352 A 25 N 2m 1m O dm Q 4N The diagram shows the light rod PQ. MODELL ING Q7 A particle moves across a horizontal surface with a constant, non-zero acceleration of (pi + qj) ms–2, where the i direction is due east and the j direction due north. Find the probability that his height is within 4 cm of the mean height. The force causing the acceleration has magnitude 200 N and acts on A in a direction parallel to the slope. ﬀ Resolving in the y-direction (≠): ﬀ Rearrange and solve this equation to find the magnitude of F: -17 sin 20° + 12 cos 18° + 6 sin 30° F= sin 60° 324 Chapter 15 Dynamics F sin 60° + 17 sin 20° – 12 cos 18° – 6 sin 30° = 0 = 9.93 N (to 3 s.f.) Exercise 1.2 Q1 Each of the diagrams below shows the forces acting on a particle. Find: a) the time taken for the projectile to reach its maximum height, b) the maximum height the projectile reaches above the ground. b) If the student is studying English and Maths, what is the probability that they are also studying Art? Resolving horizontally, taking right as positive: 4 s = 50, u = V cos q = 5 V, a = 0, t = t 1 s = ut + 2 at2 4 5 # 50 125 50 = 5 V × t fi t = 4V = 2V Now resolving vertically, taking up as positive: 3 125 s = 0, u = V sin q = 5 V, a = –9.8, t = 2V 1 s = ut + 2 at2 125 2 76 562.5 3V # 125 0 = 5 # 2V – 4.9b 2V l fi 0 = 37.5 – 4V 2 76 562.5 76 562.5 2 = 37.5 fi 4 # 37.5 = V 4V 2 76 562 .5 V 2 = 150 fi V = 22.6 ms–1 (3 s.f.) Q10 a) Resolving vertically, taking down as positive: s = 16, u = 3 sin 7°, a = 9.8, t = t 1 s = ut + 2 at2 16 = (3 sin 7°)t + 4.9t2 4.9t2 + (3 sin 7°)t – 16 = 0 Using the quadratic formula: - 3 sin 7° ! ^3 sin 7°h2 - ^4 # 4.9 # - 16h t= 9.8 - 0.3656... Replace f(x) with y: y = x2 + 3 Rearrange: x = y - 3 Replace with f -1(x) and x: f -1(x) = x - 3 . ﬀ Be able to simplify rational expressions by adding and subtracting algebraic fractions. MODELL Q4 Hint: When you're given a value for tan q, it often means you can use it to get sin q and cos q without needing a calculator. Q6 a) y = 10 x The graph shows that there are two solutions to 1 |4x + 2| = 10. dx ﬀ Start off by identifying u = cos x and v = sin x. This will depend on the shape of the graph — a sketch can show whether the tops of the trapeziums lie above the curve or stay below it. The examples and questions in this chapter use g = 9.8 ms–2, unless stated otherwise. Rational number A number that can be written as the quotient (division) of two integers, where the denominator is non-zero. When x = 4 , = dx dx 2 dy Q13 = –2 sin (2x) dx dy p When x = 4 , y = 0 and = –2 dx -1 1 So the gradient of the normal is - = 2 . c) P(X ≤ 10)? D a) Explain why the depth of water satisfies the differential equation dD =- k for some constant k > 0. 4 y 2 x –4 –2 0 –2 Q5 a) 4 y 2 x –4 –2 0 –2 2 4 b) If the two functions are set equal to each other they can be rearranged to make e2x + x2 = 3. f(0.255) = e0.255 – 0.2553 – (5 × 0.255) = –0.001... Learning Objectives: ﬀ Be able to use the Newton-Raphson method to find the root of an equation. 2 sin (q - 0.464) = 5 fi q - 0.464 = 1.107 and (p - 1.107) = 2.034 fi q = 1.57 and 2.50 (3 s.f.) Q9 4 sin 2q + 3 cos 2q º R sin (2q + a) º R sin 2q cos a + R cos 2q sin a fi R cos a = 4, R sin a = 3 3 tan a = 4 fi a = 0.644 (3 s.f.) R = 32 + 42 = 5 fi 4 sin 2q + 3 cos 2q º 5 sin (2q + 0.644) So solve 5 sin (2q + 0.644) = 2 in the interval 0.644 ≤ (2q + 0.644) ≤ 6.927. Reversing the differentiation gives the following three integrals. You want A + B cos q to have a maximum of 17 and a minimum of 7. b) Find the magnitude of the force F. Give your answer to 3 significant figures. N V RH = 15.035... ﬀ The values are symmetrically distributed, with a peak in the middle — this peak represents the mean of the values. Taking logs: n log 2 = log(3.84 × 1012) fi n = log(3.84 × 1012) ÷ log 2 = 41.80... In the example coming up on the next page, you have to spot that there's a hidden derivative described in words. fi a = 54.550...° = 54.6° (3 s.f.) b) F = mR FW = mW × 22 fi mW = 13.2 ÷ 22 = 0.6 c) FG = mG × 75 fi mG = 22 ÷ 75 = 0.29 (2 d.p.) Answers 475 Q6 If the ladder is on the point of slipping downwards: Q8 RC N RA A 100 N R K 75° 0.1R Resolving vertically: R = 100 N Resolving horizontally K + 0.1R = N fi N = K + 10 Taking moments about the base of the ladder: (K sin 75° × 1) + (100 cos 75° × 4) = (N sin 75° × 8) K sin 75° + 400 cos 75° = 8(K + 10) sin 75° 400 cos 75° – 80 sin 75° = 8K sin 75° - K sin 75° 400 cos 75° – 80 sin 75° = K(7 sin 75°) K = 26.253... Equating the coefficients of x2 gives: 1 = A + 2B, so B = –1. 23 - m So = 1.2901... p Q13 Find the equation of the normal to the curve y = cos (2x) when x = 4 . Calculating with 3D vectors Magnitude of a 3D vector ﬀ The magnitude (sometimes called modulus) of a vector v is written as |v|, and for a vector AB it is written AB . Modulus of a function Functions can have a modulus too — the modulus of a function f(x) is just f(x) but with any negative values that it can take turned positive. Adjusting the function to only allow integer values of y would fix this. Find: c) an expression for the position vector of particle B relative to O in terms of t, d) an expression for the position vector of particle B relative to particle A in terms of t, e) the distance between particles A and B at time t = 4 seconds. Chapter 8 Integration 187 Integration of sec2 x Tip: This integral is given in the formula booklet — it's given as # sec 2 kx dx = 1k tan kx + C. f (x) dx is found by adding the areas of all the trapeziums: b h h h f (x) dx . fi n + 5 = e–2teln 5 fi n = 5e–2t – 5 = 5(e–2t – 1) Replace both components in v = mi + nj to give: v = 70.1ti + 5^e -2t - 1h j A ms–1 as required. If v = wi + zj is the velocity of a body, then: # v dt = # _wi + zj i dt = a # w dt ki + a # z dt kj the body's position vector, r = Example 1 A particle is moving in a plane. Set x = 0: –17 = –2C + D, so C = 10. c) Calculate the magnitude of P. The rod's weight acts at its = 36.4 N (3 s.f.) midpoint, 1.65 m from A. What interval for Y would you use to approximate: a) P(X = 5)? Integration of (ax + b)n Exercise 1.1 — Integrating (ax + b)n, n ≠ –1 Q1 a) # ^ x + 10h10 dx = 1 ^ x + 10h11 + C 1× 11 = 1 ^ x + 10h11 + C 11 1 ^ h8 + b) # ^5xh7 dx = 5 ×1 8 ^5xh8 + C = 40 5x C 8 8 = 1 58 x8 + C = 5 x8 + C = 78125x + C 40 40 8 -2 1 c) = # ^3 5xh dx - 5 × - 1 ^3 5xh-1 + C 1 +C = 1 ^3 - 5xh-1 + C = 5 5 ^3 - 5xh 4 1 # ^3x - 4h- 3 dx = 1 1 ^3x - 4h- 3 + C d) 3 × a- 3 k 1 1 -1 = 1 ^3x - 4h- 3 + C = - ^3x - 4h- 3 + C = 3 +C -1 3x - 4 Q2 a) Begin by taking the constant of 8 outside of the integration and then integrate # ^2x - 4h4 dx as usual. MODELL Q12 19 ms–1 0.5 m 28° ING MODELL ING 2.5 m A cricket ball is hit from a point 0.5 m above flat, horizontal ground. Then you use the methods you've seen on pages 269-279 to find out what you need to know. So the critical region is Z > 2.326. It's not the same as the normal reaction, which acts when a body is in contact with a surface, and which always acts perpendicular to the surface. ﬀ Taking moments about B: RA = 0, so the moment of the force is also zero. The probability that an event B happens, given that an event A has already happened, is called the conditional probability of 'B given A', written P(B|A). m distance = ^24.8198...h2 + (4.9503...) 2 24.8198... P(M > 60 – m) = 0.9525 60 - m 60 - k fi PaZ > = 0.9525 3 -m ` j fi P Z > 3 = 0.9525 m fi P ` Z < 3 j = 0.9525 F(z) = 0.9525 for z = 1.67 So m = 1.67 fi m = 1.67 × 3 = 5.01 3 = 5 g to the nearest gram. ﬀ Be able to find the equations of tangents and normals to curves given implicitly. The Under 20 entry in this column is 10 (or 0.25), 0.25 10 5 so P(Under 20|Comedy) = 18 bor 0.45 l = 9 Q3 a) (i) The circles for R and T do not overlap, so they are mutually exclusive (i.e. P(R Ç T) = 0). So the interest rate needed is 7.17 % (3 s.f.). x 14 Chapter 2 Algebra and Functions x f(x) x The graph on the right isn't a function for x d R because f(x) is not defined for x < 0. q3 3 a) Show that the gradient of the curve when q = p is – 6 . metres. Smooth Describes a surface for which there is no friction between the surface and a body in contact with it. Chapter 8 Integration 193 Exercise 4.1 Q1 Find the following integrals: 3 # 2xx2 -- x1 dx # x44 x- 1 dx b) a) 4 3 2 # 3x x5 + 6 dx d) # 12xx4 ++ 218x 3x- x- 3 dx c) Q2 Find the indefinite integrals below: 2x x x # e xe+ 6 dx b) # 2^ee2x ++63eex h dx a) x # 3^eex + 3h dx c) Q3 Find the following integrals: 2x # 12+cos # cossin3x3x- 1 dx a) dx b) sin 2x x cot x + 6x sec 2 x # 3 cosec # c) d) d x 2 tan x dx cosec x - x + 4 x tan x # sec e) dx sec x + 5 Q4 Show that Q5 Hint: Try multiplying the bit inside the integral sec x + tan x) by in part a) sec x + tan x — there's a similar trick for part b) as well. Q6 The diagram below shows part of the curve y = y 3 y= ln x 2 4 a) Using the trapezium rule with 4 intervals, find an estimate 4 3 to 2 d.p. for dx . The remainder will have degree 0. = 8 a - 1 cosec ^2x + 1h + c k 2 = - 4 cosec ^2x + 1h + C Example 2 Find # 10 sec 5x tan 5x + 12 cosec 3x cot 3x - cosec 2 ^6x + 1 h dx . There is insufficient evidence at the 5% significance level to reject H0, the hypothesis that maths and science test results are not correlated in the population. Since tan x differentiates to sec2 x, you get: # sec 2 x dx = tan x + C Example 1 Find # 2 sec # 2 sec 2 2 x + 4 x dx . dx d ' in front of each term: dx d d 3 d 2 d 2x2y + y = 6x – 15 dx dx dx dx ﬀ Again, start by sticking ' Tip: Once you're happy with this method you might not need to write all the steps out separately, but you can do if you find it easier. f(z) Area = 0.7580 0 a – 13 1.8 ﬀ Use a calculator to find z for which F(z) = 0.7580. dx 2y - 1 dy At the stationary points, = 0, so: dx 2 3x + 2x = 0 fi x(3x + 2) = 0 2 fi x = 0 and x = - 3 2 When x = 0, y = y fi y(y – 1) = 0 fi y = 0 and y = 1. Exercise 2.1 Q1 Find the following indefinite integrals: # 2e x dx b) x # 4 + 7e x dx c) # e10x dx a) 7 # e -3x + x dx e) # e 2 x dx f) # e 4x - 2 dx d) # 12 e 2 - 32 x dx h) # e 4c 3x + 1m dx g) dy Q2 Find the equation of the curve that has the derivative = 10e–5x – 1 dx and passes through the origin. Taking moments about A: W cos 30° × 0.5l = RC × 0.75l 3 3 0.5l fi RC = 2 W × 0.75l = 3 W N Resolving perpendicular to the beam: RA + RC = W cos 30° 3 3 3 fi RA = 2 W – 3 W = 6 W N Resolving parallel to the beam: 1 F = W sin 30° = 2 W N Using F = mR: 1 2 W = m × RA 3 1 1 6 3 fi m = 2W ÷ 6 W = 2 × = = 3 3 3 = 1.73 (2 d.p.) Q9 R K 0.1R Resolving vertically: R = 100 N Resolving horizontally: K = N + 0.1R fi N = K – 10 Taking moments about the base of the ladder: (K sin 75° × 1) + (100 cos 75° × 4) = (N sin 75° × 8) K sin 75° + 400 cos 75° = 8(K – 10) sin 75° 400 cos 75° + 80 sin 75° = 8K sin 75° – K sin 75° 400 cos 75° + 80 sin 75° = K(7 sin 75°) K = 180.80... –3 3 x Exercise 2.3 Q1 State whether each function below is one-to-one or many-to-one. N ﬀ Resolve parallel to the plane ( ) for A: 4g sin 50° – FA – T = 4a 24.989... N Resolving vertically: RV + T sin 20° = 1.5g + 0.5g fi RV = 19.6 - 6.125 = 13.475 N R 13.475 N q 16.828... f (x + h) - f (x) d f (x) = lim h dx h"0 f (x) ﬀ And so for the quotient g (x) , this becomes f (x + h) - f (x) d c f (x) m = + g lim (x h) g (x) dx g (x) h"0 h ﬀ Put the top of the fraction over a common denominator (g(x + h)g(x)) and multiply this common denominator by the h. So there are two equations to solve: 1 2(x – 4) = x fi x = 8 (valid since 8 > 4) 8 8 2 –2(x – 4) = x fi x = 3 (valid since 3 < 4) So the regions that satisfy the inequality are 8 x ≤ 3 and x ≥ 8. dy d 2 2x y + 3y2 = 12x + 0 dx dx ﬀ Use the product rule on the term in x and y, where u(x) = 2x2 and v(y) = y: 2x2 Tip: Take your time using the product rule as it's easy to forget terms if you're not careful. ﬀ You can estimate the probability that a value will fall within a range by using the normal distribution functions on your calculator. Q9 Let f(x) = 2x and g(x) = 3 – 4x, then dy 1 1 = 2 × f'(x)ef(x) – 2 × g'(x)eg(x) = 2 × 2e2x – 2 × –4e3 – 4x dx 2x 3 – 4x = 4e + 2e d2 y = 4 × f'(x)ef(x) + 2 × g'(x)eg(x) = 4 × 2e2x + 2 × –4e3 – 4x dx 2 = 8e2x – 8e3 – 4x d2 y At a point of inflection, =0 fi 8e2x – 8e3 – 4x = 0 dx 2 fi e2x = e3 – 4x 1 fi 2x = 3 – 4x fi 6x = 3 fi x = 2 d2 y d2 y 1 1 > 0 if x > 2 and < 0 if x < 2 dx 2 dx 2 d2 y 1 changes sign from negative to positive at x = 2 . ﬀ PQ = OQ – OP = (x2i + y2j + z2k) – (x1i + y1j + z1k) = (x2 – x1)i + (y2 – y1)j + (z2 – z1)k So the magnitude of the vector PQ = (x2 – x1)i + (y2 – y1)j + (z2 – z1)k is ^ x2 - x1h2 + ^ y2 - y1h2 + ^ z2 - z1h2 , by Pythagoras' theorem. Chapter 8 Integration 197 This will probably make more sense if you have a look at some examples: Example 1 Evaluate # 12x3(2x4 – 5)2 dx. 2 ﬀ Now solve this equation for m. 266 Chapter 11 Probability Werewolf Zombie Ghost Total Boys 0.2 0.15 0.15 0.5 Girls 0.1 0.2 0.2 0.5 Total 0.3 0.35 0.35 1 7 ÷ 20 = 0.35 20 ÷ 20 = 1 c) Another child is picked at random. The centre of mass of the beam is 0.6 m from A. So 6x3 + x2 – 11x – 5 º (3x2 – x – 5)(2x + 1). Example 2 1 Find the inverse of the function f(x) = 2x - 1 , with domain x ≥ 2 and range f(x) ≥ 0. The graph is the same as y = arctan x but 1 stretched vertically by a factor of 2 , so the y-coordinates of the asymptotes are halved. Similarly, you can define the integral of a curve f(x) between two points a and b using limits: # b f (x) dx = lim / f (x) × dx b dx " 0 a Tip: You should have seen the formula for differentiating from first principles back in Year 1. = 2.36 N (3 s.f.) R T 472 c) When A hits the pulley, A and B are both travelling at 0.4 ms–1. 1 She says, "The probability that it lands on any given face is 6 ." Q2 Harriet shows a selection of 10 playing cards to a volunteer. Label x0, x1 and x2 on each diagram where possible. The number of bacteria in the colony, n, is directly proportional to the diameter. Find the force K in terms of i and j. ﬀ Be able to find the mean and standard deviation of a normal distribution given some probabilities for the distribution. ﬀ Start by finding du and then rearrange to get dx on its own: dx u = 3x + 2 fi du 1 = 3 fi dx = 3 du dx ﬀ If you substitute for u and dx, you end up with an x still in the integral: # x(3x + 2)3 dx = # xu3 13 du ﬀ To get rid of that x, you have to rearrange the equation for u: u = 3x + 2 fi x = Tip: Note that this integral cannot be done using methods from previous sections. dr dV dV dr = × = 8 × 10–5pr2 m3 °C–1 dq dr dq You could have converted to mm instead, and given your answer in mm3 °C–1. 2 log10 x Evaluate f(1) and f(100). Using equation 1 , 9 – m = 0.14995...s fi m = 9 – 0.14995... You've been told that k = 0.1, so substitute these values into the equation above and solve for t: 1 1 –0.1t fi 2 = e–0.1t 2 P0 = P0e 1 fi ln 2 = –0.1t fi –0.6931 = –0.1t fi t = 6.931 ﬀ So to the nearest day, t = 7. Chapter 9 Numerical Methods 237 3. 6 2 = 1 – 4x + 10x2 – 20x3 + ... Just put the values for x and y into dy : dx dy 12^ 2h - 4^ 2h^1 h 16 = = 11 dx 2^ 2h2 + 3^1 h2 Exercise 10.1 dy for each of these curves: dx a) y + y3 = x2 + 4 b) x2 + y2 = 2x + 2y c) 3x3 – 4y = y2 + x d) 5x – y2 = x5 – 6y e) cos x + sin y = x2 + y3 f) x3y2 + cos x = 4xy g) ex + ey = x3 – y h) 3xy2 + 2x2y = x3 + 4x Q1 Use implicit differentiation to find 174 Chapter 7 Differentiation dy , for each of these curves given below: dx a) x3 + 2xy = y4 b) x2y + y2 = x3 c) y3x + y = sin x d) y cos x + x sin y = xy e) ex + ey = xy f) ln x + x2 = y3 + y g) e2x + e3y = 3x2y2 h) x ln x + y ln x = x5 + y3 Q2 Find the gradient, Q3 a) Show that the curve C, defined implicitly by ex + 2 ln y = y3, passes through (0, 1). dx b) Show that y = –2 ± 2 3 when x = –2. This just means that when x is negative there is no real value that f(x) can take. ﬀ The weight of the rod (mg = 50.902... b) Show that a root of g(x), a, lies between 1 and 2. ﬀ To find the value of the test statistic: — Calculate the sample mean, x . = 0.739 (3 s.f.) 0 0.64 b) P(Z > –0.42) ﬀ Draw a sketch of the area you need to find. ﬀ The gradient of the tangent at t = p is equal to dy at that point, i.e. 2. 288 For the discrete random variable X ~ B(n, p): n ﬀ P(X = x) = d n × p x × (1 – p)n – x ﬀ Probabilities can be found using the binomial distribution functions on your calculator. Repeat this for each of your iterations. After 10 seconds, it is moving due south with a speed of 2 ms–1. Simple. You're Reading a Free Preview Pages 272 to 278 are not shown in this preview. To do this, put the x-limits into the equation for u to find the corresponding values of u. 18 a) f(x) = x3 b) f : x Æ sin 2x –180° < x ≤ 180° c) f(x) = log10 x –2 ≤ x < 0 x+2 d) f(x) = (0≤x≤2 2–x 2 e) f(x) = (1 Chapter 2 Algebra and Functions x xdR x>0 x≥0 x –3, range g(x) > 0 x 3 ﬀ First work out the composite function in terms of x: fg(x) = f(g(x)) 2 = fa 1 k = 2a 1 k + 1 x+3 x+3 Tip: Working out the domains and ranges of composite functions can be tricky — but sketching a graph always helps. However, a force can also have a turning effect, usually when a body is pivoted at a point. ÷ 122.98... Tip: She has used the relative frequency of the event to estimate the probability — you might have seen this at GCSE. b) Given that gf(2) = –8, find the value of fg(2). A new friend has joined the game and Bree thinks this will change the amount she wins. b) Calculate the magnitude and direction of the reaction force acting on the plank at A. d) Solve f(x) = f –1(x). ƒ Be able to add and subtract threedimensional vectors and multiply them by scalars. º (cosec x + cos x)(cosec x – cos x) ^sec x - tan xh^tan x + sec xh sec 2 x - tan 2 x Q4 º cosec x - cot x cosec x - cot x ^1 + tan 2 xh - tan 2 x 1 º º cosec x - cot x cosec x - cot x Multiply top and bottom by (cosec x + cot x)... ﬀ First, integrate the expression for acceleration with respect to time to find an expression for velocity: v= # a dt = ` # ^4 - 18th dt ji + ` # 72t dt jj = (4t – 9t2)i + 36t2 j + C ﬀ When t = 2, v = –20i + 135j. Q6 + RA RB T T FB B FA A 0.2g 0.9g 20° 40° Particles A and B, of mass 0.2 kg and 0.9 kg respectively, are connected by a light inextensible string. q = tan–1 a 9.842... 168 Chapter 7 Differentiation dh a) Find an expression for , the rate at which the water level dt falls in the cylinder. 1 3 = 1 – 2 (4x) + a 8 k(16x2) + ... 2 (1 + 2 ln 2) x2 = 2.395369..., x3 = 2.393518..., x4 = 2.393517... dy = f'(x)ef(x) – 6 = 3e3x – 6 dx 1 3x 3e – 6 = 0 fi e3x = 2 fi 3x = ln 2 fi x = 3 ln 2 Take ln of both sides to get rid of the exponential. Find: a) the ball's speed of projection, b) the height of the ball above the ground 2 seconds after being struck by the golf club. Chapter 8 Integration 205 Example 3 Find the area enclosed by the curve y = x2 – 2x and the line y = 2x. Area = 0.2 –3 0 Area = 0.2 3 ﬀ Now solve the equation for s: s = 3 ÷ 0.84162... Assuming that all other forces remain constant, calculate the instantaneous acceleration of the truck. ﬀ So the ball is exactly 5 m above the ground 0.314... ﬀ Because of the symmetry of the curve, P(Z ≤ z) = P(Z ≥ –z). In a random sample of 20 weeks over the past year, the mean number of hours he spent studying was recorded as 34. ﬀ An equation connecting variables with their rates of change (i.e. with a derivative term) is called a differential equation. 1 1 c) The expansion of ^1 - 5xh6 is valid for |x| < 5 . ﬀ Now put in the x-value of your stationary point: 1 d 2 y 6 ln e 2 - 5 3-5 2 = = = –0.27... Many-to-one function A function where some values in the range correspond to more than one value in the domain. For example: f(x) = (2x + 3 x2 x≤0 x>0 So f(2) is 22 = 4 (because x > 0), but f(–2) is 2(–2) + 3 = –1 (because x ≤ 0). Points of Inflection Exercise 1.1 — Points of inflection 5 1 1 1 Q1 a) y = 6 x3 – 2 x2 + 4 x + 9 dy 1 2 d2 y 1 fi = 2 x – 5x + 4 fi =x–5 dx dx 2 2 d y The graph is concave when 0 fi 3x2 + 18x – 21 > 0 fi x2 + 6x – 7 > 0 fi (x + 7)(x – 1) > 0 fi x < –7 or x > 1 So f(x) is convex for x < –7 and x > 1, and concave for –7 1, x3 > 1, so 3 < 2, so is positive. N 11 N (–2, 8) B 2.263... Both np and n(1 – p) are much greater than 5 (see p.285), so X can be approximated by Y ~ N(m, s2). sin A cos B k - a cos A sin B k a cos A cos B cos A cos B º cos A cos B sin A sin B k + a cos A cos B k a cos A cos B sin A - a sin B k a cos k cos B A º sin A sin B + 1 a cos A ka cos B k Now use tan = sin / cos... 4x n 6x n Starting with x0 = 2.5, 2.5 4 - 2^2.5h3 - 5 = 2.3875 x1 = 2.5 4^2.5h3 - 6^2.5h2 x2 = 2.373982..., x3 = 2.373799..., x4 = 2.373799... a) 2x x2 + 1 ﬀ Differentiate the denominator to see what it gives: d 2 (x + 1) = 2x dx This is the numerator. 1 for {x : x > –1}. x0 x2 x3 x1 4) T hen another horizontal line to y = x, and so on. You can use Pythagoras' theorem and trigonometry to find the magnitude of any vector. x Don't forget to label each iteration. a) Given that the child is a girl, what is the probability that they're dressed as a zombie? Sketching Iterations Learning Objectives: ﬀ Be able to sketch cobweb and staircase diagrams. dx c m dy ﬀ Then use dy 1 = to get the final answer: dx dx c m dy dy 1 1 = = 3 cos2 (3y) dx 3 sec 2 (3y) Once you get the hang of it, you don't need to use the chain rule every time. Modelling Problems Exercise 5.1 — Real-life problems Q1 a = 60, d = 3 nth term = a + (n – 1)d = 60 + (n – 1)3 = 3n + 57 12th term = 3(12) + 57 = 93 So she'll earn £93 in her 12th week. (vi) P(A|B) = P^ A + Bh 0.2 1 = 0.6 = 3 P^Bh Tip: You can also find P(B) by doing: P(B|A)P(A) + P(B|A')P(A') That's because P(B|A)P(A) = P(B Ç A), P(B|A')P(A') = P(B Ç A') And P(B Ç A) + P(B Ç A') = P(B) b) Say whether or not A and B are independent. f(2.0) = ln (2.0) – 22.0 + 3 = –0.306... ! 313.73... 1 2 y dy = 2(1 + x) dx # 1y dy = # 2(1 + x)2 dx And integrate: 2 fi ln |y| = 3 (1 + x)3 + C Now to find the particular solution, work out the value of C for the given values of x and y: 2 ln 4 = 3 (1 + (–1))3 + C fi ln 4 = C 2 so ln |y| = 3 (1 + x)3 + ln 4 ﬀ Separate the variables: ﬀ ﬀ Chapter 8 Integration 223 Example 3 Find the general solution of (x – 2)(2x + 3) dy = xy + 5y, where x > 2. ﬀ Be able to use a normal approximation to a binomial random variable. 0800 1712 712 • www.cgpbooks.co.uk Proof Chapter 1 1. Q3 The marks obtained in a language test by students in a school have followed a normal distribution with mean 70 and variance 64. a This means that x can be written as x = b c and xy can be written as xy = d , where a, b, c and d are all non-zero integers. ﬀ It's best to take complicated questions like this step by step. Q5 T A 2m 30° C 110 N 30° 1.75g B 2.25g A uniform plank, AB, of mass 1.75 kg and length 8 m, is freely hinged to a vertical wall at A. A body of mass 0.5 kg is placed at the beam's midpoint. Question Hints Occasional hints to point you in the right direction. PROBLE M SOLVING Q6 Find the unit vector in the direction of v, where v = 4i – 4j – 7k. T Taut Describes a string or wire which is experiencing a tension force and is tight and straight. Tip: Correlation was covered in Year 1 — take a look back at your notes if you need a reminder. ﬀ Draw a diagram to show what's going on: Tip: When the bucket is at its maximum mass, the ladder is on the point of slipping — i.e. in limiting equilibrium. The package is being pushed up the slope by a force of magnitude 110 N acting horizontally. Quotient Rule 158 7. y Just sketch the two graphs on a set of axes. c) The upper and lower bounds are 3.5765 and 3.5775. ﬀ r is close to –1, so this means the data has a strong negative correlation. You need to know whether the ground and supports are rough or smooth, and you should also remember that the normal reaction at a support will always act perpendicular to the body. – The model does not account for the differences between patients or other conditions such as the presence of other germs or chemicals. So both answers are equivalent. Answers 401 Q6 a = 5, r = 0.6, nth term: un = arn – 1 = 5 × (0.6)n – 1 10th term: u10 = 5 × (0.6)9 = 0.050388 (6 d.p.) 15th term: u15 = 5 × (0.6)14 = 0.003918 (6 d.p.) Difference: 0.003918 – 0.050388 = – 0.04647 (5 d.p.) You could also have: Difference: 0.050388 – 0.003918 = 0.04647 (5 d.p.) Q7 a = 25 000, r = 0.8 nth term: un = arn – 1 = 25 000 × (0.8)n – 1 to find the first term in the sequence less than 1000, solve: un = 25 000 × (0.8)n – 1 < 1000 25 000 × (0.8)n – 1 < 1000 1000 fi (0.8)n – 1 < 25 000 = 0.04 fi log(0.8)n – 1 < log0.04 fi (n – 1)log(0.8) < log0.04 log 0.04 fi n–1> log 0.8 fi n – 1 > 14.425... b) Given that the function doesn't cross the x-axis between these limits, state whether the area between the curve and the x-axis for 1 ≤ x ≤ 2 lies above or below the x-axis, justifying your answer. Dividing the relevant entry by the total for that row or column will give you the conditional probability. Q4 A cube has side length x. nth term: un = arn – 1 = 24 × (0.5)n – 1 9th term: u9 = ar8 = 24 × (0.5)8 = 0.09375 Q4 First term: a = 1.125 nth term: un = arn – 1 = 1.125rn – 1 14th term: 9216 = u14 = 1.125r13 9216 = 1.125r13 fi r13 = 9216 = 8192 1.125 fi r = 13 8192 = 2 1.1 Q5 Common ratio: r = 1 = 1.1, first term: a = 1. W 5 2 5 2 u = (- 2 i + 2 j), v = – 2 j, a = (pi + qj), t = 10 v = u + at 5 2 5 2 - 2 j = (- 2 i + 2 j) + 10(pi + qj) 5 2 7 2 2 i - 2 j = 10pi + 10qj 5 2 2 fi 10p = 2 fi p = 4 7 2 7 2 10q = - 2 fi q = - 20 2. Chapter 15: Dynamics 1. ﬀ Resolving vertically, taking up as positive: s = h – y, u = U sin q, v = 0, a = –g, Using v = u + 2as: 2 2 0 = (U sin q)2 + (2 × –g × (h – y)) 0 = U2 sin2 q – 2gh + 2gy 2gh = U2 sin2 q + 2gy h= U2 sin 2 q +y 2g b) Find the horizontal range of the particle, x, in terms of y, U and q. b) Find the gradient of the curve at this point. Example 2 Divide x3 + 5x2 – 18x – 18 by x – 3. dy Q10 If y has a stationary point, the gradient will be 0. Example 1 a R Tip: Remember — the normal reaction force is always perpendicular to the surface. Normal distribution: X ~ N(m, s2) Conditions: ﬀ The values are continuous. dx 1 = 2 dt x ^t + 1 h dV = x3 fi = 3x2 dx dV 3 dV dx 3x 2 So = × = 2 = dx dt dt x ^t + 1 h t + 1 dV 3 3 b) = + fi # 1 dV = # + dt dt 1 t 1 fi V = 3 ln (t + 1) + C V = 15 at t = 0 fi 15 = 3 ln (1) + C fi C = 15 fi V = 3 ln (t + 1) + 15 So V = 18 fi 18 = 3 ln (t + 1) + 15 3 fi 3 = ln (t + 1) fi t = e1 – 1 = 1.72 seconds (3 s.f.) Q4 a) Answers 445 dy 1 = k(p – y) fi # - dy = # k dt p y dt fi –ln (p – y) = kt + ln a fi ln (p – y) = –kt – ln a fi p – y = e–kt – ln a = e–kt e– ln a 1 1 e–kt eln a = a e–kt = Ae–kt –kt fi y = p – Ae Q5 a) b) If p = 30 000 and y = 10 000 at t = 0, then 10 000 = 30 000 – Ae0 = 30 000 – A fi A = 20 000 fi y = 30 000 – 20 000e–kt t = 5, y = 12 000 fi 12 000 = 30 000 – 20 000e–5k fi e–5k = 18 000 ÷ 20 000 = 0.9 fi –5k = ln 0.9 fi k = –0.2 ln 0.9 fi y = 30 000 – 20 000e0.2t ln 0.9 So y = 25 000 fi fi fi fi 20 000e0.2t ln 0.9 = 5000 e0.2t ln 0.9 = 0.25 0.2t ln 0.9 = ln 0.25 t = 5 ln 0.25 ÷ ln 0.9 = 65.79 66 days y c) 30 000 0.2ln 0.9t y = 30 000 – 20 000e 10 000 0 t Remember that (0.2 ln 0.9) is negative when sketching the graph. ﬀ You assume that the discrete value X = 1 is 'spread out' over the interval 0.5 < Y < 1.5. X= = X = 0 = –0.5 YY = 1 0.5 2 1.5 3 2.5 4 3.5 ﬀ Then to approximate the discrete probability P(X = 1), X= 0 1 2 < Y < 31.5). y = 5 sin x 5 y PROBLE M SOLVING 4 3 2 1 0 y = 4 sin x y = 3 sin x y = 2 sin x y = sin x x Chapter 8 Integration 207 8. p p Q3 Use the substitution x = 2 sin q , where - 2 ≤ q ≤ 2 , 3 1 to find the exact value of # 3 dx. 3t2 – 2t – 4 = 3t2 – 4t + 3 2t = 7 fi t = 3.5 s This is the value of t for which the horizontal and vertical components of velocity are equal. Algebraic fraction A fraction made up of algebraic expressions. ﬀ Be able to simplify rational expressions by multiplying and dividing algebraic fractions. Q2 Differentiate with respect to x: a) y = x2(x + 6)3 b) y = x3(5x + 2)4 c) y = x3ex x2 d) y = xe4x e) y = xe f) y = e2x sin x Q3 Find f'(x) if: x2 x-7 e) f(x) = 2x3cos x 1 a) f(x) = x3(x + 3) 2 d) f(x) = 4x ln x2 b) f(x) = c) f(x) = x4 ln x f) f(x) = x2 cos (2x) Q3b) Hint: Remember 1 that 1 = x - 2 . Hence give an integer root of f(x). ﬀ Remember, if N' is conditional on M', then M' is conditional on N'. You might have to work out the range of a mapping from the domain you're given. Find T and the distance d. 0.3410... cos a ms–1 Vertical component = 6.1111... Q3 y = cos–1 x The only negative solution in that domain is 5p p p– 6 = 6 . ﬀ The population of rabbits is a discrete variable while the model is continuous — choosing a function that limits the possible values of P to integers would solve this problem. Q4 a) Sketch the graphs of y = x + 1 and y = 2x on the same axes. You can label the branches of a tree diagram using set notation like this: 1st trial (event A) P(A) A P(A') A' 2nd trial (event B) P(B|A) B P(A Ç B) P(B'|A) P(B|A') B' B P(A Ç B') P(A' Ç B) P(B'|A') B' P(A' Ç B') Tip: If the events are independent, then the probabilities for the second trial are just P(B) and P(B') — it doesn't matter whether or not A has happened. Find the range of values that the frictional force, F, can take, given that the coefficient of friction between the particle and the plane is 0.4. R Tip: So F can take any value up to 47.04 N, depending on the magnitude of P. Integrating using the reverse of the chain rule In Chapter 7, you saw the chain rule for differentiating a function of a function. The Normal Distribution In this section you'll be introduced to the normal distribution and use it to find different probabilities. Tip: Taking moments about A means you don't have to worry about the reaction force when finding x. 1 Q11 a) f(x) = - with domain x > 3 has a range f(x) > 0. In this case, you might need to use a continuity correction. b) (i) (1 + 6x)–1 = 1 – 6x + 36x2 – ... n = 2 , so n + 1 = 2 . The probability that he's late for school is 0.4. If he's late, the probability that he's shouted at is 0.7. If he's on time, the probability that he's shouted at is 0.2. MODELL ING Given that Horace is shouted at, what is the probability that he was late? The magnitude of the normal reaction force from the plane on the particle is 15 N. c) Verify that the value found in part b) is a root of the equation x3 – 2x2 – 5 = 0 to 1 decimal place. d) f(x) has domain {x : x ≥ 2} and range {f(x) : f(x) ≥ 3}. 2x 2 Find the coordinates of this point, and show that it is a maximum. Tip: Points of inflection are where the curve changes between concave and convex — see pages 135-136. Inverse Functions Inverse functions 'undo' functions. d) P(160 < L < h) = 0.9746 160 - 300 h - 300 fi Pa < Z < 50 k = 0.9746 50 h - 300 fi P a - 2.8 < Z < 50 k = 0.9746 h - 300 fi P a Z < 50 k – P(Z ≤ –2.8) = 0.9746 h - 300 fi P a Z < 50 k = 0.9746 + P(Z ≤ –2.8) h - 300 fi P a Z < 50 k = 0.9746 + 0.0026 h - 300 fi P a Z < 50 k = 0.9772 F(z) = 0.9772 for z = 2.00 (3 s.f.) h - 300 So 50 = 2 fi h = 2 × 50 + 300 = 400 Q5 a) Let H = height in cm. Turning point A stationary point that is a (local) maximum or minimum point of a curve. Some graphs, like tan x, have an asymptote where the line jumps from positive to negative without actually crossing the x-axis, so you might think there's a root when there isn't. –2 = –1(1) + c fi c = –1 so the equation of the normal is y = –x – 1. ﬀ First, solve the differential equation to find the general solution: dP 1 = –kP fi P dP = –k dt dt 1 fi # P dP = # –k dt fi ln P = –kt + C ﬀ At t = 0, P = P0. 2 seconds after being struck, the ball is travelling upwards at an angle of 10° to the horizontal. PROBLE M SOLVING Chapter 14 Kinematics 319 Chapter 15 Dynamics 1. You've contradicted the assumption that there is an odd number x for which x2 is even. 13 N 32° x° The particle is in equilibrium. rough Example 2 A ladder rests against a smooth vertical wall at an angle of 65° to rough horizontal ground. Q8 The graph of k : x Æ tan x is shown below. x 180° 360° –1 ﬀ And if a < 0, the graph is also ﬀ The graph of f(ax) is f(x) y = a f(x) 1 stretched parallel to the y-axis (i.e. vertically) by a factor of a. ﬀ So, P(D|C') = 7 outcomes in D Ç C' number of outcomes corresponding to D 7 = 13 total number of possible outcomes Example 3 Tip: The probability that Vikram catches the bus, given that he walks to the bus stop, is 0.3. 260 Chapter 11 Probability Vikram either walks or runs to the bus stop. = 12.7 m (1 d.p.) Q5 AO = OA = 1 2 + ^- 4h2 + 3 2 = 452 26 BO = OB = ^- 1h + ^- 3h + 5 = 35 It's pretty clear that the magnitude of AO is going to be the same as OA so there's no need to find AO . c) Show that the negative solution of the equation e2x + x2 = 3 lies between x = –2 and x = –1, and find this solution to 1 d.p. Chapter 9 Numerical Methods 233 2. Get rid of this fraction by multiplying the top and bottom by 2x. ﬀ Use the cosine rule to find the angle q from these lengths. Learning Objectives: ﬀ Be able to integrate functions of the fl(x) form , including f (x) multiples of these functions. Find the magnitude of the force F. Q4 Hint: Take extra care with the units here. Parametric and Cartesian Equations 88 93 1. Chapter 16 Moments 2m 3m D B xN 13 N D Mg 6N C 2m C 49 N 356 2N 1.5 m A A B D C b) 10 N ym B RD A uniform rod, AB, of length 5 m, rests horizontally in equilibrium on supports at C and D, as shown. There is a sign change (and the function is continuous in this interval) so there is a root in this interval. dy dy d fi 2x + 2xy – 2y = 10 + 4 dx dx dx Use the chain rule to dy d y dy differentiate y2 and 4y. What percentage of the athletes have a slower personal best time than Gary? Q3 It is thought that the rate of increase of the number of field mice (N) in a given area is directly proportional to N. Suppose f(x) = –6, then |f(x)| = 6. The 25 N force acts vertically downwards, and the rod makes an angle of 40° with the horizontal. At this point, the x-value is x1, the value of your first iteration. T 3m A R 2m B 2m 30 N 40 N Find the tension T in the wire and the force R. 1 f(0.75) = (2 × 0.75) – 0.75 = 0.16... There is a sign change between the upper and lower bounds (and the function is continuous in this interval) so this value is correct to 1 d.p. 8 Q3 a) x2 + 3x – 8 = 0 fi x2 = 8 – 3x fi x = x - 3 b) f(–5) = (–5)2 + (3 × –5) – 8 = 2 f(–4) = (–4)2 + (3 × –4) – 8 = –4 There is a sign change (and the function is continuous in this interval) so there is a root in this interval. Learning Objectives: ﬀ Be able to integrate products of the form du l f (u) using the dx chain rule in reverse. 1 x –1 The domain of f (x) is the range of f(x): {x : x < 1}. z z As before, you can use a calculator to find probabilities. y= 1 x -1 x=1 x domain x > 1 Exercise 2.2 Q1 State whether or not each of the graphs below shows a function, and if not, explain why. ﬀ It'll help to think about what the derivative actually means. = 5.19 kg (3 s.f.) ﬀ Now draw a diagram showing the forces at A. 1 0.01 < 6 so the approximation is valid. Chapter 7 Differentiation 149 The equations for differentiating trig functions can be combined with the chain rule to differentiate more complicated functions. Example 1 Two particles, A and B, of mass 6 kg and 8 kg respectively, are connected by a light, inextensible string. (p, –1) is a minimum point on the graph, so this will be repeated every 2p. p 1 #0 p cosec 2 ` 2x - p4 j dx = >- 11 cot ` 2x - p4 jH cot x = tan x a2k 0 p p Divide by the x p 1 = - 2 8cot ` - jB = - 2 x p 4 = G 2 0 coefficient of x. y=4 x y=4 –1 3x Stretch horizontally by a 1 factor of 3 and translate by e 2 –2 P(–2, –3) c) y 3 0 o -1 P(2, 3) y = –f(x) i.e. 1 down. You can use this method to integrate trig functions by writing them as fractions: ﬀ You can work out the integral of tan x using this method: d sin x tan x = cos x , and (cos x) = –sin x dx ﬀ The numerator is –1 × the derivative of the denominator, so sin x # tan x dx = # cos x dx = –ln|cos x| + C Tip: The integral of tan, – ln|cos x|, is the same as ln|sec x| by the laws of logs. V is the volume in the container and it is equal to Vin – Vout. Find the particle's speed and direction of motion at time t = 7.5 seconds. Leave your answer in surd form. Find P, given that the resultant of the three forces acts at 45° above the positive horizontal. 68% 95% 99.7% standard deviations (±2s) of the mean. In the exam, you'll only have to divide by a linear factor (i.e. with a degree of 1). Differentiating functions multiplied together To differentiate two functions multiplied together, use the product rule: ﬀ Be able to use the product rule together with other rules for differentiation in order to differentiate more complicated functions. + 86 sin 10° = 5.4g + Mg fi M = 8.595... b) Use the cos addition formula: 2p 2 cos bq - 3 l – 5 sin q = 0 2p 2p 2bcos q cos 3 + sin q sin 3 l – 5 sin q = 0 3 m – 5 sin q = 0 2c - 1 cos q + 2 2 sin q –cos q + 3 sin q – 5 sin q = 0 –cos q + ^ 3 - 5h sin q = 0 cos q = ^ 3 - 5h sin q sin q 1 = = tan q ^ 3 - 5h cos q So q = –0.296... This means it's a 'many to one' function — there's more about these on page 18. ax + b 1 The functions ex and x are pretty easy to integrate — it's just the opposite of differentiating ex and ln x. You usually need to make a few assumptions, which will affect the accuracy of your model. There are four negative solutions for 2x in the interval 0 ≤ x ≤ 720° at 124.849..., (360° – 124.849...°), (360° + 124.849...°) and (720° – 124.849...°) and each of these needs to be divided by 2 to give x. 1 Q9 The function m(x) = 2 - is undefined when x = 2 and x 4 when x = –2. Using the conditional probability formula: P^S + Ch P(S|C) = . Q9 Find the possible values of |5x + 4|, given that |1 + 2x| ≤ 3. 1 3^0.3h 27^0.3 2h 189^0.33h ^1 + 3 (0.3)h4 ≈ 1 + – + 4 32 128 = 1.1889 (4 d.p.) ^ 4 1.9 h - 1.1889 d) 4 × 100 = 1.26% (3 s.f.) ^ 1.9 h -1 3x 2 5x 3 Q3 a) ^1 - 2xh 2 = 1 + x + 2 + 2 + ... Tip: Under H0, X ~ N(12, 4), so X ~ N(12, 20) X ~ N(12, 0.2). The random variable X ~ B(800, 0.4) is to be approximated using the normally distributed random variable Y ~ N(m, s2). Equation in s. u-2 3 # x(3x + 2)3 dx = # a u -3 2 k u 13 u - 2u = # du 9 ﬀ So 3 4 du 3 1 u5 u4 = 9 a 5 - 2 k + C u5 u4 = 45 - 18 + C = Tip: The expression in brackets is often the thing you substitute. – 11.810... c) P(X < 300) is approximated by P(Y < 299.5). = 1.265... 1 + cos x dx (1 cos x) 2 cos x Q10 Find any turning points on the curve y = 4 3 cos x in the range 0 ≤ x ≤ 2p. The string makes an angle of 20° with the beam. Let X be the sample mean of Bree's winnings. y d) y=x 5 4 3 2 y = f(x) 1 x1 x2 0 1 2 3 x0 5 6 7 x A divergent cobweb diagram. Answers 411 Q12 y = (2x – 3)7, so let y = u7 where u = 2x – 3 dy du fi = 7u6 = 7(2x – 3)6, =2 du dx dy dy du = × = 7(2x – 3)6 × 2 = 14(2x – 3)6 dx du dx d2 y Use the chain rule again to find : dx 2 dy dy 6 6 = 14(2x – 3) , so let = 14u where u = 2x – 3 dx dx du d c dy m fi = 84u5 = 84(2x – 3)5, =2 du dx dx 2 d y d c dy m d c dy m du = = × = 84(2x – 3)5 × 2 dx dx dx du dx dx 2 = 168(2x – 3)5 d2 y At a point of inflection, =0 fi 2x – 3 = 0 dx 2 3 fi x= 2 2 d y 3 When x < 2 , (2x – 3) < 0 fi 2 , (2x – 3) > 0 fi >0 dx 2 2 d y 3 So the sign of changes at x = 2 , dx 2 so this is a point of inflection. 2) D raw a horizontal line to y = x. A light, inextensible string attached at the opposite corner applies a tension force T to the lamina as shown. x Answers 409 Q5 a) f(x) = x3 + 2x2 + 3x + 3 fi f'(x) = 3x2 + 4x + 3 fi f''(x) = 6x + 4 At a point of inflection, f''(x) = 0 2 fi 6x + 4 = 0 fi x = - 3 2 When x > - 3 , 6x + 4 > 0 2 and when x < - 3 , 6x + 4 < 0 So the graph of y = f(x) has one point of inflection, 2 at x = - 3 . - 1 a- 1 - 1k 2 2 -1 1 2 + ^ h b) (4x)2 + ... Give your answer to 3 d.p. Q4 The shape of an aeroplane's wing is modelled by the curve y = ln x , where x and y are measured in metres. Simplifying Expressions Exercise 1.1 — Simplifying algebraic fractions 4 4 = = 2 2x + 10 2^ x + 5h x + 5 5x = 5x Q2 x 2 + 2x x (x + 2) x + 2 6x 2 - 3x = 3x (2x - 1) = 2x - 1 Q3 x 3x 2 3x 2 Q1 4x 3 = 4x 3 = 4x x 3 + 3x + 6 = 3 (x + 2) = 3 Q5 x 2 + 3x + 2 (x + 1) (x + 2) x + 1 x 2 + 3x = x (x + 3) = x Q6 x 2 + x - 6 (x - 2) (x + 3) x - 2 2x - 6 = 2 (x - 3) = 2 Q7 x 2 - 9 (x - 3) (x + 3) x + 3 5x 2 - 20x = 5x (x - 4) = 5x Q8 2x 2 - 5x - 12 (2x + 3) (x - 4) 2x + 3 3x 2 - 7x - 6 = (3x + 2) (x - 3) = 3x + 2 Q9 2x 2 - x - 15 (2x + 5) (x - 3) 2x + 5 x 3 - 4x 2 - 19x - 14 = (x + 1) (x + 2) (x - 7) = + Q10 x 2 x 2 - 6x - 7 (x + 1) (x - 7) To factorise the cubic, try a few different values for x — once you've spotted that f(–1) = 0 (and so (x + 1) is a factor) you can take that out and see what's left to factorise. 3x 2 x = 1 + 5 + 50 + ... b) At the 1% level, you'll have a different critical region, but everything else is the same. = m × 35 fi m = 0.19 (2 d.p.) Q3 a) 2.25g Resolving horizontally: RH + (27.237...) cos 30° = 110 sin 30° fi RH = 31.411... Proof by Contradiction 1 Chapter 2 — Algebra and Functions 1. l b1 + l Q9 a) gf(x) = gb 1 x = ln x 1 Domain: {x : x > 0} (domain of g = range of f = {x : x > 0}) Range: {gf(x) : gf(x) > 0}. You can use the chain rule in reverse to integrate special products of functions and their derivatives. ﬀ Is the model appropriate? When a bucket of water of mass 18 kg is placed at point C, the plank is in equilibrium, and is on the point of tilting about B. The function f(x) = x + 7 is one-to-one, so it does have an inverse. You're given information about two events: Let L = 'Horace is late' and let S = 'Horace is shouted at'. The Trapezium Rule 230 234 238 241 245 251 254 1. s 2.273... Example 2 The mapping shown here is a function, because any value of x in the domain maps to only one value in the range. Don't just multiply the top by 2x and the bottom by x. Putting this into the Newton-Raphson formula gives: x 4 - 2x 3 - 5 xn + 1 = xn - n 3 - n 2 . N Vertical component = 15 sin 30° = 7.5 N ﬀ So the resultant force has a horizontal component of 20 + 12.99... This means that, in the i direction: 1 x3 3 3 3 3 = 9t fi x = 27t = 3 t = 3t ﬀ In the j direction: dy 2 = y fi dt # y dy = # 2 dt fi y2 2 = 2t + D ﬀ y = 0 when t = 0, so D = 0. f(x + h) f(x) It can therefore be rewritten as a sum of the areas of the "extra bits" on the diagram: f(x + h)g(x + h) – f(x)g(x) = Area(A) + Area(B) Area(A) = g(x + h)[f(x + h) – f(x)] A B g(x) g(x + h) Area(B) = f(x)[g(x + h) – g(x)] g (x + h) [f (x + h) - f (x)] + f (x) [g (x + h) - g (x)] m h + + g (x h) [f (x h) f (x)] m f (x) [g (x + h) - g (x)] m = lim c + lim c h h h"0 h"0 ^ fghl(x) = lim c h"0 Tip: g(x)f'(x) + f(x)g'(x) is just another way of writing u dv + v du . The resultant is shown by the hypotenuse of the triangle. The objects are being pushed up a rough slope inclined at an angle q to the horizontal, by a force of 20 N acting parallel to the slope. 1 = (1 + 7x)-4 ^1 + 7xh4 - 4^ - 4 - 1h^ - 4 - 2h The x3 term is (7x)3 1#2#3 - 4 # -5 # -6 = (343x3) 6 Q5 a) = –20(343x3) = –6860x3 So the coefficient of the x3 term is –6860. b) By writing the first derivative of y = (x – 2)2(x + 4)3 in the form d2y dy = (Ax2 + Bx + C)(x + D)n, find and hence identify dx dx 2 the nature of the stationary points of the curve. Inverse function An inverse function, e.g. f –1(x), reverses the effect of the function f(x). so x = –1.7720 (5 s.f.) Here, you could find the exact roots using the quadratic formula. x = cos–1(0.52631...) = 58.2° (1 d.p.) There is another positive solution in the interval 0 ≤ x ≤ 360° at (360° – 58.2°), so x = 301.8° (1 d.p.) Remember, you can use the graphs or the CAST diagram to find other solutions in the interval. ms–2 ﬀ List the variables: u = 0, v = v, a = 3.454..., t = 3 v = u + at = 0 + 3.454... Example 2 Differentiate e2x 2x - 3 with respect to x. Conditional Probability Learning Objectives: ﬀ Be able to use set notation to represent probabilities. The vertical component of its velocity is unaffected by the impact. np = 98 but n(1 – p) = 2, so a normal approximation would not be suitable. 2 b ﬀ The area of the trapezium is an approximation of the integral # a b f (x) dx. So if the claim is correct, the height of the leeks after 28 days will be the 15th term of a geometric progression with first term 5 and common ratio 1.15. This is bigger than the domain of f(x), so the domain of g(x) will need to be restricted. MODELL ING A O The toy can be modelled as a large cuboid with a smaller cuboid section missing. n = 9 doesn't satisfy the inequality. So |x| < 5 fi –5 < x < 5. So P(171 < H < 179) = 0.499. Given that the particle is on the point of moving, calculate the coefficient of friction if: a) W = 20 N, P = 5 N b) W = 15 N, P = 8 N Q3 A particle of mass 12 kg is placed on a rough horizontal surface. When x = 1, y = 12 – 1 = 0, 1 so (1, 0) is a point of inflection of y = x2 – 1 . db 1 = 4 b gives: db = 4 dt dt b ﬀ Integrate both sides: #b fi 2b 2 = 4t + C -1 2 db = # 4 dt 1 ﬀ In this case, it's easier to find C for the given values of b and t before you rearrange the equation: 2 b = 4t + C fi 2 900 = 4(12) + C fi 60 = 48 + C fi C = 12 ﬀ Now rearrange to get the form b = f(t): 2 b = 4t + 12 fi b = 2t + 6 fi b = 4t2 + 24t + 36 224 Chapter 8 Integration Exercise 11.2 Q1 Find the general solutions of the following differential equations where x ≥ 0. Differentiating cosec, sec and cot Remember from page 55 the definitions of these trig functions: 1 cosec x º sin x 1 sec x º cos x 1 cos x cot x º tan x º sin x ﬀ Since cosec, sec and cot are just reciprocals of sin, cos and tan, the quotient rule can be used to differentiate them. a) X ~ N(48, s2) and P(X < 53) = 0.8944 b) X ~ N(510, s2) and P(X < 528) = 0.7734 c) X ~ N(17, s2) and P(X > 24) = 0.0367 d) X ~ N(0.98, s2) and P(X < 0.95) = 0.3085 e) X ~ N(5.6, s2) and P(X > 4.85) = 0.8365 Chapter 12 The Normal Distribution 279 Q4 X ~ N(68, s2). As the magnitude of the force which is trying to move the object increases, the magnitude of the frictional force will also increase. a) Prove that AB = OA . Give your answers in the form y = f(x). The ladder is in limiting equilibrium. Integration of f (x) Fractions in which the numerator is the derivative of the denominator are pretty easy to integrate — there is a general formula which comes from the chain rule. Tip: To see visually why this formula works, draw a diagram with two right-angled triangles like the one on the previous page. dx dx ﬀ Put these into the formula: # 2 - 3 x x(x – 1)–3 dx = 8- 2 ^ x - 1h 2 B2 - 3 # 2 3 - 1 ^ x - 1h-2 dx 2 - 3 - 3 x 1 = 8- 2 ^ x - 1h 2 B2 - 2 6^ x - 1h 1 @2 3 2 1 = :- 2 ^ 2h-2 + 2 ^1 h-2 D - 2 62 -1 - 1 -1 @ 3 1 1 = :- 8 + 1 D - 2 : 2 - 1 D 5 1 7 = 8 + 4 = 8 ﬀ Now you can evaluate the original integral: 3 x2 2 3 x2(x – 1)–4 dx = :- 3 ^ x - 1h 3 D + 3 # x^ x - 1h 3 dx 2 2 2 3 x2 2 7 = :- 3 ^ x - 1h 3 D + 3 b 8 1 2 9 4 7 = ;a - 3 ^ 2h 3 k - a - 3 ^1 h 3 kE + 12 9 4 7 = :- 24 + 3 D + 12 23 7 37 = 24 + 12 = 24 # 3 Exercise 9.2 Q1 Use integration by parts twice to find: Q3 Hint: Since x ≥ 0 and e–2x > 0 for all x, the area will be entirely above the x-axis. Answers 385 y c) y = cot c) x 4 y 5 y = 2 + 3 cosec x x –180° 360° Q5 a) y = 2 + sec x is the graph of y = sec x translated vertically up by 2: y = 2 + sec x y 1 2 3 2 2 x b) The minimum points are at (–2p, 3), (0, 3) and (2p, 3). e4 So the equation of the tangent is y = 6 (x – 4). ﬀ Resolving perpendicular to the plane ( ): R – 100 cos 35° = 0 fi R = 100 cos 35° = 81.9 N (to 3 s.f.) b) The sledge remains stationary. a) b) c) 21 N 3.2 N 10 N x 45° x x Q3-4 Hint: Finding the direction and magnitude of a vector given in components was covered in Year 1. MODELL ING 7x cm 4x cm 4x cm After t minutes, the triangle that forms the base of the prism has base length 7x cm and height 4x cm, and the length of the prism is also 4x cm. c) If the student is not studying Maths, what is the probability that they are studying English? Q7 State the largest possible domain and range of each function: a) f(x) = 3x – 1 b) g(x) = (ln x)2 Q4 Hint: 'Evaluate' is just another way of asking you to 'find the value of'. x 102 78 f(x) P(X ≤ 78) 1 – P(X ≤ 78) ﬀ Input the upper bound x = 78, a lower bound, m = 102 and s = 144 = 12 into the normal cdf 78 102 function on your calculator. ﬀ Use the binomial cdf on your calculator, with n = 20 and p = 0.15. If the domain of f(x) does not fully include the range of g(x) then the domain of g(x) will have to be restricted further. 1.277... Tip: Make sure you know how to calculate normal probabilities on your calculator. d) Suggest two adaptations that would allow the model to better represent the movement of an object underwater. ﬀ Be able to model A projectile is an object which has been projected (e.g. thrown or fired) through the air. There is a sign change between the upper and lower bounds (and the function is continuous in this interval), so a solution to 1 d.p. is x = 2.8. Q8 Upper and lower bounds are 0.65 and 0.75 1 f(0.65) = (2 × 0.65) – 0.65 = –0.23... So the gradient is 0 when x = 1. You may remember these derivatives from Chapter 7, but here's a recap: d dx (cosec x) = –cosec x cot x d dx (sec x) = sec x tan x d 2 dx (cot x) = –cosec x Tip: You'll be given these derivatives in the formula booklet. b) The distribution of cars' speeds on a different stretch of road is shown in the diagram below. = 82.8 N (3 s.f.) cos 15° - 0.1 sin 15° 3. dx 2 2 d y So changes sign at x = 1. ﬀ The chain rule you met on p.139 can be used to differentiate parametric Tip: On page 139, you saw this given in the dt form '× ' rather than dx dx '÷ ', but it means dt the same thing. Chapter 14 Kinematics 315 Example 3 A body is moving on a plane. = 35.168... = 1 – 6x + 36x2 – ... The particles collide 2 seconds after they are projected. A mapping is just a set of instructions that tells you how to get from one value to another, and a function is a special kind of mapping. 28 ÷ 2 = 14, so there are 14 lots of 2 days in 28 days In 28 days, the height should increase 14 times by 15%. Q3 a) Show that the equation x4 – 5x + 3 = 0 has a root between x = 1.4 and x = 1.5. 3 b) Use the iterative formula xn+1 = 3 5 - xn and x0 = 1.4 to find iterations x1 to x6 to 3 decimal places. b) r = –0.8925 is close to –1, so it suggests strong negative correlation in the sample, i.e. higher weight tends to mean worse kidney function. or t = 2.273... Tip: Remember — d 1 ln y = y . Inverse Trig Functions Exercise 3.1 — Arcsin, arccos and arctan Q1 a) If x = arccos 1 then 1 = cos x so x = 0. b) Find the equations of the tangent and normal to the curve at the other point. Evaluate fg(90°) and gf(90°). Tip: The expression f l(x) g (x) - f (x) gl(x) (g (x)) 2 is just another way of writing the quotient rule du - dv v u dx dx . Using symmetry, P(Z ≤ –z) = 0.0574 ÷ 2 = 0.0287 fi P(Z ≤ –z) = 0.0287 for z = –1.90 (3 s.f.) So z = 1.90 (3 s.f.) b) P(z < Z < –1.25) = 0.0949 fi P(Z < –1.25) – P(Z ≤ z) = 0.0949 fi P(Z ≤ z) = P(Z < –1.25) – 0.0949 fi P(Z ≤ z) = 0.1056 – 0.0949 = 0.0107 F(z) = 0.0107 for z = –2.30 (3 s.f.) Exercise 1.5 — Finding the mean and standard deviation of a normal distribution 23 - m k = 0.9332 Q1 a) P(X < 23) = 0.9332 fi P a Z < 6 F(z) = 0.9332 for z = 1.50 (2 d.p.) 23 - m fi = 1.5 fi m = 23 – 1.5 × 6 = 14 6 57 - m k = 0.9970 b) P(X < 57) = 0.9970 fi P a Z < 8 F(z) = 0.9970 for z = 2.75 (2 d.p.) 57 - m fi = 2.75 fi m = 57 – 2.75 × 8 = 35 8 528 - m c) P(X > 528) = 0.1292 fi P a Z > 100 k = 0.1292 528 - m fi P a Z # 100 k = 1 – 0.1292 = 0.8708 F(z) = 0.8708 for z = 1.13 (2 d.p.) 528 - m fi = 1.13 fi m = 528 – 1.13 × 100 = 415 100 11.06 - m k = 0.0322 d) P(X < 11.06) = 0.0322 fi P a Z < 0.4 F(z) = 0.0322 for z = –1.85 (2 d.p.) 11.06 - m fi = –1.85 0.4 fi m = 11.06 – (–1.85) × 0.4 = 11.8 Answers 455 1.52 - m e) P(X > 1.52) = 0.9938 fi P a Z > 0.02 k = 0.9938 m - 1.52 fi P a Z < 0.02 k = 0.9938 F(z) = 0.9938 for z = 2.50 (2 d.p.) m - 1.52 fi 0.02 = 2.5 fi m = 2.5 × 0.02 + 1.52 = 1.57 Here you use the fact that P(Z < z) = P(Z > –z) for the standard normal distribution. Normal reaction The reaction force from a surface acting on an object. Q11 Hint: Remember the log laws from Year 1. ﬀ Use Pythagoras' theorem to find the magnitude: 1.354... Q6 a) When the toy is moving due north, the velocity in the i-direction is zero. ﬀ When integrating a function like this, you integrate each term separately, so: # 6x 2 - 4x + 3e x dx = # 6x 2 dx - # 4x dx + # 3e x dx ﬀ The first two terms are of the form xn so they integrate to 2x3 and 2x2 respectively. ﬀ Write down the suvat values as usual: u = 0.5i – 1.2j, v = 0, a = a, t = 4 Tip: Not all constant acceleration problems involve gravity — you can use the suvat equations whenever you're told to assume that acceleration is constant. Modulus 6. To find the probabilities, divide every entry by the new total of 20. R – 0.1g cos 20° = 0 R = 0.98 cos 20° = 0.921 N (to 3 s.f.) Chapter 15 Dynamics 327 Example 3 MODELL ING Two brothers are fighting over a sledge of weight 100 N. For example, if you compare the values of P(B|A) and P(B), you can see that: Tip: Another way of getting P(B'|A) is to do 1 – P(B|A), since B|A is the complement of B'|A. Find: a) the horizontal range of the particle, b) the speed and direction of the particle when it lands. The coefficient of friction between A and the plane is 0.7. B is released from rest and falls d m vertically to the ground in 2 seconds. Putting this into y = mx + c gives 2 the equation of the tangent as y = 3x + 3. ﬀ When something changes over time, the derivative is d of that variable.) = 64 – 960x + 6000x2 – ... # x4 sin (x5) dx = 15 # 5x4 sin (x5) dx ﬀ Now split up the integral: du = 5x4 and f'(u) = sin (x5) dx ﬀ Use the formula. This means that a frictional force, F, acts to try and prevent any motion. P^F + Gh a) P(G|F) = P (F) 2 5 P(F Ç G) = 11 and P(F) = 11 2 P^F + Gh = 2 So, P(G|F) = P (F) = 11 5 5 11 P^F + Gh and P(G) = 3 . 0 X=1 X=2 X=3 X=4 X The interval you need to use with the normal distribution depends 1.5 to find. Find f(x). Q2 A stone is catapulted with speed 12 ms–1 at an angle of 37° above the horizontal. # cot5 x dx = 15 ln|sin x| + C # # 2(cosec x + sec x) dx # 2(cosec x + sec x) dx = # 2 cosec x + 2 sec x dx d) 2 ln|sec x + tan x| + C = –2ln|cosec x + cot x| + 2ln|sec x + tan x| + C Use log laws to simplify. In part c), 1 + (–1) = 0, so (x – 1) is a factor of the cubic. Find a. Q4 Evaluate the following definite integrals, giving exact answers: # e dx b) a) 3 2 p 2 2x #p e c) - 2 p - 2x dx d) # # 0 -1 3 6 12e 6 12x + 12 dx 1 e x + 3 x dx e Q4 d) Hint: Remember: x x n = en e 1 n e x x = e- n Chapter 8 Integration 183 Integrating 1x and 1 ax + b The method for integrating xn and (ax + b)n on p.180 doesn't work for n = –1. Just replace each x in the function with –2 and calculate the answer: f(–2) = (–2)2 – 1 = 4 – 1 = 3 b) Find the value of x for which f(x) = 12 for the function f : x Æ 2x – 3. 8.2 N b) a) 4N c) 11 N 5N 45° 39° 7 N d) y Hint: You can answer these questions either by resolving the forces, or drawing a vector triangle and using trigonometry and Pythagoras' theorem. b) The root in part a) can be estimated using the iterative formula 2 xn+1 = xn + 5. Q12 a) The distances she runs form a geometric sequence with a = 12, r = 1.03. Put each fraction over the common denominator ﬀ Multiply both top and bottom of each fraction by the same factor — whichever factor will turn the denominator into the common denominator. Often used to describe a body in equilibrium. ING Q7 Target 1 24 ms–1 ING Target 2 70° h MODELL A ball is fired from a machine with speed 24 ms–1 at an angle of 70° above the horizontal. Q7 Hint: To differentiate parametric equations, use dy dy dx = ' dx dt dt (see p.169). R Radian A unit of measurement for angles. 2 –0.02 = U sin 5° U cos 5° – 4.9a U cos 5° k 0.958... The two components of velocity When you project a body through the air with initial speed u, at an angle of q to the horizontal, it will move along a curved path: projectile motion under gravity, and understand the modelling assumptions. Differentiation A method for finding the rate of change of a function with respect to a variable — the opposite of integration. Equating coefficients Making the coefficients of equivalent terms on each side of an identity equal in order to calculate the value of unknowns in the identity. Q4 A particular type of toy car uses two identical batteries. 2 x 3 –3 c) f(x) 1 Range: –1 ≤ f(x) ≤ 1 x 90° 180° 270° 360° –1 d) f(x) Range: 0 ≤ f(x) ≤ 5 5 Answers Q10 a) It is not a function because it is not defined for all values of x in the domain (it's not defined at x = 0 or x = 4). ﬀ You're told that the number of bacteria (b) increases as time (t) increases db — so that's the rate of change of b with respect to t, or . f(x) Area = P(X ≤ a) = P(X ≤ b) P(a ≤ X ≤ b) ﬀ The probability of the variable taking a value between two limits is the area under the curve between those limits. dy 2 Q6 = f'(x)ef(x) = 4xe 2x dx dy At x = 1, = 4 × 1 × e2 = 4e2 and y = e2 dx y = mx + c fi e2 = (4e2 × 1) + c fi c = –3e2 So the equation of the tangent is y = 4e2x – 3e2. 0.64 43.5 - 42 Now x = 43.5, so z = = 1.875 0.64 You're interested in the higher end of the distribution, so find the critical value, z, such that P(Z > z) = 0.05. So at the stationary points: dx x2 + 2xy – y2 = 10x + 4y – 21 fi x2 + 2x(5 – x) – (5 – x)2 = 10x + 4(5 – x) – 21 fi x2 + 10x – 2x2 – 25 + 10x – x2 = 10x + 20 – 4x – 21 fi –2x2 + 14x – 24 = 0 fi x2 – 7x + 12 = 0 fi (x – 3)(x – 4) = 0 fi x = 3 or x = 4 x = 3 fi y = 5 – 3 = 2 x=4fiy=5–4=1 Put each value of x into y = 5 – x to find the corresponding y-coordinate. 0.958... ÷ 3.5 = 114.521... Putting this into y = mx + c gives: 12 31 1 = 5 (3) + c fi c = - 5 so the equation of the normal is y = 12 x - 31 5 5 or 12x – 5y – 31 = 0. Chapter 10 Learning Objectives Showing which bits of the AQA specification are covered in each section. Q4 a) On the same axes sketch the graphs of |f(x)| and |g(x)|, where: f(x) = 5x + 10 and g(x) = x + 1 b) Hence find any solutions of the equation |f(x)| = |g(x)|. Q7 A package of mass 9 kg is moving up the line of greatest slope of a rough plane inclined at an angle of 22° to the horizontal. You know sin q = 13 , 12 so using the triangle method, cos q = 13 . Example 2 A ball is kicked from a point on horizontal ground. Show that: g 1 = 3 tan a – 2 cos 2 a PROBLE M SOLVING Q16 A PROBLE M SOLVING 2U 45 m B U q 15 m Particle A is projected horizontally from a point 45 m above ground level. 8 x 3 28 x3 x = 5x × 12 = 5x × 12 3 2 2× x2 2x 2 = 5 × 3 = 15 Cancel all common factors. Connected Rates of Change Exercise 8.1 — Connected rates of change Q1 dx = –0.1 (it's negative as the cube is shrinking), dt dV and V = x3, so = 3x2 dx dV dV dx = × = –0.3x2 cm3 min–1 dt dx dt dV dx = 90x2, and = 0.15. 9 - x 2 for x d R , Q6 Hint: 9 – x2 cannot be negative, as you can't take the square root of a negative number, so work out the values of x for which 9 – x2 ≥ 0 and use these as the domain. 2 Q9 Find any stationary points for the curve y = xe x - x . ﬀ Compare this value of r to the critical region — r = –0.28 is greater than the

critical value of –0.3783, so r is not in the critical region and the result is not significant. Q7 a) Find f’(x) if f(x) = 3 sin x + 2 cos x. So x = 63.4° and 153.4° (1 d.p.) Q10 (cosec x – 3)(2 tan x + 1) = 0 means that either 1 1 cosec x = 3 (and so sin x = 3) or tan x = - 2 . You can either keep the converted limits in the same places as the corresponding original limits and carry on as normal or swap them so the higher value is the upper limit and stick a minus sign in front of the whole integral. Example 3 A curve defined implicitly by sin x – y cos x = y2 passes through two points (p, a) and (p, b), where a < b. Then just multiply as before. 2^0h + 2 dy 2 At (0, 3), = = 3 dx 2 ^3 h - 3 The y-intercept is 3 (as it goes through (0, 3)). b) Find the equations of the normals to the curve at each of these points and explain why these normals cannot intersect. 31° 31° F 14 N Q4 A particle is acted on by a force of magnitude 9 N in the positive horizontal direction and a force of magnitude 10 N at 30° above the positive horizontal. One end of a light, inextensible string is attached to B, and the other end is fixed to the wall, directly above A, as shown. Finding Area using Integration Learning Objective: ƒƒ Be able to use integration to find the area between two curves, or the area between a line and a curve. a) Gary’s personal best time is 49.3 seconds. Chapter 8 Integration 225 You may also have to identify limitations of the model, as well as suggest possible changes that would improve it. Then under H0, X ~ N(70, 64), 64 so X ~ N a70, 40 k = N^70, 1.6h. 3x + 1 Trig identities Tip: If you need a reminder of the trig identities, they’re given in Chapter 3. p Also, because cos is symmetrical, C = – 6 will give you the same answer — either one is fine. Q5 Q8 dy = f’(x)ef(x) = (3 × e3x) = 3e3x dx When it crosses the y-axis, x = 0, so y = e3 × 0 + 3 = 4 dy = (3 × e3 × 0) = (3 × 1) = 3 dx 1 fi Gradient of the normal = – 3 1 y = mx + c fi 4 = (– 3 × 0) + c fi c = 4 1 So the equation of the normal is y = – 3 x + 4. radians s = rq = 8 × 0.802... A horizontal force of 8 N acts on the lamina in the negative x-direction at B, and a vertical force of 11 N pulls upwards on the lamina at C. The Binomial Expansion 120 2. You need to find how many are needed to exceed 1000 (£10 in pence). b) Evaluate when t = –1. ƒƒ When the particle is directly above O, the i-component of its position vector will be zero: 8t + t2 – 10 = 0 ƒƒ Solve for t using the quadratic formula: 8 2 - ^4 # 1 # - 10h 2 fi t = 1.099... Example The curve C is defined by the parametric equations x = t2 – 1 and y = t3 – 3t + 4. a) Find T, the magnitude of the tension in the wire. Make sure you don’t get caught out here — the brackets are in the form (qx + p)n, not (p + qx)n. Chapter 9 Numerical Methods 239 Exercise 3.1 Q1 Hint: You’ll need to draw the line y = x on each diagram. Putting this into the Newton-Raphson formula gives: h (xn) sin (xn) + xn3 - 1 = xn xn + 1 = xn hl(xn) cos (xn) + 3xn2 Q2 Differentiating f(x) gives f’(x) = 4x3 – 6x2. The rod is uniform, so its mass acts at its midpoint, x m from D. 178 Chapter 7 Differentiation Q2 A curve is defined implicitly by x3 + x2 + y = y2. (3x + 1)–2 = 1 – 6x + 27x2 + ... PROBLE M SOLVING Q10 Differentiate y = cos2 x by: a) Using the chain rule directly. 6 ƒƒ Here you’re given the numbers of outcomes, so you have to find the probability of the events you’re interested in. b) Verify that a root of the equation x2 + 3x – 8 = 0 lies in the interval (–5, –4). Q2 y = x + 4 is a map with domain {x : x d N , x ≤ 7}. Questions like the following examples can seem a bit overwhelming at first, but follow things through step by step and they shouldn’t be too bad. 1 x), multiply the whole algebraic fraction (i.e. top and bottom) by 1 the same factor to get rid of it (for x , you’d multiply through by x). Exercise 1.5 Q1 For each of the following, use the information to find m. Chapter 12 The Normal Distribution 277 In the first two examples you found the mean, but you can find the standard deviation (s.d.) in exactly the same way. dx 2 d2 y = 2x3 – 2x = 2x(x2 – 1) = 2x(x + 1)(x – 1) dx 2 When x < –1, 2x is negative, x + 1 is negative and x – 1 is d2 y negative, so is negative for x < –1. Area Area = a x b ƒƒ Values of the cumulative distribution function (cdf) Tip: For any continuous random variable X, P(X = a) = 0 for any value of a. MODELL ING Q5 OA = i – 4j + 3k and OB = –i – 3j + 5k. The large cuboid section is an enlargement of the smaller, ‘missing’ cuboid, by a scale factor of 2, centred at point O. y = f(x) ƒƒ The y-values are found by putting the x-values into the equation of the curve — so y1 = f(x1). N Resolving vertically: RV + 1.75g + 2.25g = (27.23...) sin 30° + 110 cos 30° fi RV = 69.681... 2.449 - 1 2.5 - 1 and so on, until you find after 16 iterations that the value of the root, to 2 d.p., is: ƒƒ Get the x2 on its own by dividing by x − 1: x2 = x = 2.47 b) Show that x3 − x2 − 9 = 0 can also be rearranged into x = and use this to make an iteration formula. Pulley A wheel, usually modelled as fixed and smooth, over which a string passes. a) Find an expression for the rate of change of radius with respect to temperature. × 3) = 12.4 ms–1 (3 s.f.) Q2 a) R 1.2 ms–2 26 N F 4g b) Resolving vertically: R – 4g = 0 R = 4g = 39.2 N c) Resolving horizontally: Fnet = ma 26 – F = 4 × 1.2 F = 21.2 N d) The box is moving, so friction takes its maximum value, i.e. F = mR: 21.2 = 39.2m fi m = 0.54 (2 d.p.) Q3 a) 0.8 ms–2 R P F 7g b) Resolving vertically: R – 7g = 0 fi R = 7g = 68.6 N c) Resolving horizontally: Fnet = ma P – F = 7 × 0.8 P – mR = 5.6 P = 5.6 + (0.2 × 68.6) = 19.3 N (3 s.f.) Q4 a) R 70 N a F 40° mg b) Resolving perpendicular to the plane (): R – mg cos 40° = 0 fi R = mg cos 40° Resolving parallel to the plane (): Fnet = ma 70 – F – mg sin 40° = 3.2m 70 – mR – mg sin 40° = 3.2m 70 – 0.35(mg cos 40°) – mg sin 40° = 3.2m 70 = m(3.2 + 0.35g cos 40° + g sin 40°) m = 70 ÷ 12.126... Answers 451 Q6 A vector is parallel to another if it is multiplied by the same scalar in each direction. dx 3x - 1 du dv 2 u = 2 + 3x , v = 3x – 1 fi = 6x, =3 dx dx du - dv v u 7^3x - 1h × 6x A - 6^2 + 3x 2h × 3 @ dy = dx 2 dx = dx v (3x - 1) 2 2 2 (18x - 6x) - (6 + 9x) 9x 2 - 6x - 6 = = (3x - 1) 2 (3x - 1) 2 Since the denominator is squared, it is always positive. Normal Approximation to a Binomial Distribution Learning Objectives: ƒƒ Be able to apply a continuity correction when using a normal distribution to approximate a discrete random variable. Give your answer 2 in the form y = mx + c, using exact values for m and c. x+1 a) Sketch the graphs of f(x) and f –1(x) on the same set of axes. The graph shows that there are two solutions to 2|x – 4| = x, one where x > 4 and one where x < 4. ƒƒ If the sum of the moments about a particular point is not zero, the rod will turn clockwise or anticlockwise about that point. Particle A is held 20 cm from the pulley and, when released from rest, accelerates towards the pulley at a rate of 0.4 ms–2. So after 10 years, u11 = ar10. The resultant of the three forces has no horizontal component. Here, the component of the object’s weight acting down the slope could cause it to slide down the plane. Does this data provide evidence at the 1% level of significance that the machine is dispensing a mean volume that is different from 30 ml? f(3.5775) = e3.5775 - (10 × 3.5775) = 0.0089... Taking moments about A: (0.1g × x) + (2 × 6) = 16 cos 70° × 3 0.1gx = 4.416... Use a 5% and a 1% level to test Kyle’s belief. N If the block is on the point of sliding down the plane, then friction acts up the plane: R D F R a g 50° b) Resolving perpendicular to the plane (): R = g cos 50° Resolving parallel to the plane (): Fnet = ma g sin 50° – F = a g sin 50° – mR = a g sin 50° – (0.4 × g cos 50°) = a fi a = 4.9875... ƒƒ Be able to identify diagrams of iterations that show convergence or divergence. Tip: The curve y = x2 – 2x crosses the x-axis at x = 0 and x = 2. Example 3 Divide (2x3 − 7x2 − 16x + 11) by (x − 5). b) Find P(J’ Ç K’). Exercise 1.3 Simplify the following: 2x 5x 6x 3 2 Q1 Hint: Remember — Q1 a) 3 × 4 b) 7 × x2 cancelling before you 2 multiply will make things 8x 4 6 y 8x 2 x 3 c) d) × 2 × a whole lot simpler. 1 s = ut + 2 at2 = 3 cos 7° × 1.770... Here are the two methods... 5 sin (q – 56.1°) = 29 fi q – 56.1° = 68.2° and (180° – 68.2°) = 111.8° fi q = 124.3° and 167.9° (1 d.p.) c) f(x) = 29 sin (x – 56.1°). + bn (n ! N) 1 2 r n n! n where d n = C r = r! ^n - r h ! r n (n - 1) 2 n (n - 1) ... Tip: To get maximum marks you need to include all the steps above and write a conclusion that links the answer to the context of the question. Q7 86 N 10° A B 5.4g Mg A non-uniform plank, AB, of mass 5.4 kg and length 11 m, rests with A against a rough, vertical wall. The particle is held in place by a light, inextensible string inclined at an angle of 30° to the plane. c) Hence write down an approximation of the root from part a) to 2 decimal places. dx 2 When –1 < x < 0, 2x is negative, x + 1 is positive and d2 y x – 1 is negative, so is positive for –1 < x < 0. ƒƒ In some questions you might be asked to sketch the graphs of Tip: Setting the equations equal to each other and then rearranging them to get f(x) = 0 gets you to where you were in the previous section. This will come in handy in other parts of maths, so it’s a pretty important skill. Q6 Find the domain and range of f –1(x) for the following functions: a) f(x) = log10 (x – 3), x > 3 c) f(x) = e) f(x) = tan x, 0° ≤ x < 90° x for {x : x < 2} x-2 b) f(x) = 4x – 2, 1 ≤ x ≤ 7 d) f(x) = 3x – 1 for {x : x ≥ 2} f) f(x) = ln (x2) for {x : 3 ≤ x ≤ 4} Q7 Find the inverse f –1(x) for the following functions, giving the domain and range: a) f(x) = ex + 1, x d R b) f(x) = x3, x < 0 1 c) f(x) = 2 – log2(x), x ≥ 1 d) f(x) = - for {x : x ≠ 2} x 2 Q8 f(x) = 2x + 3, x d R . Q1 a) P(Z ≤ 1.87) = 0.969 (3 s.f.) b) P(Z < 0.99) = 0.839 (3 s.f.) c) P(Z > 2.48) = 0.00657 (3 s.f.) d) P(Z ≥ 0.14) = 0.444 (3 s.f.) e) P(Z > –0.24) = 0.595 (3 s.f.) f) P(Z > –1.21) = 0.887 (3 s.f.) g) P(Z < –0.62) = 0.268 (3 s.f.) h) P(Z ≤ –2.06) = 0.0197 (3 s.f.) i) P(Z ≥ 1.23) = 0.109 (3 s.f.) Q2 a) P(1.34 < Z < 2.18) = 0.0755 (3 s.f.) b) P(0.76 ≤ Z < 1.92) = 0.196 (3 s.f.) c) P(–1.45 ≤ Z ≤ 0.17) = 0.494 (3 s.f.) d) P(–2.14 < Z < 1.65) = 0.934 (3 s.f.) e) P(–1.66 < Z ≤ 1.66) = 0.903 (3 s.f.) f) P(–0.34 ≤ Z < 0.34) = 0.266 (3 s.f.) g) P(–3.25 ≤ Z ≤ –2.48) = 0.00599 (3 s.f.) h) P(–1.11 < Z < –0.17) = 0.299 (3 s.f.) For some of these questions, you could have used the symmetry of the distribution to work out the answer. x dx = –cot x + C Chapter 8 Integration 201 Example 3 a) Find # tan 2 x - 1dx sec2 x º 1 + tan2 x ƒƒ Rewrite the function in terms of sec2x: tan2 x – 1 = (sec2 x – 1) – 1 = sec2 x – 2 # tan ƒƒ Now integrate: b) Find 2 x - 1 dx = # sec 2 x - 2 dx = tan x - 2x + C # cot 3x dx 2 ƒƒ Get the function in terms of cosec2x: cot2 3x = cosec2 3x – 1 # cot c) # cos 3 cosec2 x º 1 + cot2 x 2 3x dx = # cosec 2 1 3x - 1dx = - 3 cot 3x - x + C x dx ƒƒ You don’t know how to integrate cos3 x but you can split it into cos2 x and cos x and use identities. Hint: Again, you might find it useful to draw diagrams for these questions. ƒƒ Differentiating separately gives: du dv = −sin x and = cos x (see page 150) dx dx Tip: ‘Show that’ questions on trig functions often involve using a common identity, so make sure you know them — see Chapter 3. – 1600g = 2000a fi a = 1.69 ms–2 (3 s.f.) Q6 a) Resolving perpendicular to the 40° plane () for A: RA = 0.2g cos 40° Using F = mR: FA = 0.7 × 0.2g cos 40° = 0.14g cos 40° Resolving parallel to the 40° plane () for A: T – FA – 0.2g sin 40° = 0.2a T – 0.14g cos 40° – 0.2g sin 40° = 0.2a fi 5T – 0.7g cos 40° – g sin 40° = a 1 Resolving perpendicular to the 20° plane () for B: RB = 0.9g cos 20° Using F = mR: FB = 0.05 × 0.9g cos 20° = 0.045g cos 20° Resolving parallel to the 20° plane () for B: Fnet = ma 0.9g sin 20° – FB – T = 0.9a 0.9g sin 20° – 0.045g cos 20° – T = 0.9a fi g sin 20° – 0.05g cos 20° – (1.11...)T = a 2 Q4 a) Resolving perpendicular to the plane () for P: R = 60g cos 50° Using F = mR: F = 0.1 × 60g cos 50° = 6g cos 50° Resolving parallel to the plane () for P: Fnet = ma 60g sin 50° – F – T = 0 T = 60g sin 50° – 6g cos 50° = 412.63... ƒƒ Resolving horizontally: F = N sin 28° = (36.421...) sin 28° = 17.1 N (3 s.f.) Exercise 2.2 Q1 T A 20° B 12g 366 Chapter 16 Moments A uniform rod, AB, of mass 12 kg and length 16 m, rests against a rough, vertical wall at A. ƒƒ Substitute u for one of the functions of x to give a function using substitutions that require trig identities. Tip: Always check the answer seems sensible. ƒƒ First, deal with the terms in x and constant terms — in this case that’s the two terms on the right-hand side: d 2 d 3 2x y + y = 12x + 0 dx dx ƒƒ Now use the chain rule on the term in y. Tip: R is the normal reaction of the wall on the rod — don’t confuse it with the reaction in the hinge in the examples in the previous section. Find the value of m to the nearest gram. fg (x + h) - fg (x) f (x + h) g (x + h) - f (x) g (x) = lim h h h"0 The numerator of this fraction can be seen as the area of a rectangle f(x + h) by g(x + h) minus the area of a rectangle f(x) by g(x). dr dq dr dr dV = × dq dV dq dV 1 k 1 = × =×k= cm °C–1 dq dV 2pr 2 2pr 2 dr 3 6 b) V = 4, so r = p = 1.240..., and k = 1.5. So Q5 The surface area of the tank, dA A = (2pr2) + 3r(2pr) = 8pr2, so = 16pr, dr dH and = –2 (it’s negative as heat is lost). ƒƒ If the steps are moving further and further away from the root, the sequence is diverging. ƒƒ Using the conditional probability formula: P(M’|N’) = ƒƒ Using the probabilities you found above: P(N’) = 1 – P(N) = 0.68 P(M’ Ç N’) = P(M’)P(N’|M’) = 0.8 × 0.7 = 0.56 ƒƒ So, P(M’|N’) = P^Ml + Nlh P^Nlh P^Ml + Nlh 0.56 56 14 = 0.68 = 68 = 17 P^Nlh Two-way tables Probabilities can also be shown in two-way tables, like this: A A’ Total B P(A Ç B) P(A’ Ç B) P(B) B’ P(A Ç B’) P(A’ Ç B’) P(B’) Total P(A) P(A’) 1 Tip: You might also see tables with numbers of objects or equallylikely outcomes instead of probabilities (like in the example on the next page). Chapter 11 Probability 265 Two-way tables can make finding conditional probabilities quite easy — if you’re trying to find a probability “given A”, simply look in the A column (or row) of the table. Q3 Integrate the function e8y + 5 with respect to y. # esin x cos x dx = du dx f’(u) esin x + C f(u) = eu c) # x4 sin(x5) dx ƒƒ Here, u = x5. x=3 Range: f(x) ≥ 5 y y = 2|x| + 5 x Q3 a) 2 b) 1 Q4 y c) 4 d) 3 1 4 1 3 –1 x 2 –2 4 x Answers 377 Q5 a) b) y |f(x)| y y = |4x + 2| 5 10 5 3 –5 f(x) 2 b) y 5 y=2 x 5 3 x –½ |f(x)| The line y = 2 intersects with the line |3x – 5| in two places so there are 2 solutions to |3x – 5| = 2. It’s called a random variable because it takes different values with different probabilities. c) The domain is x d R , x ≠ np (where n is an integer). 5 3 5 b) Hence evaluate A between the values x = 2 and x = 2 . p So the equation of the normal is y = 4x + 2 . dx Q5 The curve C is defined by the parametric equations x = t2 sin t, y = t3 sin t + cos t. Exercise 2.2 Q1 Which of the binomial distributions described below would a normal approximation be suitable for? dx dx 154 Chapter 7 Differentiation As h → 0, x + h → x so this bit is just g(x). or –4, 1, –4, 1, ... ƒƒ Start by transforming X to Z. Critical value The value of the test statistic at the edge of the critical region. ƒƒ P(Z ≤ 0) = P(Z ≥ 0) = 12 , so you’d expect f(z) 0 z P(Z ≤ z) = P(Z ≥ –z) f(z) z 1 P(Z ≤ z) to be < 2 for negative values of z 1 and > 2 for positive values of z. So # (n + 1)f’(x)[f(x)]n dx = [f(x)]n + 1 + C fi # 3(8x3)(2x4 – 5)2 = # 24x3(2x4 – 5)2 dx = (2x4 – 5)3 + C ƒƒ Divide everything by 2 to match the original integral: # 12x3(2x4 – 5)2 dx = 1 4 3 2 (2x – 5) + C Example 2 Tip: This one looks pretty horrific, but it isn’t too bad once you spot that –cosec2x is the derivative of cot x. Since x is rational, it can be written as a x = b , and since it is positive, a and b are both positive integers (or both negative, in which case you can simplify the fraction by dividing top and bottom by –1 to get a and b positive). f(2.2) = ln (2.2) – 22.2 + 3 = –0.806... So C(0) + D = 0 (longest day) fi D = 0 C(6) + D = p (shortest day) p fi 6C + 0 = p fi C = 6 p So g(t) = 12 + 5 cos b 6 t l Because cos is periodic, you might have found D = 2p. From the calculator, F(z) = 0.975 for z = 1.96 to 2 d.p. 19.84 - m So = 1.96 Equation in m. b) (1 + 6x)3 = 1 + 18x + 108x2 + ... ƒƒ ‘Find the gradient of the tangent’ means you have to differentiate. b) Find the speed of A when it hits the pulley. x4 = –1.690415... Using the modulus means you’ll never end up taking ln of a negative value. There’s no hinge here — it’s friction which keeps the rod from slipping on the wall. ƒƒ n is small and p isn’t close to 0.5, so a normal approximation is not appropriate. Q3 sin x + 7 cos x º R sin (x + a) fi sin x + 7 cos x º R sin x cos a + R cos x sin a fi 1 R cos a = 1 and 2 R sin a = 7 2 ÷ 1 gives tan a = 7 fi a = 1.21 (3 s.f.) 1 2 + 2 2 gives: Q4 R2 cos2 a + R2 sin2 a = 12 + ^ 7 h = 8 fi R = 8 = 4×2 = 4 × 2 = 2 2 So sin x + 7 cos x º 2 2 sin (x + 1.21). - p (0) - 3 (- 1) dy 3 When q = p, = - 6. Arithmetic Series 4. ƒƒ So going back to the hedgehog weights on page 269, you could define a random variable, W ~ N(m, s 2), where W represents hedgehog weight. s Resolving horizontally, taking right as positive: s = s, u = 15, a = 0, t = 0.063... The point R lies on the line PQ such that PR : RQ = 3 : 1. The formula for integrating by parts is: ƒƒ Be able to integrate functions where integration by parts has to be applied more than once. At time t seconds, the particle’s position relative to a fixed origin O is given by r = [(t3 – 3t)i + (t2 + 2)j] m. 9-m So s = 0.14995... Proof by Contradiction In Year 1, you saw a few different kinds of mathematical proof. Another trigonometric function which is easy to integrate is the derivative of tan x, sec2 x. 0.2 m A spanner is attached to a bolt at a point, O. ƒƒ Draw a diagram to show the forces acting on the rod: Tip: The peg is smooth, so there is no frictional force at C. F C 6N A xm Find the sum of the moments about A if: a) x = 10, y = 6, q = 90° b) x = 1, y = 5, q = 0° c) x = 4, y = 8, q = 270° d) x = 7, y = 6, q = 180° e) x = 5, y = 5, q = 30° f) x = 1, y = 11, q = 120° Q7 A uniform rectangular lamina of mass m has corners at the following coordinates: A (0, 0), B (–2, 8), C (2, 9), D (4, 1). b) Explain why it wouldn’t be suitable to use a normal approximation here. Q5 Find f’(x) if f(x) = ex cosec x. Let X be the sample mean of the scores. Example 2 j 4.8 N 28° i Tip: Draw an arrow to show which direction you are taking as positive. As it cools, the surface area of the cube is modelled as decreasing at a constant rate of 0.027 m2 s–1. Q5 a) There are 2 solutions in the interval –p ≤ y ≤ p, at y = 0.464 (3 s.f.) and y = 0.464 – p = –2.68 (3 s.f.) 5 Q7 First find cos q. Q4 The parametric representation of a curve is given by x = t ln t, y = t3 – t2 + 3. = 0.47 (2 d.p.) Q8 You need to consider the situations where the block is on the point of sliding up the plane, and when it is on the point of sliding down the plane. What time do they have to beat to be selected for the programme? So under H0, X ~ 30 Z= ~ N (0, 1) 3 20 296 The volume (in ml) of a cleaning fluid dispensed in each operation of a machine is normally distributed with mean m and standard deviation 3. Then x + 3 = 3k + 3 = 3(k + 1) is also a multiple of 3 and is larger than x, which contradicts the initial assumption. ƒƒ The normal reaction of the wall on the rod acts perpendicular to the wall. fi a = 10.7 (3 s.f.) Area = 0.8 0.1 0.1 a – 13 1.8 0 z b – 13 1.8 b - 13 ƒƒ And using symmetry, 1.8 = 1.281... dx s + 3s ln s s 2 + 3s 2 ln s From a), when x = 0, s = 1. 10% 5% 2.5% 1% 0.5% 20% 10% 5% 2% 1% n 4 0.8000 0.9000 0.9500 0.9800 0.9900 ... When t = 2p, cos t = 1. Hypothesis Tests of the Mean of a Population Exercise 2.1 — Hypothesis tests of a population mean Q1 a) Let the mean weight in grams of all plums from the tree this year be m. Modulus Sometimes in maths you want to work with numbers or functions without having to deal with negative values. f l(x) Remember that differentiating ln |f(x)| gives . dv du 1 = (x – 1)–4. So you can interchange the ≤ and < signs. c) No, because a value in the domain can map to more than one value in the range. Differential equations ƒƒ A differential equation is an equation that includes a derivative term such dy dP d s d V as (or etc, depending on the variables), as well as , , dx dt d t d r other variables (like x and y). # 1 dx = 1a ln|ax + b| + C ax + b # Example 4 Find # 1 dx . The reaction forces at A and C are both perpendicular to the beam. 2 6y0 + 2 ^ y1 + y2 + ... The particle is moving in a plane, and at time t = 0 the particle passes through the origin, O. The Product Moment Correlation Coefficient The product moment correlation coefficient (PMCC) measures correlation. Example 1 1 x with domain {x : x ≥ 0} and g : x Æ - with x 1 1 domain {x : x > 1}, solve the equation fg(x) = 2 and state the range of fg(x). ƒƒ Start by differentiating u with respect to x: u = x2 – 2 fi du = 2x dx ƒƒ Now rearrange the equation for du to get dx on its own: dx du = 2x fi du = 2x dx dx 1 fi dx = 2x du du isn’t really a fraction, but you dx can treat it like one for this bit. dy d2 y Q7 = f’(x)ef(x) – 1 = 2e2x – 4 – 1, = 2 × f’(x)ef(x) = 4e2x – 4 dx dx 2 2 d y 4e2x – 4 > 0 for all values of x, so is always positive, dx 2 2x – 4 which means the graph of y = e – x is always convex. , pn – 1, pn, so pn is the largest prime number. Find the length of the rod. Newton’s Laws of Motion Exercise 3.1 — Newton’s laws of motion Q1 a) Resolving horizontally: Fnet = ma 10 cos 45° = 3a a = 7.071... There is a sign change between the upper and lower bounds (and the function is continuous in this interval), so a solution to 2 d.p. is x = 0.25. F has magnitude 10 N, and acts ym at an angle of q to the horizontal, as shown. ms–1 ƒƒ Next, resolve horizontally for A (Æ) to find its deceleration: –23.002... ex Q12 y2x + 2xy – 3x3 = x2 + 2 passes through two points where x = 2. ƒƒ All you need to do now is write your conclusion. Since 0.76 > 0.5494, the result is significant. ƒƒ Know that ƒƒ An inverse function does the opposite to the function. The last three iterations round to the same answer to 2 d.p., so to 2 d.p. x = 1.56 Q6 a) f(3) = e3 – (10 × 3) = –9.914... dy cos p + 0 sin p = =1 2 (0) + cos p dx ƒƒ So the gradient of the tangent at (p, 0) is 1. For large x, x + 2 ≈ x + 1 so as x Æ •, x+2 Æ 1. d) Given that the radius of the cylinder is 20 cm, calculate the rate at which the volume of water in the tank is decreasing when t = 10 s. dV 1 c) = 3(V – 1) fi - dV = 3 dt dt V 1 1 fi # - dV = # 3 dt V 1 fi ln|V – 1| = 3t + ln k fi V = ke3t + 1 So when t = 0 and V = 5, 5 = k + 1 fi k = 4 fi V = 4e3t + 1 dy tan y 1 1 d) = x fi tan y dy = x dx dx 1 fi # cot y dy = # x dx fi ln|sin y| = ln |x| + ln k fi sin y = kx p So when x = 2 and y = 2 , 1 x 1 = 2k fi k = 2 fi sin y = 2 dx 1 e) = 10x(x + 1) fi dx = 10 dt dt x ^ x + 1h 1 1 1 Using partial fractions, º x – + , x 1 x ^ x + 1h 1 1 so # x - + dx = # 10 dt x 1 fi ln |x| – ln |x + 1| = 10t + ln k x fi ln = 10t + ln k x+1 x fi + = ke10t x 1 1 So when t = 0 and x = 1, 2 = k 1 10t x fi + = 2 e x 1 dV 1 Q3 a) = a – bV fi dV = dt dt a bV 1 fi # dV = # 1 dt a bV 1 fi - b ln|a – bV| = t + C fi ln|a – bV| = –bt – bC b and C are both constants, so let –bC = ln k: fi ln|a – bV| = –bt + ln k fi a – bV = ke–bt fi bV = a – ke–bt a fi V = b – Ae–bt (letting A = k ÷ b) a b) When t = 0 and V = 4b , 4a - a 3a a a a a 4b = b – A fi A = b – 4b = 4b = 4b c) As t gets very large, e–bt gets very close to zero, a so V approaches b . 182 Chapter 8 Integration This method gives you a general rule for integrating functions of the form eax + b: # e ax + b dx = 1a eax + b + C for a ≠ 0 This means you just need to divide by the coefficient of x and add a constant of integration — the eax + b bit doesn’t change. = 1 + 8x + 48x2 + 256x3 + ... State the range of each. So n! = 1 × 2 × ... Thin Describes a body which is modelled as having no thickness. 2 Q6 a) 3 sin x + cos x º R sin (x + a) fi 3 sin x + cos x º R sin x cos a + R cos x sin a fi 1 R cos a = 3 and 2 R sin a = 5 5 2 ÷ 1 gives tan a = 3 1 2 + 2 2 gives: R2 cos2 a + R2 sin2 a = 32 + 52 = 34 fi R = 34 So 3 cos 2x + 5 sin 2x º 34 cos (2x – a), 5 where tan a = 3 . ƒƒ Be able to use the quotient rule alongside other methods for differentiating complex functions. If it was zero, then the particle wouldn’t be moving parallel to i at this time — it would be stationary. ƒƒ The rod is in limiting equilibrium, so friction is at its maximum: F = mR 35.168... Convergent sequence/series A sequence/series that tends towards a limit. The range is y ≥ 1 or y ≤ –1. ÷ 5 = 1.607... dy 1 b) Show that = 0 when x = - 2 cot (2x) dx (you do not need to solve this equation). It is suggested that the weight of plums has increased. Using the product law: P(A Ç B’) = P(A)P(B’|A) = 0.6 × 0.5 = 0.3 Example 2 Tip: To find the probability of an event, divide the number of equally likely outcomes corresponding to that event by the total number of possible outcomes. 3-1 1- 3 2 3-4 º º º 3 –2 × 2 + 3 1 3 1 p p b) cos x = cos x cos 6 – sin x sin 6 3 1 fi cos x = 2 cos x – 2 sin x fi ^2 - 3 h cos x = – sin x sin x fi cos x = tan x = 3 – 2 p From a), tan b - 12 l = 3 – 2, p so one solution for x is - 12 . ar 4 0.75 = Dividing gives: r3 = ar = 0.015625 48 fi r = –0.25 b) ar = –48 fi a = 48 = 48 = 192 - 0.25 r c) | r| < 1 so you can find the sum to infinity: 192 = 192 = 153.6. S¥ = a = 1 - r 1 - ^ - 0.25h 1.25 Q8 The sum of terms after the 10th is S3 - S10. Friction Q5 R F Exercise 2.1 — Friction Q1 The particle is on the point of slipping, so F = mR. Have a look back at pages 20-21 for more on how to find them. Tip: You’re told that the resultant force acts in the x-direction. dy f (x + h) - f (x) cos (x + h) - cos x F = lim < F Then = lim < dx h " 0 (x + h) - x (x + h) - x h"0 Using the cos addition formula: dy cos x cos h - sin x sin h - cos x F = lim < dx (x + h) - x h"0 - = lim ; cos x (cos h 1) sin x sin h E h h"0 - 1) sin x sin h cos x (cos h E = lim ; h h h"0 Using small angle approximations, 1 cos h ≈ 1 – 2 h2 and sin h ≈ h, so: dy b- 1 h2l = lim > cos x × 2 - sin x × h H dx h"0 h h h x cos = lim :- 2 - sin x D = –sin x h"0 Q10 a) y = cos2 x, so let y = u2 where u = cos x. dy dy ye x 2 dy ex b) ex + y = 3y2 fi = = 3- . Calculate the thrust force in the rod. When it does, the string will go slack and there will be no tension force acting on A. ƒƒ Now put in x = p and y = 0 to find the gradient at (p, 0). PROBLE M SOLVING Chapter 2 Algebra and Functions 1. T2 30° T1 8g The diagram shows a body of mass 8 kg held in equilibrium by two light, inextensible strings. ƒƒ Be able to sketch the The modulus of a number, x, is written |x|. (seems okay so far). Q4 A bird is observed diving into the sea. y A1 is the area under the curve y = x2 between 0 and 1, so integrate between these limits to find the area: 3 1 1 #0 x 2 dx = : x3 D 0 = 13 - 0 = 13 y = x2 2 1 A1 A2 y=2 − x 1 Tip: A2 is just a triangle with base 1 and height 1, so you could also calculate its area using the formula for the area of a triangle. 10% ... The examples below show a convergent and a divergent staircase diagram. It’s a u-shaped quadratic, so the quadratic is positive when k > 1413.7 (ignoring the negative solution). Find: a) the length of time that the ball is in the air, b) the fielder’s horizontal distance from where the ball was struck, c) the speed of the ball at the point where it is caught. dx dx = 0.6. dt dV dV dx = × = 6 3 x mm3 per day dt dx dt dV c) x = 0.5 fi = 3 3 = 5.20 mm3 per day (2 d.p.) dt Q8 a) n is directly proportional to D, so you can write this as n = kD, where k is a constant. f(x) x Finding ranges and domains using graphs Sketching a graph can also be really useful when trying to find limits for the domain and range of a function. From your calculator, P(Z ≤ z) = 0.05 for z = –1.645, so the critical region is Z < –1.645. ƒƒ There are no real solutions for the square root of a negative number. Draw the mapping diagram. b) Find the gradient of C when t = 1. Describe the effect this has on the magnitude of the acceleration of the particle. Algebraic division Dividing one algebraic expression by another. b) Using your graph from part a), explain how you know that the equation e2x + x2 = 3 has two solutions. f(–1.5) = e(2 × –1.5) + (–1.5)2 – 3 = –0.70... So f(x) = 1 is an asymptote. 1 x 2 Find out where the curve and line meet by solving x2 = 2 – x. = 1 cot ^6x + 1h 6 ƒƒ Putting these terms together and adding the constant gives: # 10 sec 5x tan 5x + 12 cosec 3x cot 3x - cosec 2 ^6x + 1h dx = 2 sec 5x - 1 cosec 3x + 1 cot ^6x + 1h + C 6 6 Exercise 3.3 Q1 Find the following integrals: # cosec 2 11x dx b) a) # - cosec ^ x + 17h cot ^ x + 17h dx c) # 13 sec ` p4 - x j tan ` p4 - x j dx e) Q2 Find # 10 cosec 2 d) # 5 sec 10q tan 10q dq # - 3 cosec 3x cot 3x dx `a - 2x j - 60 sec ^a - 6xh tan ^a - 6xh dx Q3 Integrate the function 6 sec 2x tan 2x + 6 cosec 2x cot x with respect p p to x between the limits of x = 12 and x = 8 . ƒƒ A normal distribution is continuous, so can easily model continuous normal distribution to real-life situations. Product rule A method for differentiating a product of two functions. PROBLEM SOLVING In a 3D board game, players take turns to position counters at points inside a cuboid grid. dx dx ƒƒ Putting these into the formula gives: # 3 x2 x2(x – 1)–4 dx = :- 3 ^ x - 1h 3 D 2 3 2 # 3 2 3 x2 2 = :- 3 ^ x - 1h 3 D + 3 2 ƒƒ # 2 - 2x ^ x - 1h-3 dx 3 # 2 3 x^ x - 1h 3 dx - 3 x(x – 1)–3 dx is still tricky to integrate. Chapter 12 The Normal Distribution The conditions for the normal distribution are useful when you’re given a diagram and asked to find m and s. Q2 A projectile is fired from a height of 5 m directly above a fixed point O, which is on flat, horizontal ground. Q8 k : x Æ tan x , x d R . The other critical value will be –2.575. You're Reading a Free Preview Pages 149 to 169 are not shown in this preview. 2 dV dV Q10 a) = k, V = 3 pr3, so = 2pr2. A bucket of water is hung 3 m from the base of the ladder. Integrating (ax + b)n, n ≠ –1 In Year 1, you learnt to think of integration as the opposite of differentiation. 2 4 c) A translation of 4 right / by the column vector e o. Q2 6 cos x – 5 sin x º R cos (x + a) fi 6 cos x – 5 sin x º R cos x cos a – R sin x sin a fi 1 R cos a = 6 and 2 R sin a = 5 5 2 ÷ 1 gives tan a = 6 fi a = 39.8° (1 d.p.) 1 2 + 2 2 gives: R2 cos2 a + R2 sin2 a = 62 + 52 = 61 fi R = 61 So 6 cos x – 5 sin x º 61 cos (x + 39.8°). Given that the magnitude of the normal reaction of the plane on the particle is four times the magnitude of F, find the weight of the particle. b) The normal approximation has m = np = 1000 × 0.05 = 50 s2 = np(1 – p) = 1000 × 0.05 × 0.95 = 47.5 So approximate X with Y ~ N(50, 47.5). Q12 Find f’(x) if f(x) = ln (x + sec x). Q2 a) Let T ~ N(51, 2.12) represent the personal best time taken to run 400 m in seconds. dx 2 (- 4) - 0 + 3 So the gradient of the tangent is 1 and the gradient of the normal is –1. When this is the case, you can just integrate ‘normally’ without separating the variables. k = –24.0° (3 s.f.) b) a = tan–1a 9 i.e. 24.0° below the horizontal. b) Find the approximate bearing on which the toy is moving at t = 2.5 s. V Unit vector A vector of magnitude one unit. + 2 ^ yn - 1 + ynh a = h 6y0 + 2^ y1 + y2 + ... Write each force in the form (ai + bj) N. ƒƒ The smallest possible value of x2 is 0. 12 25 q 1 1 a) (i) cos2 a 2 k = 2 (1 + cos q) = 2 b1 + 13 l = 26 q 25 5 So cos a 2 k = 26 = . fi MA = 23.979... × 2) = 8.771... You need to be able to combine what you learned in Year 1 with what you’ve seen so far in this chapter. The centre of mass is at the centre of the lamina, so the horizontal distance is 3 ÷ 2 = 1.5 m and the vertical distance is 1 ÷ 2 = 0.5 m Chapter 16 Moments 355 Tip: There will also be a reaction force in the pivot at A (you can resolve forces to find it), but since you’re taking moments about A, it doesn’t affect the answer. M 23 8

of the curve is a function of x. = –ln|cos 3x + 2| – ln k = –ln|k(cos 3x + 2)| The minus sign is just to avoid fractions in the logarithm. 2^1 2h^ - 2h - 1 dy At (1, –2), = = 1, dx 2^1 h^ - 2h - ^1 h3 so the gradient of the normal is also –1. Rigid Describes a body which does not bend. Q2 This is a one-tailed test, so the hypotheses are H0: r = 0 and H1: r > 0. So the stationary points of A are (3, 2) and (4, 1). a) The number of successful first-serves in one game, given that the player attempts 10 first-serves. Velocity The rate of change of an object’s displacement with respect to time. Connected rates of change ƒƒ Some situations have a number of linked variables, like length, surface area and volume or distance, speed and acceleration. ƒƒ Consider the vertical and horizontal motion separately. You can use trigonometry to resolve the body’s initial velocity into its horizontal and vertical components: ƒƒ Horizontal component (x): cos q = Tip: You can split the projectile’s velocity at any point along its path into horizontal and vertical components — as long as you know its speed and direction at that particular point. 1 4 =1 = 1 = 1 < 1, 4 4 1 4, r so the sequence converges. You’ve already seen resolving vectors (p.298), so some of this should be familiar: ƒƒ A component of a force is the magnitude of the force in a particular direction. f (x) The hardest bit about integrations like this is recognising that the denominator differentiates to give the numerator — once you’ve spotted that you can just use the formula. dx : dt dx = dx d A = 1 dA = 1 = - 0.00225 × × 12x × 0.027 dt dA d t dA dt a dx k ƒƒ Now use the chain rule to find ƒƒ So when x = 15, dx is a change dt in length over time. L Lamina A flat two-dimensional body whose thickness can be ignored. 7 x0 7.42 7.58 7.65 x1 x2 x3 x e) U se the Newton-Raphson formula to find the negative root, b, to 5 s.f. Start with x0 = –1. ƒƒ Is the experiment itself truly random? Tip: So X ~ N(53, 3.56²). 1 a1 - k 1 6 6 1 1 Q7 a) ^1 - 5xh6 = 1 + 6 (–5x) + 1 # 2 (–5x)2 + ... Frictional forces ƒƒ You’ve already covered friction on pages 331-334, but back then you were only concerned with objects which were being held in equilibrium. 5 6x The expansion is valid for 5 < 1, so |x| < 6 . Q2 Give GH and HG as column vectors, where KJK- 1ONO KJK 2 ONO K O K O OG = KK- 3OO and OH = KK 4 OO . f (x) = # f' (x) dx = # ^8 - 7xh4 dx 1 ^ - h5 + = - 1 ^ - h5 + C C - 7 × 5 8 7x 35 8 7x Substitute in the values of x and y at the point given to find the value of C. c) f(x) = cos x 5–x 0° ≤ x ≤ 360° d) f(x) = (x–5 Q3 State the domain and range for the following functions, giving your answers in set notation: b) a) f(x) f(x) 11 f(x) = x2 – 4x + 5 3 f(x) = 2x + 1 x 16 0≤x a. Chapter 2 Algebra and Functions 25 Q4-5 Hint: The range of f(x) is quite tricky to find — you might find it helpful to think about what happens to f(x) as x Æ 3 and sketch the graph. 300 Chapter 14 Kinematics The Constant Acceleration Equations v = u + at 1 s = ut + 2 at2 u+v s = ` 2 jt v 2 = u2 + 2as 1 s = vt – 2 at2 s = displacement in m u = initial speed (or velocity) in ms–1 v = final speed (or velocity) in ms–1 a = acceleration in ms–2 t = time that passes in s (seconds) Remember — these equations only work if the acceleration is constant. b) g(1) = 1³ – 4(1) + 1 = 1 – 4 + 1 = –2 g(2) = 2³ – 4(2) + 1 = 8 – 8 +1 = 1 There is a sign change (and the function is continuous in this interval) so there is a root in this interval. So the variable here is the mass of cereal in a box. b) No, because a value of x can map to more than one value of f(x). Tip: This approximation is appropriate because n = 90 is large and p = 0.47 is close to 0.5. ƒƒ X ~ B(n, p) where n = 90 and p = 0.47. ƒƒ Be able to sketch functions and use the sketches to find approximate locations of roots. So there are 5 terms in the series before a term has modulus less than 1. p b) Find the value of x for which f’(x) = 0 and 0 ≤ x ≤ 2 . b) The expansion of (1 + 3x)4 is valid for all values of x, since n is a positive integer. 1#2#3 27x 2 27x 3 3x = 1 – 5 + 50 – 50 + ... Here a = –4, b = 3 and n = 2. Chapter 8 Integration 209 Exercise 8.1 Q1 Find the following integrals using the given substitutions: # 12(x + 3)5 dx, u = x + 3 a) b) # (11 – x)4 dx, # 24x(x2 + 4)3 dx, u = x2 + 4 c) d) # sin5 x cos x dx, u = 11 – x u = sin x # x(x – 1)5 dx, u = x – 1 e) Q2 Use an appropriate substitution to find: # 21(x + 2)6 dx b) a) # (5x + 4)3 dx # 24x(x2 – 5)7 dx # x(2x + 3)3 dx d) c) Q3 Use the given substitutions to find the following integrals: # 6x x + 1 dx, u = x + 1 b) # x- dx, u = 4 - x a) 4 x 15^ln xh4 # x dx, u = ln x c) Q4 Find the following integrals by substitution: 2x # 1 e+ e x dx # ^ 4x- h dx b) # -1 dx c) a) 4 x 2x 1 Q5 Hint: This is the formula from p.197. Similarly, comparing the coefficients of x3 gives B – 2A = –3 (so B = –1) and comparing the coefficients of x gives –2C = 10 (so C = –5). Parallel lines and collinear points ƒƒ As with two-dimensional vectors, parallel vectors in three dimensions are scalar multiples of each other. Calculating with Vectors Q7 PQ = (q - 4) 2 + 6 2 + (2q - 3) 2 So: (q - 4) 2 + 6 2 + (2q - 3) 2 = 11 q2 – 8q + 16 + 36 + 4q2 – 12q + 9 = 121 5q2 – 20q – 60 = 0 q2 – 4q – 12 = 0 (q – 6)(q + 2) = 0 Exercise 2.1 — Calculating with 3D vectors Q1 a) 1 + 4 + 8 = 1 + 16 + 64 = 81 = 9 b) 4 2 + 2 2 + 4 2 = 36 = 6 ^- 4h2 + ^- 5h2 + 20 2 = 441 = 21 c) d) 7 2 + 1 2 + ^- 7h2 = 99 = 3 11 ^- 2h2 + 4 2 + ^- 6h2 = 56 = 4 × 14 = 2 14 e) 2 2 2 Q2 a) The resultant is: (i + j + 2k) + (i + 2j + 4k) = 2i + 3j + 6k Its magnitude is: 2 2 + 3 2 + 6 2 = 49 = 7 b) resultant: 2i + 14j + 23k magnitude: 2 2 + 14 2 + 23 2 = 729 = 27 KJK2ONO c) resultant: KK6 OO, magnitude: 2 2 + 6 2 + 9 2 = 11 KK OO 9 LJ P N KK 2 OO d) resultant: KKK 5 OOO , magnitude: 2 2 + 5 2 + 14 2 = 15 K14O L P JK1ONO K O e) resultant: KKK 2 OOO , magnitude: 10 2 + 2 2 + 14 2 = 10 3 K14O L P 2 Q3 a) (5 - 3) + (6 - 4) 2 + (6 - 5) 2 = 2 2 + 2 2 + 1 2 = 4 + 4 + 1 = 9 = 3 ^- 11 - 7h2 + ^1 - 2h2 + ^15 - 9h2 b) = 324 + 1 + 36 = 361 = 19 ^6 - 10h2 + ^10 - ^- 2hh2 + ^- 4 - ^- 1hh2 c) = 16 + 144 + 9 = 169 = 13 ^7 - 0h2 + ^0 - ^- 4hh2 + ^14 - 10h2 d) = 49 + 16 + 16 = 81 = 9 ^2 - ^- 4hh + ^4 - 7h + ^- 12 - 10h e) = 36 + 9 + 484 = 529 = 23 2 2 2 ^ - 7h2 + ^9 - ^- 1hh2 + ^- 6 - 4h2 f) 30 = 529 + 100 + 100 = 729 = 27 Q4 |2m – n| = |–6i – 5j + 10k| = (- 6) 2 + (- 5) 2 + 10 2 = 12.68857... Their teacher says, “The probability that a child from any primary school walks to school on a given day is 0.56.” 268 Chapter 11 Probability Chapter 12 The Normal Distribution 1. So 3 cannot be written as an integer fraction, so it is irrational. b) To disprove the statement, find a counter-example. ƒ Be able to show that two 3D vectors are parallel, and that three points are collinear. Here, the block is on the point of sliding down the slope, because of its weight. dy So > 0 when 9x2 – 6x – 6 > 0 fi 3x2 – 2x – 2 > 0 dx dy Use the quadratic formula to solve = 0: dx 2 ! 28 1 ! 7 2 ! 4 - (4 × 3 × - 2) x= = = 6 3 6 Since the coefficient of x2 in 9x2 – 6x – 6 is positive, dy is u-shaped. + 4 = 1.7984... 2 ^ y0 + y1h + 2 ^ y1 + y2h + ... Q15 A projectile is moving relative to the x- and y-coordinate axes. The first term is the initial height, the second term is the height after one 15% increase and so on, so the 15th term is the height after fourteen 15% increases. dx 1 + x 2 Q7 a) When x = 1: 0 + y2 = y + 6 fi y2 – y – 6 = 0 fi (y – 3)(y + 2) = 0 fi y = 3 and y = –2. f(2.4) = e2.4 – 2 – 2.4 = –0.057... d) f(x) = x2 + 3 with domain {x : x > 0} has a range {f(x) : f(x) > 3}. Real number Any positive or negative number (or 0) including all rational and irrational numbers, e.g. fractions, decimals, integers and surds. Tip: Remember, you can solve simultaneous equations by adding or subtracting them to get rid of one unknown. ln 2.15 – (2.15 – 3)2 = 0.0429... 1 y = x y = f(x) 5 4 3 2 1 0 x2 x1 1 2 4 x0 5 6 7 x A convergent staircase diagram. dy =dx 2d b) The gradient of the normal is –6e–4. If P(X < a) = 0.9192, find the value of a to 2 s.f. ƒƒ Draw a sketch of the area you need to find. b) f(x) = ln x – 2x + 3. A B 8 8 + º º (3u + 2) (3u - 2) ^9u 2 - 4h (3u + 2) (3u - 2) So 8 = A(3u – 2) + B(3u + 2) Using substitution: 2 u = 3 fi 8 = 4B fi 2 = B 2 u = - 3 fi 8 = –4A fi –2 = A -2 2 2 2 8 + So º º (3u + 2) (3u - 2) ^9u 2 - 4h (3u - 2) (3u + 2) So the integral can be expressed: 4 4 #3 ^9u28- 4h dx = #3 (3u2- 2) - (3u2+ 2) dx 4 = : 2 ln 3u - 2 - 2 ln 3u + 2 D 3 3 3 4 = : 2 ln 3u 2 D 3 3u + 2 3 = 2 ln 10 - 2 ln 7 = 2 ln 55 3 14 3 11 3 49 11. Q5 Differentiate arccos x with respect to x. Q6 In a group of eleven footballers, five are over 6 feet tall. dx = (3x + 4)5 + c fi 15 # ^3x + 4h4 dx = (3x + 4)5 + c fi # ^3x + 4h 4 1 c dx = 15 (3x + 4)5 + 15 1 = 15 (3x + 4)5 + C Divide by the constant term to get the integral you’re after. B C A 30° WN A uniform beam, AB, of weight W N rests in limiting equilibrium at an angle of 30° to the horizontal on a rough peg at A and a smooth peg at C, where AC = 0.75AB. The example below shows a convergent cobweb diagram. x = 30.9, so z = 3 20 s n Tip: This is a two-tailed test. ƒƒ Be able to restrict the domain of a mapping to make it a function. Under certain conditions you can use a normal distribution N(m, s²) to approximate the binomial distribution — this can simplify calculations and is usually pretty accurate. 4 Chapter 2 Algebra and Functions Adding and subtracting algebraic fractions You’ll have come across adding and subtracting fractions before, so here’s a little reminder of how to do it: 1. Dividing algebraic fractions Tip: This is the same method that you’d use for normal fractions. Find: a) the magnitude of the tension in the wire, b) the coefficient of friction between the wall and the rod. Given that the mass is accelerating at 0.6 ms–2 and the coefficient of friction between the mass and the plane is 0.4, find T, the tension in the rope. Rational expression A function that can be written as a fraction where the numerator and denominator are both polynomials. ƒƒ Resolving horizontally, taking right as positive: s = sx, u = 18 cos 40°, a = 0, 1 Using s = ut + 2 at2: sx = (18 cos 40° × 1.8) + 0 = 24.8198... U Statistic A quantity that is calculated using only known observations from a sample. To find a unit vector, divide the vector by its magnitude: OQ 1 = (2i + 5j + 4k) OQ 3 5 2 5 4 = i+ j+ k 3 5 3 5 3 5 = 2 5 4 5 5 15 i + 3 j + 15 k Distance between two points in three dimensions ƒƒ The distance between any two points P(x1, y1, z1) and Q(x2, y2, z2) is the magnitude of the vector PQ . The domain of f –1(x) is x d R and the range is f –1(x) d R . 4 f (x) = # f' (x) dx = # dx 10 - 9x = 4 × 1 ln 10 - 9x + C = - 4 ln 10 - 9x + C -9 9 The curve passes through the point (1, 2), so substitute these values to find C. Q6 f(x) = x for {x : x d R }, x ≠ 1 and g(x) = x2 for {x : x d R }. Trigonometric Identities sin (A ± B) ∫ sin A cos B ± cos A sin B cos (A ± B) ∫ cos A cos B ∓ sin A sin B tan A ! tan B 1 tan (A ± B) ∫ 1 " tan A tan B (A ± B ≠ (k + 2)p) A+B A-B sin A + sin B = 2 sin 2 cos 2 A+B A-B sin A – sin B = 2 cos 2 sin 2 A+B A-B cos A + cos B = 2 cos 2 cos 2 A+B A-B cos A – cos B = –2 sin 2 sin 2 Differentiation Small Angle Approximations For small angle q, sin q ≈ q 1 cos q ≈ 1 – 2 q 2 tan q ≈ q f(x) tan kx f’(x) k sec2 kx sec x sec x tan x cot x –cosec2 x cosec x –cosec x cot x f (x) dy = f l(x) g (x) - f (x) gl(x) Quotient rule: y = g (x) , dx ^g (x)h2 Integration (+ a constant) # f (x) dx sec2 kx 1 k tan kx tan x ln |sec x| cot x ln |sin x| Integration by parts: 484 Formula Sheet f(x) # f (x) dx ln |sec x + tan x|, sec x cosec x 1 p ln tan b 2 x + 4 l –ln |cosec x + cot x|, 1 ln tan b 2 x l # u ddxv dx = uv - # v ddux dx MAT61 f(x) P G C Numerical Methods CGP 1 b-a y dx ≈ 2 h[y0 + 2(y1 + y2 + ... Putting these values into y = mx + c gives: 0 = p + c fi c = –p So the equation of the tangent at (p, 0) is y = x – p. n = 2, so n + 1 = 3. So to calculate a bound of a curve across a turning point, do a separate calculation either side of the turning point — one will use the right hand corner, and the other will use the left. Some implicit relations are either awkward or impossible to rewrite in the form y = f(x). # 2 # 0.4 # 0.3 a) x + 2 dx , 2 intervals 0 2 c) e x dx , 2 intervals 0 e) e x + 1 dx , 6 intervals 0 b) # 3 d) # p 4 f) # 1 -p 4 p 0 2 ^ln xh2 dx , 4 intervals 4x tan x dx , 4 intervals ln ^2 + sin xh dx , 6 intervals Q2 For questions 1a) and b) above, find an upper and lower bound for the area beneath the curve to 4 s.f. Q3 Use the trapezium rule with 3 intervals to find an estimate to # p 2 0 sin 3 q dq . a) Find the tension in the chain. Since you’re told the probability of M, but not that of N, show M on the first set and N on the second set. Exercise 11.3 — Applying differential equations to real-life problems dN 1 = kN fi N dN = k dt dt 1 fi # N dN = # k dt fi ln N = kt + ln A Q1 a) fi N = ekt + ln A = Aekt Note that you don’t need to put modulus signs in ln N here, as N can’t be negative — you can’t have a negative number of germs in your body. All of these can be derived from the quotient rule. So you’re looking for a critical value, z, such that P(Z > z) = 0.01. In the tests you need to carry out, the population will be normally distributed with known variance (i.e. X ~ N(m, s²)). p To describe the graph of y = 2 sin b x + 6 l, each of p these points needs to have 6 subtracted from the x-coordinates, and the y-coordinates multiplied by 2. You're Reading a Free Preview Pages 390 to 421 are not shown in this preview. are the areas under the curve to the left of x (the probability that X ≤ x) for different values of x. d2 y Again, it might help to sketch the graph of 2 . (L – R) dt ƒƒ dL is the rate of cooling — so the temperature is decreasing dt and you need a minus sign in the equation again. p p p Q7 a) There are 6 intervals of h = 6 between - 2 and 2 . 15 N Use the formula: Moment = Fd = 15 × 2 = 30 Nm clockwise Tip: Make sure you give a direction with your answer — i.e. which way the force will cause the rod to turn. + 13 = 7.5826... ƒƒ Let L = temperature of the liquid, R = room temperature and t = time. 6A - 5 = e15 e15 + 5 A= 6 As A ≥ 1, then 6A – 5 must be greater than 6 – 5 = 1, so the modulus can be removed as it’ll always be positive. Q4 An object is travelling in a plane with non-uniform acceleration. = 0.977 (3 s.f.) b) If X ~ N(102, 144), find P(84 ≤ X ≤ 106) to 3 s.f. x f(x) ƒƒ Draw a sketch showing the area you need to find. Similarly 95% of the values should fall between x = m ± 2s and nearly all of the values (99.7%) should fall between x = m ± 3s. Q2 A horizontal force of magnitude 26 N is acting on a box of mass 4 kg, causing the box to accelerate across a rough, horizontal plane at a rate of 1.2 ms–2. Limit (sequences and series) The value that the individual terms in a sequence, or the sum of the terms in a series, tends towards. y = ex is known as ‘the’ exponential function. 1) D raw a vertical line from x0 to the curve. Replace f(x) with y: y = 5(x - 2) y Rearrange: x = 5 + 2 x Replace with f –1(x) and x: f –1(x) = 5 + 2. Example 1 If the random variable X ~ N(m, 4) and P(X < 23) = 0.9015, find m. 10. From part a), this means that a is also a multiple of 3, so write a = 3k for some integer k. ƒƒ This time the force is acting at an angle — resolve the force to find its component acting perpendicular to the spanner: Tip: You can either find the component of the force acting perpendicular to the spanner (as done in the example), or you can find the perpendicular distance to the line of action of the force: O 0.2 ° 60 sin 0.2 m 20 N 60° Moment = 20 × 0.2 sin 60° 20 N Fy Fx 60° O 0.2 m ƒƒ Fx acts through O, so its moment is zero (it has no turning effect). – 3a – 23.402... – H e has assumed that the probability holds on any particular day, but factors like weather will affect the probability — children would probably be more likely to walk to school on warm, sunny days than on cold, rainy days. dx ƒƒ There’s no set rule for this — you just have to look at both parts, see which one differentiates to give something nice, then set that one as u. Integration of Trigonometric Functions Exercise 3.1 — Integration of sin x and cos x 1 1 1 Q1 a) # 7 cos x dx = 7 # cos x dx = 7 sin x + C b) # - 3 sin x dx = - 3 # sin x dx = - 3 ^- cos xh + C = 3 cos x + C i) # cos ^4x + 3h + sin (3 - 4x) dx Q2 = 1 sin 3q + cos q + C 6 p 2 p Q3 a) # sin x dx = 6- cos 02 = - cos 2 + cos 0 = 0 + 1 = 1 0 p b) #p 3 sin 3x dx = - 13 6cos 3p p 6 = - 3 ^cos 3p - cos 0h p = - 3 ^- 1 - 1h = 6 p p Q4 a) Integrate the function with respect to x within the limits 1 and 2: 2 #1 2p cos ` p2x j dx = 2pp 8sin ` p2x jB12 = 4 8sin ` p2x jB12 `2j = 4 `sin ^ph - sin ` p jj = 4 ^0 - 1h = - 4 2 b) Since the function doesn’t cross the x-axis for 1 2 so x – 2 > 0 and 2x – 1 > 3 so the modulus signs can be removed. Q10 Show that a solution to the equation 4x – 2x3 = 15 lies between –2.3 and –2.2. Q11 Show that a solution to the equation ln (x + 3) = 5x lies between 0.23 and 0.24. = 3.38 N (to 3 s.f.) Resolving vertically: mg = 8 cos 25° 8 cos 25° m= = 0.740 kg (to 3 s.f.) g Q2 Resolving horizontally: 15 sin x = 13 sin 32° 13 sin 32° k x = sin–1 a = 27.339...° = 27.3° (to 3 s.f.) 15 Resolving vertically: W = 13 cos 32° + 15 cos (27.339...°) = 24.3 N (to 3 s.f.) Q3 a) R 8N Q6 R 14.7 N 2g Resolving the forces on the block parallel to the plane: 14.7 cos q = 2g sin q sin q = 14.7 14.7 2g fi tan q = ^2 × 9.8h = 0.75 cos q Q7 TN 15 N 33° 12° mg a) The string is inclined at 45° – 12° = 33° to the plane. 1 If (i) p ≈ 2 , and (ii) n is large, Tip: Remember... dx c) At the stationary points 6t2 – 3t3 = 3t2(2 – t) = 0, so the stationary points occur at t = 0 and t = 2. Putting these values into the equation gives: 1 3 C = 72 000 3 60 = –3200(0) + C fi 1 fi 3 D3 = 72 000 – 3200t ƒƒ The tank is empty when D = 0 fi 72 000 = 3200t fi t = 22.5 s Tip: Part d) is a ‘connected rates of change’ question — see p.165 if you’ve forgotten how to tackle them. ƒƒ Now highlight the areas where the graph of |2x – 4| (the red line) is above the graph of 5 – x (the blue line): y y = |2x – 4| ƒƒ You can see that there are two regions that satisfy the inequality — the region to the left of x = –1 and the region to the right of x = 3. dx dq dV dV dx = × = 13.5x2 cm3 °C–1 dq dx dq dV When x = 3, = 121.5 cm3 °C–1 dq Q2 V = 30x3 so dA dr = 8pr, and = –1.6. dr dt dA dA dr = × = –12.8pr cm2 h–1 dt dr dt dA When r = 5.5 cm, = –221.17 cm2 h–1 (2 d.p.) dt dr Q4 = 2 × 10–2 mm °C–1 = 2 × 10–5 m °C–1, dq 4 dV and V = 3 pr3 m3, so = 4pr2 m3 m–1. dt dt Answers 443 Q5 Let the variable t represent time. P ^C h Use a tree diagram to help you find P(S Ç C). MODELL ING If the side length of the cube after t seconds is x m, dx find at the point when x = 15 m. dy = x – f’(x)ef(x) = x - 2e2x - 6 dx 2 d y = 1 – 2 × f’(x)ef(x) = 1 – 4e2x – 6 dx 2 d2 y At the point of inflection, =0 dx 2 2x – 6 2x – 6 fi 1 – 4e = 0 fi 1 = 4e fi ln (1) = ln (4e2x – 6) fi 0 = ln 4 + ln (e2x – 6) = ln 4 + 2x – 6 fi 2x = 6 – ln 4 1 fi x = 3 – 2 ln 4 = 3 – ln 2 1 1 Remember from laws of logs that 2 ln 4 = ln 4 2 = ln 2. Example 2 The function f(x) = x2, x d R is a many-to-one function, as two elements in the domain map to the same element in the range, as shown: f(x) = x2 f(x) 9 Both 3 and −3 map to 9. b) Find the equation of the normal to the curve when y = 0. q = tan–1 a 1.96 k = 49.106...° (measured anticlockwise from the upward vertical) So direction = 90° + 49.106...° = 139° (3 s.f.). A girl of mass 50 kg begins to climb the ladder. So the number of successful firstserves in one game, G, can be modelled with a binomial distribution — G ~ B(10, 0.65) b) There are 100 ‘trials’, each with a probability of success of 0.65. Tip: If you used a continuity correction here, you’d work out P(Y > 49.5) instead, which is 0.278 (3 s.f.). ƒƒ Choose u and dv : dx Let u = ln x and u = ln x fi ƒƒ Put u, v, dv = x3 dx du 1 = x dx dv x4 = x3 fi v = 4 dx Tip: If you have a product that has ln x as one of its factors, let u = ln x, as ln x is easy to differentiate but quite tricky to integrate (see next page). From your calculator, P(Z ≤ z) = 0.995 for z = 2.575. -5 The range of this is |f(x)| ≥ 0 y = f(|x|) y y = f(|x|) = 5|x| – 5 ƒƒ Step 1: Draw the graph of y = 5x – 5 as before. x ^t 1h a) Show that the volume (V) is increasing at a rate which satisfies the dV 3 differential equation = + . a) Given that A begins to accelerate up the slope when the particles are released from rest, find the tension in the string. dx 2 1 1 56 1 When x = –1, y = – 10 + 3 – 2 + 4 = 15 2 d y and = –2 + 2 = 0. ƒƒ Know and be able to use a result for integrating products of the form f’(x)[f(x)]n. dV dV Q2 a) µ V fi = –kV, for some k > 0 dt dt 1 fi V dV = –k dt 1 fi # V dV = # –k dt fi ln V = –kt + ln A fi V = Ae–kt t = 0, V = V0 fi V0 = Ae0 = A fi V = V0e–kt b) Using years as the unit of time: 1 1 t = 1, V = 2 V0 fi 2 V0 = V0e–k 1 fi 2 = e–k 1 fi ln 2 = –k fi k = ln 2 fi V = V0e–t ln 2 So V = 0.05V0 fi fi fi fi fi 0.05V0 = V0e–t ln 2 0.05 = e–t ln 2 ln 0.05 = –t ln 2 t = ln 0.05 ÷ –ln 2 = 4.322 years t = 4 years, 4 months (or 52 months) You could have used months as the units of time instead, and started with t = 12. Find the maximum mass of an egg that could end up in one of farmer Elizabeth’s sponge cakes. º (1 + tan2 x) + (1 + cot2 x) º sec2 x + cosec2 x 2 2 Q3 cot x + sin x º (cosec2 x – 1) + (1 – cos2 x) º cosec2 x – cos2 x This is the difference of two squares... dx 152 Chapter 7 Differentiation PROBLE M SOLVING Differentiating by using the chain rule twice Sometimes you’ll have to use the chain rule twice when you have a function of a function of a function, like sin3 (x2). you find the continuous probability P(0.5 4.5 4 0.5 4.5 discrete value X = 1.5 2 is spread2.5out over3.5 the interval Y =the–0.5 ƒƒ Similarly, 1.5 < Y < 2.5, so P(X = 2) is approximated by P(1.5 < Y < 2.5), and so on. 2x + 1 dx u ƒƒ You can see that y is a quotient in the form of v . If you consider what happens as x Æ 3 you’ll see that f(x) approaches 3. The peak at the mean reflects the fact that values close to the mean are most likely. Q7 In a particularly wet village, it rains almost continuously. There is a sign change (and the function is continuous) between 1.85 and 1.9, so the root is at x = 1.9 (to 1 d.p.). So if the claim were true, the leeks would be 35.4 cm tall (to 1 d.p.). You're Reading a Free Preview Pages 470 to 479 are not shown in this preview. 2.46 2.47 2.48 ƒƒ f(2.4715) and f(2.4725) have different signs, so a root must lie between them. If X represents the time taken in seconds, then X ~ N(600, 105). 1 The solutions for sin x = 3 are x = 19.5° or x = 180° – 19.5° = 160.5°. 7 - x03 = 7 - 13 b) x1 = 3 3 = 1.414, x2 = 1.179, x3 = 1.337, x4 = 1.240 Q2 a) x1 = 2 + ln x0 = 2 + ln 3.1 = 3.1314, x2 = 3.1415, x3 = 3.1447, x4 = 3.1457, x5 = 3.1460 b) a = 3.146 Q3 a) f(1.4) = 1.44 – (5 × 1.4) + 3 = –0.1584 f(1.5) = 1.54 – (5 × 1.5) + 3 = 0.5625 There is a sign change (and the function is continuous in this interval) so there is a root in this interval. 35x 525x 2 1 – 2 + 4 – ... s = 2, u = 0, a = 4.9875..., t = t 1 s = ut + 2 at2 1 2 = 2 (4.9875...)t2 t2 = 0.8020... You don’t actually need to check that both components are positive — if one is, the other one will be too. To show that events A and B are independent, you just need to show that one of the following statements is true: ƒƒ P(B|A) = P(B) [or P(A|B) = P(A)] ƒƒ P(A) × P(B) = P(A Ç B) Example 1 For events A and B, P(A) = 0.25, P(B|A) = 0.8 and P(A’ Ç B) = 0.4. a) Find: (i) P(A Ç B), (iv) (B|A’), (ii) P(A’), (v) P(B), (iii) P(B’|A), (vi) P(A|B). So you know that P(F < a) = 0.7580. y Q5 a) 2 y = |x + 2| y = |2x| –2 Q6 |4x + 1| = 3 fi 4x + 1 = 3 or –(4x + 1) = 3 1 fi x = 2 or x = –1 1 1 If x = 2 , 2|x - 1| + 3 = 2 - 2 + 3 = 1 + 3 = 3 = 4 If x = –1, 2|x - 1| + 3 = 2|-2| + 3 = 4 + 3 = 7 Q7 a) |x| < 8 fi –8 < x < 8 b) |x| ≥ 5 fi x ≤ –5 and x ≥ 5 c) |2x| > 12 fi 2x < –12 and 2x > 12 fi x < –6 and x > 6 d) |4x + 2| ≤ 6 fi –6 ≤ 4x + 2 ≤ 6 fi –8 ≤ 4x ≤ 4 fi –2 ≤ x ≤ 1 e) 3 ≥ |3x – 3| fi –3 ≤ 3x – 3 ≤ 3 fi 0 ≤ 3x ≤ 6 fi 0 ≤ x ≤ 2 f) 6 – 2|x + 4| < 0 fi 6 < 2|x + 4| fi 3 < |x + 4| x + 4 < –3 and x + 4 > 3 fi x < –7 and x > –1 g) y y = 3x + 8 x The graph shows that there are two solutions, one where both x + 2 and 2x are positive, and one where x + 2 is positive but 2x is negative. p Chapter 10: Vectors 1. a) 7 N b) 2 m 1.5 m 2m 2.5 m X 7m X Hint: Remember to choose either anticlockwise or clockwise as the positive direction each time. dx dy When = 0, y = 5 – x. 1 Q5 Show that # 0 2 3 -^t + 3h 5 7 dt = 2 ln 3 - 3 ln 2. b, then a = kb for some constant k. 0.8 m B A RA 1.2 m C RB 18g Mg Find the value of M and the magnitude of the reaction at B. Example 1 a) Give the value of f(–2) for the function f(x) = x2 – 1. dx 2 You could also use the fact that 9e3x is always positive to show that it’s a minimum without needing to substitute. ƒƒ Start by defining a random variable to represent the force needed. b) x d R , x ≠ 0, x ≠ 4 Exercise 2.3 — Types of function It helps to sketch the graph of each function to identify its type. sin ^ 2x + 7h Q5 Prove that: a) # sec x dx = ln|sec x + tan x| + C b) # cosec x dx = –ln|cosec x + cot x| + C Q6 Find the following integrals: a) # 2 tan x dx b) # tan 2x dx c) # 4 cosec x dx d) # cot 3x dx # 12 sec 2x dx f) e) # 3 cosec 6x dx Q7 Find 194 Chapter 8 Integration # cosec 2x cot 2x - 1 sec 2 x dx. Find: a) P(B|A) b) P(A|C) c) P(C’|B) Q5 Events J and K are such that P(J) = 0.4, P(J|K) = 0.64 and P(K|J) = 0.2. a) Find P(J È K). So on the 45th day she’ll have over £10. ƒƒ The range of f(|x|) will be the same as the Tip: Don’t get the two types of modulus graph mixed up. 3 p b) Find the equation of the normal at the point c 2 , 6 m. 3 #1 2^ln xh2 dx ≈ 2h [y0 + 2(y1 + y2 + y3) + y4] 1 = 4 [2(ln 1)2 + 2(2(ln 1.5)2 + 2(ln 2)2 + 2(ln 2.5)2) + 2(ln 3)2] 1 = 4 [0 + 2(0.3288 + 0.9609 + 1.6792) + 2.4139] = 2.09 (3 s.f.) c) h = (0.4 – 0) ÷ 2 = 0.2, so the x-values are 0, 0.2, 0.4. 0 .4 #0 e x dx ≈ 2h [y0 + 2y1 + y2] 0.2 = 2 [e0 + 2e0.04 + e0.16] = 0.1[1 + 2.0816 + 1.1735] = 0.426 (3 s.f.) p p -- p j ` d) h = 4 4 ÷ 4 = 8, so the x-values are - p , - p , 0, p , p . Using your graph write down the number of roots of the equation f(x) = 0. 1.25 34 - 32 = 1.789 (3 d.p.) x = 34, so z = 1.25 First test at the 5% level: You’re interested in the higher end of the distribution, so find the critical value, z, such that P(Z > z) = 0.05. = m × 38.941... b) Given that the block is accelerating at a rate of 3.2 ms–2, find the mass of the block. Binomial Expansion and Partial Fractions 132 Chapter 7 — Differentiation 1. This is a one-tailed test. 1 1 1 1 ^4 + xh2 = 4 2 `1 + x j2 = 2 `1 + x j2 c) 4 4 1 -1 1 x j 2 # 2 ` x j2 ` `1 + 4x j2 = 1 + 1 + 2 4 1#2 4 1 #- 1 #- 3 2 2 x 3 + 2 1 # 2 # 3 ` 4 j + ... a) Formulate a differential equation for N. y Tip: In this case, the iterations alternate between being below the root and above the root, but are getting closer each time. k fi q = –39.615...° tan q = a 17 i.e. 39.6° (3 s.f.) below the horizontal. 6 in partial fractions. For example, if you divide x2 + 4x – 3 by x + 2, the divisor is x + 2. Tip: Take a look at your Year 1 notes if you can’t remember how these proofs work. The range for this function, in set notation, is {f(x) : f(x) ≥ 0}. ƒƒ Be able to identify whether a function is one-to-one or manyto-one for a given domain. So P(320 < Y ≤ 350) = 0.485 (3 s.f.) This is one of the reasons why the normal distribution is so useful — because it can be used to approximate other distributions. In Year 1 you saw the binomial distribution B(n, p). Chapter 10 Vectors 1. Z ~ N(0, 1) ƒƒ The curve is symmetrical about the mean 0. m xm Q7a) Hint: You’ll need to use Pythagoras or trig. –4 = 0 + c (so c = –4) for both, since x = 0, so the equation of the tangent is y = x – 4 and the equation of the normal is y = –x – 4. Q3 The Venn diagram below shows the probabilities for events R, S and T: R 0.2 S 0.05 T 0.11 0.04 a) Which events are: b) Find: 0.35 0.25 (i) mutually exclusive? There are 3 other positive solutions for 2x in the p p interval 0 ≤ 2x ≤ 4p, at b p + 4 l, b 2p + 4 l p p 5p 9p 13p and b3p + 4 l, so x = 8 , 8 , 8 and 8 . 11th term: 2362 = u11 = a(0.85)10 2362 = a(0.85)10 fi a = 236210 = 11 997.496... You’re looking for the probability that X > 75. So the root is between 1.8 and 1.9 f(1.85) = ln (1.85) – 21.85 + 3 = 0.010... Event A has happened and for B|A to happen, B will also happen. 1 + 2p 1 c) Show that the tangents intersect at the point b 2 , 2 l . The Newton-Raphson Method Exercise 4.1 — The Newton-Raphson method Q1 a) Differentiating f(x) gives f’(x) = 10x. ƒƒ Be able to calculate Components of forces Learning Objectives: force vectors into horizontal and vertical components. The reciprocal is 1 ÷ the original thing — for fractions you just turn the fraction upside down. ƒƒ The scatter graph of B shows two clusters of data — there appears to be negative correlation overall, but none within the clusters. 480 Glossary Q Quotient The result when you divide one thing by another, not including the remainder. 1 tan–1 - 2 = –26.6°, so the other solution is x = –26.6° + 180° = 153.4°. dx 1 + x 2 Q6 Hint: You’ll need to use the identity sec2 q º 1 + tan2 q. dx u = ex and v = cot x fi So du dv = ex and = −cosec2 x dx dx dy dv du =u + v = (ex × −cosec2 x) + (cot x × ex) dx dx dx = ex (cot x − cosec2 x) Exercise 7.1 Q1 Differentiate with respect to x: a) y = cosec (2x) b) y = cosec2 x d) y = cot 7 x e) y = x4 cot x g) y = cosec (x2 + 5) h) y = e3x sec x c) y = cot (7x) f) y = (x + sec x)2 i) y = (2x + cot x)3 Q2 Find f’(x) if: a) f(x) = sec x x+3 1 b) f(x) = sec x c) f(x) = sec x Q3 Find f’(x) if f(x) = (sec x + cosec x)2. (see page 19). ms–1, at an angle of 45° above the horizontal. The rod is supported in equilibrium by two vertical reaction forces of magnitude 145 N and 90 N, as shown in the diagram below. b) If the block is released from rest at the top of the plane, find how long it will take to reach the bottom of the plane. This can make implicit relations tricky to differentiate — the solution is implicit differentiation: Learning Objectives: ƒƒ Be able to differentiate functions defined implicitly. ÷ 9.8 = 0.8770... x mean = ƒƒ You can see that the shape of the normal curve approximately fits the shape of the histogram for hedgehog weights above. X - 24 So under H0, Z = ~ N (0, 1). fi m = 20.42 (2 d.p.) 2 Tip: So X ~ N(20.42, 4). = 1 – 2x + 6x2 – ... You're Reading a Free Preview Pages 88 to 89 are not shown in this preview. ƒƒ Be able to identify the limitations of a model and suggest refinements to address them. So f –1(x) has domain {x : ln 9 ≤ x ≤ ln 16} and range {f –1(x) : 3 ≤ f –1(x) ≤ 4}. 1 Q1 a) On the same axes, sketch the graphs of y = x and y = x – 2. Q2 R a 20° 16 N 7N 49 N A particle of weight 49 N initially at rest on a horizontal plane is acted on by a constant horizontal resistive force of 7 N and a force of 16 N acting at an angle of 20° to the horizontal, as shown. Tip: Just replace B with B’ in the product law. ƒƒ Disproof by counter-example: giving one example that shows that the statement is not true. – 9.387... dx ƒƒ Start off by rewriting the function as y = (sin x)3 ƒƒ Now you have a function of a function and can carry out the chain rule in exactly the same way as before: y = (sin x)3, so let y = u3 where u = sin x Tip: Don’t get (sin x)3 confused with sin x3 — for this, you’d take u = x3, so end up with 3x2 cos x3 when you differentiate. Product Rule Learning Objectives: ƒƒ Be able to use the product rule for differentiation and recognise when it’s needed. Proving this rule is a lot trickier than anything covered in this section (and you won’t need to know how to do it in the exam), but it goes like this: You need the formula for differentiation from first principles: f l(x) = lim h"0 f (x + h) - f (x) h So when you have a product to differentiate, this can be written as: (fg)l(x) = lim h"0 Tip: The length of the green arrow is f(x + h) – f(x) and the length of the orange arrow is g(x + h) – g(x). It’s fully up to date for the latest AQA course, with brilliant study notes, tips, examples and practice questions for every topic. The parameter here is r, the PMCC of the population. When an object is resting against a surface, the surface will exert a normal reaction on the object perpendicular (or normal) to the surface and, if the surface is rough, friction will oppose any motion. N Using F = mR: 58.800... PROBLE M SOLVING Q5 For each of the following, find m and s. Q2 a) y y = cosec x 1 –1 2 y = sec x –1 2 3 2 2 x 3p b) There is a maximum at b 2 , -1 l p and a minimum at b 2 , 1 l. Here, ex won’t change whether you integrate it or differentiate it, so you just need to think about whether the integration would be made easier by differentiating 2x or by integrating it. 294 Q3 A doctor checked the kidney function of 9 of her patients to see if it was related to their weight. Tip: A is “on the point of sliding” — this is another way of describing limiting equilibrium, which means that the frictional force takes its maximum possible value: F = mR. Find an expression for , the rate of change dt of volume with respect to time. If the signs are different, there’s a root somewhere between them. 360° 180° y = f(x) Tip: In this case, the transformation is a stretch when 0 sin x × b 2 h l + cos x × h H = lim h h h"0 approximations (see p.49) 1 2 sin q ≈ q and cos q ≈ 1 – 2 q . There’s enough information to write an expression for the volume of the prism: Tip: If you’ve forgotten the formulas for areas and volumes of 3D solids, you should brush up on them now. So f–1(x) has domain x ≤ 2 and range f–1(x) ≥ 1. y = f(x) y Tip: It doesn’t matter if you use rectangles above or below the curve — the result will be the same. Find the magnitude of the frictional force, F, on the particle if: a) W = 5 N, m = 0.25 b) W = 8 N, m = 0.3 c) W = 15 N, m = 0.75 Q2 P W A particle of weight W is at rest on a rough horizontal surface. Using more strips (i.e. increasing n) gives you a more accurate approximation. 228 Chapter 8 Integration MODELL ING Q1 A virus spreads so that t hours after infection, the rate of increase of the number of germs (N) in the body of an infected person is directly proportional to the number of germs in the body. a) Write the position vectors OP OQ as column vectors. Q9 Hint: The amount of the water remaining is decreasing at a rate of 0.3 cm3 s–1 — so this value should be negative. You can work this out by doing the transformations on the minimum point of the graph y = x2 (which is (0, 0)). So substitute (xn+1, 0) into the equation: 0 – f(xn) = f’(xn) (xn+1 – xn) - f (xn) xn+1 – xn = l f (xn) f (xn) xn+1 = xn – l f (xn) Tip: This formula is in the formula booklet. 1 For example, x3 - 2x + 2 is a polynomial, but 3 x -3 - 2x 2 is not as it has a negative power and a fractional power of x. Tree diagrams are really useful for seeing how the result of one event affects the probability of the other event happening. # cosec ^ax + bh cot ^ax + bh dx = - 1a cosec ^ax + bh + C # sec ^ax + bh tan ^ax + bh dx = 1a sec ^ax + bh + C # cosec 2 ^ax + bh dx = - 1 cot ^ax + bh + C a Tip: The ax + b bit has to be the same in each trig function — e.g. you couldn’t integrate sec x tan 3x using these formulas. Q7 A particle moves so that it traces a curve C, as defined by the 1 parametric equations y = ln t and x = 3 . dv ƒƒ Start by working out what should be u and what should be dx du — choose them so that v is easier to integrate than 2x ex. Q9 A toy rocket is launched at an angle of 60° to the unit vector in the i direction, with a velocity v = (i + aj + k) ms–1. Glossary 481 Index A acceleration 300-318 due to gravity 300 non-uniform 313-318 suvat equations 300-305 addition formulas 68-73 proof 68-70 algebraic division 8-11 algebraic fractions 3-7 arcs 44-47 arcsin, arccos and arctan 51-54 area under a curve 181, 204-206 B binomial distributions 284-289 approximating with a normal distribution 284-287 binomial expansions 120-133 as approximations 129, 130 combinations of expansions 124 expanding (p + qx)n 126-128 percentage error 130 using partial fractions 132, 133 validity 121, 123, 124, 133 where n is a positive integer 120, 121 where n is negative or a fraction 122, 123 C Cartesian equations 93, 94 centres of mass 354-356 chain rule 139-142, 153 using the chain rule twice 153 cobweb diagrams 238 coefficient of friction 331-334 composite functions 19-22 conditional probability 259-266 connected particles 343-346 connected rates of change 165-167 constant acceleration equations 300-305 in vector form 309-311 continuity corrections 284 482 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The area C2 is the area under the blue line between 0 and 4 minus the area under the red curve between 2 and 4. Reading off the graph, these are: x = 0.5 and x = –1. Q8 Q8 Hint: You’re told that the reaction forces at A and C both act perpendicular to the beam. You're Reading a Free Preview Pages 104 to 116 are not shown in this preview. The ball leaves the ground with speed V ms–1, at an angle 3 of q above the horizontal, where tan q = 4 . Using a calculator: P(T < 28) = 0.0912 (3 s.f.) b) 56 × 0.0912 = 5.1702 So approximately 5 blood donors replace their blood cells more quickly than Edward. Differentiation of e , ln x and a 143 4. c) Find the rate of change of the volume of the crystal with respect to time when x = 0.5. Q8 The growth of a population of bacteria in a sample dish is modelled by the equation: D = 1 + 2lt where D is the diameter of the colony in mm, t is time in days, and l is a constant. seconds after being thrown, and again 2.273... The rod is on the point of slipping to the left, as viewed in the diagram, so the frictional force at A acts to the right. Now multiply. Differentiating using the product rule gives x(1 + 2 ln x). ƒƒ The frictional force acts parallel to the wall, in the opposite direction to potential motion at the wall. 56 - 53 = 3 P(X > 56) = P ` Z > f(z) s j P ` Z > s j = 0.2 ƒƒ Draw a sketch to show the information. ƒƒ When you’re drawing your diagram, you can work out in which directions the components act by considering the other forces — because the rod is in equilibrium, the horizontal components and vertical components of all the forces acting on the rod must sum to zero. a) It takes Edward 28 days to replace his red blood cells. dx ƒƒ The two factors are 2x and ex, so try them both ways round: dv du = ex, then v = 2ex dx dx dv du If u = ex and = 2x, then v = x2ex dx dx If u = 2x and dv = ex dx ƒƒ So let u = 2x and ƒƒ Put u, v, Easier to integrate than 2xex. + 1#2#3 1 #- 2 1 x x 2 = 1 + 3`2j + 3 2 3 `2j 1 #- 2 #- 5 3 3 ` x j3... ƒƒ As usual, identify u and v then differentiate them separately: u = x2 and v = (2x – 1)3 du dv dv fi = 2x and = 2 × 3(2x – 1)2 (using the chain rule for) dx dx dx ƒƒ Then put it all into the product rule formula: dy dv du =u + v = [x2 × 6(2x – 1)2] + [(2x – 1)3 × 2x] dx dx dx = 6x2(2x – 1)2 + 2x(2x – 1)3 Tip: In a ‘show that’ question it’s good to look at the final answer so that you know you’re on the right track. N So the range for which the block is in equilibrium is 12.049... N ≤ D ≤ 44.160... N Exercise 3.3 — Connected particles, pegs and pulleys Q1 a) Resolving vertically (Ø) for B: Fnet = ma 8g – T = 8 × 0.5 fi T = (8 × 9.8) – 4 = 74.4 N b) Resolving vertically (≠) for A: R – 10g = 0 fi R = 98 N A is sliding, so F takes its maximum value, i.e. F = mR = 98m Resolving horizontally (Æ) for A: Fnet = ma T – F = 10 × 0.5 98m = 74.4 – 5 fi m = 0.71 (2 d.p.) Q2 Resolving parallel to the plane () for the lower block: Fnet = ma 20 – T – F – mg sin q = ma 1 Resolving parallel to the plane () for the upper block: Fnet = ma T – F – mg sin q = ma 2 Substituting 1 into 2 : T – F – mg sin q = 20 – T – F – mg sin q 2T = 20 fi T = 10 N Q3 a) s = 10, u = 0, a = a, t = 30 1 s = ut + 2 at2 1 10 = 2 × a × 302 fi a = 0.0222... a Q3 a) Rearrange the equation x2 + 3x – 8 = 0 into the form x = x + b where a and b are values to be found. y=x 3) T hen a vertical line to the curve, etc. = 1.96 s (3 s.f.) Example 5 Tip: This is the general equation of the path of a particle projected from a point on a horizontal plane. Proof by contradiction Assuming that a statement is false, then showing that this assumption is impossible, to prove that the statement is true. This means that 68% of the values should fall between x = m ± s. 5-5 11 - 5 P(5 < X < 11) = P a 4 < Z < 4 k = P(0 < Z < 1.5) ƒƒ Draw a sketch showing the area you need to find. pn - 1 pn — call this number P. xn = b y1 = f(x1) ... = 1 + 18x + 216x2 + 2160x3 + ... ƒƒ Use the addition law: P(A È B) = P(A) + P(B) – P(A Ç B) 0.6 = 0.5 + P(B) – P(A Ç B) 0.1 = P(B) – P(A Ç B) ƒƒ Since the events are independent, you know that P(A Ç B) = P(A)P(B). 15 N Chapter 15 Dynamics 329 Q3 Hint: The ‘line of greatest slope of the plane’ just means straight down (or up) the slope of the plane. 1 S n = 2 n (n + 1) 1 2+ 2 (n n) > 1000 n² + n > 2000 fi n² + n – 2000 > 0 Putting the quadratic equal to zero and solving using the quadratic formula gives n = 44.2 (ignoring the negative solution), which by looking at the shape of the quadratic graph gives n > 44.2 as the solution to the inequality. ƒƒ Using the result from part a), and substituting x = 45 m, y = 10 m and a = 45°: y = x tan a – gx 2 2u cos 2 a 2 10 = 45 tan 45° – 10 = 45 – 9.8 # 45 2 2u 2 # cos 2 45c 19 845 u2 19 845 = 35 u2 fi u = 23.811... Chapter 12 The Normal Distribution 281 b) Find the range of values that includes the middle 80% of forces needed to snap the length of elastic. Taking moments about the midpoint: 0.5RD = 49 × 1 RD = 98 N b) Resolving vertically: Mg = 49 + RD M = (49 + 98) ÷ 9.8 = 15 kg Q3 When the rod is about to tilt about D, TC = 0. a) Differentiate y = (x + 1)2(x2 – 1). Q11 f(x) = 26 Chapter 2 Algebra and Functions 5. b) Find U, the speed that the second dart is thrown at. Q8 Hint: Trig identities will help here — see Chapter 3 for a recap. ƒƒ So P(Z > z) = 0.15 for z = 1.04 (3 s.f.) Area = 0.15 Area = 0.85 0 z Area = 0.15 Exercise 1.4 Q1 If X ~ N(12, 0.64), find the value of x such that: a) P(X < x) = 0.8944 b) P(X ≤ x) = 0.0304 c) P(X ≥ x) = 0.2660 d) P(X > x) = 0.7917 Q2 If X ~ N(25, 4²), find the value of x such that: a) P(X < x) = 0.975 b) P(X > x) = 0.4 Q3 If X ~ N(48, 5²), find the value of x such that: a) P(53 < X < x) = 0.05 b) P(x < X < 49) = 0.4312 Q5c) Hint: Draw a graph of the area you’re given, then use the symmetry of the graph to find the values of z. P(Z < –1.645) = 0.05 –1.645 0 z ƒƒ Since z = –1.788... Resolving vertically, taking down as positive: s = 6, u = 12.7, a = 9.8, t = t 1 s = ut + 2 at2 6 = 12.7t + 4.9t2 fi 4.9t2 + 12.7t – 6 = 0 Using the quadratic formula: - 12.7 ! 12.7 2 - ^4 # 4.9 # - 6h 9.8 fi t = 0.4081... New material is deposited only on the rectangular faces of the prism (i.e. the height does not change), so that x increases at a rate of 0.6 mm per day. a) b) 2.5 m 6 N c) 23 N A 30° A 4m 3N m 0.8 18° Q2 For each of the following light rods, find the sum of the moments about X. d) Find the coefficient of friction, m, between the box and the plane. Q5 Use the iterative formula xn+1 = 2 – ln xn with x0 = 1.5 to find the root of the equation ln x = 2 – x to 2 decimal places. MODELLING A Problem Solving Problem solving examples and questions involve skills such as combining different areas of maths or interpreting information given to identify what’s being asked for. Geometric sequence/series A sequence or series in which you multiply by a common ratio to get from one term to the next. ƒƒ An inverse function maps an element in the range to an element −1 in the domain — the opposite of a function. a) b) c) 10 ms–1 6.8 ms–1 –1 18 ms 20° 21.6° 65° d) e) f) 19.7° 84° 24 ms–1 9.7 ms–1 Q1 Hint: If you’re not asked to give answers to a specific accuracy, rounding to 3 significant figures is fine for most situations. Find the mean and standard deviation of the daily rainfall. Find the horizontal and vertical components of its velocity. Tip: For y = |f(x)| graphs you reflect the dotted line in the x-axis, but for these y = f(|x|) graphs you reflect the solid line in the y-axis. Finding iteration formulas ƒƒ The iteration formula is just a rearrangement of the equation, leaving a single ‘x’ on one side. The y-values are sometimes called ordinates. This is an example of a convergent staircase diagram: y y=x 2) T hen a horizontal line to y = x. For example, y = x2, x d R has the range y ≥ 0, as the squares of all real numbers are positive (or zero). You’d still get the same final answer by cancelling down. ƒƒ The particle is in equilibrium, so the resultant force on the particle is zero. Then 1 = 2 and v1 = sin x. ƒƒ The domain of the inverse is the range of the function, and the range of the inverse is the domain of the function. ƒƒ Be able to find the range of values within which a normally distributed random variable will fall with a given probability. Iterative Methods 3. – 0.5 = 0.433 (3 s.f.) z 1.5 Tip: You can check your answers by using X ~ N(5,16) and finding P(5 < X < 11) on your calculator. 3. N ƒƒ Resolve perpendicular to the plane ( ): Fnet = ma R – 20 cos 15° = 20 g ×0 R = 20 cos 15° = 19.318... b) Find the magnitude of this force. A horizontal force is applied to the particle such that the particle is on the point of slipping. Then H0: m = 70 and H1: m < 70. y y = |10 – 5x| If you wanted to solve this graphically, you would see y = |3x – 6| that the graphs look like this: 2 x x The shaded regions shows the values of x that satisfy the inequality. c) Draw a graph to show this particular solution. So you want to find P(X > 620). b) Calculate the magnitude of the normal reaction force exerted by the plane on the particle. You're Reading a Free Preview Pages 60 to 82 are not shown in this preview. ƒƒ First, you need to find the length of time that the stone is in the air before it lands. Exercise 4.2 — Combining the methods Q1 a) (x + 2)g(x) = (x + 2)(x³ – 4x + 1) = x4 – 4x2 + x + 2x³ – 8x + 2 = x4 + 2x³ – 4x² – 7x + 2 = f(x) So x = –2 is a root of f(x). 1 b) The expansion is valid if |x| < 6 . 292 The significance level of a test (a) determines how unlikely your data needs to be under the null hypothesis (H0) before you reject H0. Q11 A golf ball is struck by a golf club from a point on flat, horizontal ground. l = 12.8° (to 3 s.f.) above the horizontal Example 2 ƒƒ Resolving in the y-direction (≠): 9 sin 35° + 12 sin 50° – 13 = 1.354... Continuing the iterations: x2 = 6 × 7.4161... = 0.246 (3 s.f.) f(z) c) P(0.12 < Z ≤ 0.82) ƒƒ Draw a sketch of the area you need to find. ƒƒ Step 2: Use the chain rule for the terms in y only. How was the data sampled? 1#2#3 -3 # -4 = 1 + 18x + (36x2) 2 - 3 # - 4 # -5 + (–216x3) + ... Even where it looks like the curve and the top of the trapezium are the same, if you zoom in far enough there will always be a gap. 1 s = ut + 2 at2 s = 19 cos 28° × 1.558... N ƒƒ Draw a diagram to show the forces acting between the ladder and the ground, and the ladder and the wall: ƒƒ Taking moments about the base of the ladder: N sin 65° × 5x = (4.5g cos 65°× 2.5x) + (1.3g cos 65° × 4x) (4.531...)xN = (46.593..)x + (21.536...)x R fi N = (68.130...)x ÷ (4.531...)x = 15.034... The triangles will have sides whose lengths are the distances between the two points in the x, y and z directions. ƒƒ Substituting everything back into the original integral gives: # 2 7 2 x 2x - 3 dx = # 1 1 = 2 2 u2 + 3 2 × u × u du # 1 2 u4 + 3u2 du 2 1 u5 = 2 : 5 + u 3 D 1 25 23 15 13 36 3 33 = : 10 + 2 D - : 10 + 2 D = 5 - 5 = 5 Sometimes when you convert the limits of a definite integral, the upper limit converts to a lower number than the lower limit does. = 19.00... 1 Q9 y = 5x - 1 , so let y = u 2 where u = 5x – 1 dy 1 1 1 du 1 fi = 2 u- 2 = 2 , =5 du 5x - 1 dx dy dy du 1 1 5 = × = 2 × 2 x5= dx du dx 5x - 1 2 5x - 1 dy 5 5 when x = 2, = =6 dx 2 10 - 1 and y = 10 - 1 = 3 5 4 y = mx + c fi 3 = (6 × 2) + c fi c = 3 5 4 So the equation of the tangent is y = 6x + 3 . Cover image © iStock.com/Pobytov MAT61DK With thanks to Alison Palin for the proofreading. ƒƒ Here, input the lower bound x = 17, an upper bound (e.g. 9999), m = 13 and s = 5.76 = 2.4 into the normal cdf function. x2 = = 2.491... b) the coefficient of friction between A and the surface. Using them for projectiles means that you consider the horizontal and vertical components of the motion separately. Tip: These facts show how values close to the mean are much more likely than those further away. 9-m P(X < 9) = P a Z < s k = 0.5596 ƒƒ Draw a sketch to show the information. So f(x) ≈ 1 + x + x2 – 3(1 – x + x2) + 8(1 – 2x + 4x2) = 1 + x + x2 – 3 + 3x – 3x2 + 8 – 16x + 32x2 = 6 – 12x + 30x2 c) f(0.01) = 6 ^1 - 0.01h^1 + 0.01h^1 + 2 (0.01)h = 5.8829... Justify your answers. ƒƒ The situation being modelled is the approach of winter — the model could give a limit on t to show at which point the model stops being appropriate. Q2 a) y = 2 arccos x y 2 x –1 1 The graph is the same as y = arccos x but stretched vertically by a factor of 2, so the y-coordinates of the endpoints and y-intercept are doubled. The numbers of equally likely outcomes corresponding to the events are shown. 1 ƒƒ When the population of rabbits has halved, P = 2 P0. Arcs and Sectors Exercise 1.1 — Radians 7p d) 18 Q2 a) 45° 3p b) 4 5p e) 6 b) 90° d) 450° e) 135° Q1 a) p 3p 2 5p f) 12 c) 60° c) f) 420° Exercise 1.2 — Arc length and sector area Q1 s = rq = 6 × 2 = 12 cm 1 1 A = 2 r2q = 2 × 62 × 2 = 36 cm2 Q2 Get the angle in radians: 46 × p 46° = 180 = 0.802... The lamina is held in equilibrium by a vertical force F acting at point C, as shown. Q2 The value, x, of a house is increasing over time, t, at a rate inversely proportional to the square of x. Repeated use of integration by parts du which is dx simpler than the function you started with, but still tricky to integrate. The coefficient of friction between A and the plane is 0.4. a) Find M, given that A is on the point of sliding up the plane. Q2 + T a B 8g RB RA T T mg 20 N mg Q2 Hint: The thrust force acts just like a tension force, but it pushes against the blocks rather than pulling on them. ƒƒ m is a number greater than or equal to 0, and is a measure of the effect of friction between the object and the surface — the higher the number, the greater the effect of friction. These are: ƒƒ Component of weight parallel to the plane: mg sin q ƒƒ Component of weight perpendicular to the plane: mg cos q Where mg is the weight of the object and q is the angle of the incline of the slope. 11 23 35 62 1 1 , x > - 2 , evaluate g(0) and g(2). ƒƒ The unit vector in the direction of the z-axis is k, Learning Objectives: ƒƒ Understand how to represent vectors in three dimensions, using both unit vectors and column vectors. c) Putting this into y = mx + c gives: 6 1 1 - 6 64 = 64 ln 2 + c fi c = 64 + 64 ln 2. Differentiating: dy dy dy 1 - 2xy 2xy +x2 + 3y2 =1 fi = 2+ 2 dx dx dx x 3y dy 1 - 2 (- 2) (1) 5 At (–2, 1), = = dx (- 2) 2 + 3 (1 2) 7 7 so the gradient of the normal is - 5 . Immediately after the propelling force has stopped, the particle has speed 3 ms–1. s = 1.56 s (3 s.f.) b) Resolving horizontally, taking right as positive: s = s, u = 19 cos 28°, a = 0, t = 1.558... ƒƒ The null hypothesis (H0) is a statement about the value of the population parameter, r. ƒƒ Proof by exhaustion: breaking a statement down into two or more cases that cover all possible situations, then showing that the statement is true for all of them. or –1414.7... Find: a) the acceleration of the particle, b) the speed of the particle after 7 seconds, c) the magnitude of the normal reaction with the plane. 2 So the curve has equation y = –2e– 5x – 1 + e . 3 Q11 tan2 x + 5 sec x + 7 = 0 fi (sec2 x – 1) + 5 sec x + 7 = 0 fi sec2 x + 5 sec x + 6 = 0 fi (sec x + 2)(sec x + 3) = 0 1 So sec x = –2 fi cos x = - 2 1 and sec x = –3 fi cos x = - 3 Solving over the interval 0 ≤ x ≤ 2p gives: x = 1.91, 2.09, 4.19 and 4.37 (2 d.p.) Q12 Drawing a right angled triangle will help to solve this question: 60 tan q = 11 60 11 Notice that 180° ≤ q ≤ 270° — this puts us in the 3rd quadrant of the CAST diagram so sin will be –ve, cos will be –ve and tan will be +ve. MODELL ING a) Explain why this situation can be modelled by the differential equation dP = –kP, where t is the time in days and k is a positive constant. 640x 3 20x 160x 2 (3 + 4x)5 = 243(1 + 3 + 9 + 27 + ...) = 243 + 1620x + 4320x2 + 5760x3 + ... = 9.8 fi t = 1.770... a) Draw a diagram to illustrate the forces acting on the particle. It basically tells you how close to a straight line the points on a scatter diagram lie. Taking moments about A: (5.4g × x) + (Mg × 11) = 86 sin 10° × 11 fi x = 69.724 ÷ 52.92 = 1.32 m (3 s.f.) 476 N So the possible range of values for K is 3.882... a) b) 1 1 1 1 2 2 2 6 3 3 2 4 4 c) 2 5 3 3.5 4 4.5 3 Graphs of functions Tip: If you’ve got a discrete set of values that x can take (e.g. x d {1, 2, 3}) then draw a mapping diagram instead of a graph. into equation 1 gives: m = 4 – 1.5 × 0.198675... Q5 Hint: Remember that ln ex = x. O The toy can be modelled as a large cuboid with a smaller cuboid section missing. y = f(x) x xn ƒƒ Similarly, if you try a point where f(x) has a shallow gradient, the tangent meets the x-axis a really long way away from the root, which could cause the iterations to diverge: y y = f(x) root xn+1 xn x Exercise 4.1 Hint: Look back at Chapter 7 if you need help differentiating any of these functions. Q1 Ella throws a wooden cuboid into the air. Limits (integration) The numbers between which you integrate to find a definite integral. a) Two of the forces are given by (6i – 2j) N and (5i + 5j) N and the resultant force is (7i + j) N. ƒƒ Often you’ll be given an approximation to a root and be asked to show that it’s correct to a certain accuracy. BA = OA – OB = (i – 4j + 3k) – (–i – 3j + 5k) = 2i – j – 2k BA = 2 2 + (- 1) 2 + (- 2) 2 = 3 Triangle AOB is right-angled because: 2 2 2 AO + BA = ^ 26 h + 32 = 26 + 9 = 35 2 = ^ 35 h 2 = BO Q6 |v| = 4 2 + (- 4) 2 + (- 7) 2 = 9 1 4 4 7 v so the unit vector is v = 9 v = 9 i – 9 j – 9 k 2. 1 #-2 1 = 1 + 3 (4x) + 3 2 3 (16x2) + ... Transformations of Graphs Exercise 6.1 — Transformations of graphs Q1 d) a) b) y a) c) x b) c) y = -3 3 Q6 y 2 c) 1 -5 –4 -3 2 x –1 a) Turning point at (0, 0) 0 b) Turning point at (0, 3), translation vector e o 3 380 Answers –1 90° 180° b) a) 270° 360° x d) The minimum points on c) are (90°, 0) and (270°, 0). Chapter 8 Integration 213 9. Example 3 Find ƒƒ # 3x 2 - x 2 dx, using a suitable substitution. No other forces act on the particle. The tension in any strings supporting the rod at any other point will also be zero. = 87.2 N (3 s.f.) ƒƒ Substituting R into equation 2 : Q cos 10° = 0.7 × 87.232... When you’re doing projectile questions, you’ll have to consider the motion of the projectile in two dimensions. The ƒƒ next step is to work out the values of the constants (A, B, etc.). ƒƒ Use Newton’s second law to find the new resultant force: Fnet = ma Fnet = 3(7i – 2j) = 21i – 6j ƒƒ Finally subtract the two original forces from Fnet to find K: (12i – 4j) + (–3i + 16j) + K = (21i – 6j) K = (21i – 6j) – (12i – 4j) – (–3i + 16j) = (12i – 18j) N Chapter 15 Dynamics 337 Exercise 3.1 Q1 R 10 N a 45° 3g A particle of mass 3 kg is being accelerated across a horizontal plane by a force of 10 N acting at an angle of 45° to the horizontal, as shown above. Resultant force/vector The single force/vector which has the same effect as two or more forces/vectors added together. – The model does not account for people entering or leaving the town. ƒƒ A has the coordinates (3, 2, 4), B has the coordinates (2, 6, –5). So under H0, X - 12 Z= ~ N (0, 1) 0.2 ƒƒ Now find the value of your test statistic: x-m = 11.2 12 = –1.7888... x2 1 - x2 Use the substitution x = cos q where 0 ≤ q ≤ p. Then there must be an odd number x for which x2 is even. n = –4, so n + 1 = –3. c) u = x4, v = ln x fi x + 2, v = x + 7 du dv 1 1 fi = , = dx 2 x + 2 dx 2 x + 7 dy x+2 x+7 dv du =u +v = + dx dx dx 2 x+7 2 x+2 Q6 u = dy 4 9 13 + = = dx 2 9 2 4 12 13 23 y = mx + c fi 6 = b 2 × 12 l + c fi c = 6 23 13 So the equation of the tangent is y = 12 x + 6 . Q3 Integrate sec2(x + a) + sec2(3x + b) with respect to x, where a and b are constants. x2 = - 3 2 7 3b - 3 l - 3 f(xn) f’(xn) x3 = –2.279020068..., x4 = –2.279018786..., x5 = –2.279018786... ƒƒ Significance levels can be given as a decimal or percentage. Differentiating Trig Functions 149 5. with the values you’ve found. b) Using the iterative formula xn+1 = ln (10xn) with an appropriate starting value, find values for x1, x2, x3 and x4 to 3 d.p. c) Verify that the value of the root from part a) is x = 3.577 to 3 d.p. d) Describe what happens when you use the alternative formula e xn xn+1 = 10 with x0 = 3. ÷ 2 = 27.237... 330 Chapter 15 Dynamics 2. If they want a particular solution, they’ll give you x and y values for a certain point. Chapter 13 Hypothesis Testing On page 369 there’s a table of critical values. This is one step. b) When a particle of mass 16 kg is placed on the beam, the beam remains horizontal and in equilibrium, but is on the point of tilting anticlockwise about C. p b) Find the equation of the normal to the curve when x = 2 . 6 y = |3x – 4| y 3 y = |f(x)| 2 4 1 –2 –1.5 –1 –0.5 0 – 1 0.5 1 1.5 2 b) There are two solutions to |4x + 1| = 3. Then the situation can be modelled by a binomial distribution, B(n, p). = 6.11 ms–1 (3 s.f.) Horizontal component = 6.1111... – T he model doesn’t account for the age of the child — it might be that older children are more likely to walk while younger children get driven to school. ƒƒ Resolving parallel to the plane ( ): Fparallel = 4 cos 30° = 3.46 N (to 3 s.f.) ƒƒ Resolving perpendicular to the plane ( ): 4N 30° 20° Fperpendicular = –4 sin 30° = –2 N Chapter 15 Dynamics 321 Exercise 1.1 Q1 Find the horizontal and vertical components of each of the forces shown: y y b) a) c) y 20 N 17 N 25° 40° 6.5 N d) 10° x x y e) y f) 30° 3N 5N x y x x 13 N 60° x Q2 Each of the following diagrams shows a force and the angle it makes with the horizontal. c) There is no constant term to take out here, so use the general formula: 1 1 # dx = 7 ln 7x + 2 + C 7x + 2 # 1 -43x dx = 4a -13 ln 1 - 3x + ck d) = - 4 ln 1 - 3x + C 3 Q2 # 1 - 20 dx = 1 # 1 dx - 20 # 1 dx x x 8 x 8x 1 159 + = ln x - 20 ln x + C = 8 ln x C 8 1 - 20 = 1 - 160 = - 159 You could also notice that 8x x 8x 8x and 1 159 integrate 8x using the method for x . x 3 - 3x + 5 ƒƒ This means the iteration formula is xn + 1 = xn - n 2 -n 3xn 3 ƒƒ From the question we take x0 = –2, so (- 2) 3 - 3 (- 2) + 5 - 7 = x1 = - 2 3 3 (- 2) 2 - 3 3 7 7 b - l - 3b - l + 5 3 3 7 = - 2.280555556... So the range of f is f(x) > –1 b) ln x is not defined for x ≤ 0, so the largest possible domain of g(x) is x > 0. Q7 a) 2 cos x + sin x º R cos (x – a) º R cos x cos a + R sin x sin a fi R cos a = 2, R sin a = 1 1 tan a = 2 fi a = 26.6° (3 s.f.) R = 22 + 12 = 5 fi 2 cos x + sin x º 5 cos (x – 26.6°). Modulus The modulus of a number is its positive numerical value. Tip: Breaking it into steps like this means you’re less likely to go wrong. Here’s how the hypothesis test works: ƒƒ Be able to conduct a hypothesis test about the population mean of a normal distribution when the variance is known. Tip: Don’t worry about trying to make your sketch perfect, the important things are that the graphs are the correct shape and that they cross the axes in the right places. b) t = 0, N = 200 t = 8, N = 400 fi fi fi fi 200 = Ae0 = A N = 200ekt 400 = 200e8k ln 2 = 8k 1 fi k = 8 ln 2 t fi N = 200e 8 ln 2 3 ln 2 So t = 24 fi N = 200e = 1600 c) Some possible answers are: – The number of germs doubles every 8 hours, so over a long time the model becomes unrealistic because the total number of germs will become very large. 2 x 3 8y a) 2y × 3 3 8y 2 4y = 2xy × 3 1 x 3 × 4y 4x 3 y = 1× 3 = 3 b) Tip: In cases like these, it’s a lot easier to do all the factorising and cancelling before you multiply. Find the number of boxes that you would expect to contain less than 780 g. Imagine that the x- and y-axes lie flat on the page. dA We can write this as = –0.027. Example 1 R a 7N 25º 18 N A particle of weight 18 N is being pulled along a smooth horizontal surface by a force of 7 N acting at an angle of 25° to the horizontal, as shown. This is just the vector describing the body’s displacement from the origin. Tip: Only sketch the function over the given domain and range. You want the smallest integer value of n that satisfies the inequality, so round up. ƒƒ Your hypotheses will be: H0: m = 30 and H1: m ≠ 30 ƒƒ The significance level is 1%, so a = 0.01. = 23.002... The plank is horizontal, and a bird lands on the other end, applying a downward force of magnitude 15 N, as shown. If you don’t like using this method, you can use the triangle method to work out sin y and cos x. Since 1.789 > 1.645, the result lies in the critical region and is significant. 270 Chapter 12 The Normal Distribution Describing a normal distribution If a continuous random variable X is normally distributed with mean m and variance s 2, it is written like this: X ~ N(m, s2) ‘N’ stands for ‘normal’ and ‘~’ is short for ‘is distributed’. Q4 Hint: See p.258 for a reminder of finding probabilities of multiple events. Using your calculator, P(Y ≥ 50) = 0.251 (3 s.f.) b) A student takes a sample of 11 robin chicks from her garden. × r=0 × × × x Example 1 Marcus records the amount of exercise (in hours) that 30 people do in a month, and the length of time (in mins) it takes them to complete a lap of a cycle route. c) Your answers to part a) and part b) should be the same. Example 1 A teacher thinks that test scores are related to the number MODELL ING of hours spent revising. This bit has no h’s in it, so the limit as h → 0 is just f(x). ƒƒ Using a calculator, P(Z < 0.5) = 0.691 (3 s.f.) 274 Chapter 12 The Normal Distribution 0 0.5 z b) If X ~ N(5, 16), find P(5 < X < 11). 5. A sample of 40 students is taken and the mean score is 67.7. Assuming the variance remains unchanged, test the teacher’s claim at the 5% level of significance. Q6 A curve is defined implicitly by x2y + y2x = xy + 4. dy To find dx for an implicit relation between x and y: ƒƒ Step 1: Differentiate terms in x only (and constant terms) with respect to x, as normal. Q5 Solve, either graphically or otherwise: Q5 Hint: You can solve these equations algebraically if you prefer, using the squaring method. You’ll have seen this before, but here’s a recap of the method: ƒƒ The gradient of the tangent is the same as the gradient of the curve at that point. l = 75.9° (to 3 s.f.) above the positive x-axis Chapter 15 Dynamics 323 Example 3 y 4N P 60° 5N x 70° Q The diagram shows all the forces acting on a particle. 1 Q8 y = cos x = (cos x)–1, so let y = u–1 where u = cos x dy 1 du fi = –u–2 = - , = –sin x du cos 2 x dx dy dy 1 sin x du = × =– × –sin x = = sec x tan x dx du dx cos 2 x cos 2 x 1 Remember that cos x = sec x. Exercise 1.1 Q1 Prove, by contradiction, that there is no largest multiple of 3. Conditional Probability 2. b) What is the probability that it’s sunny, given that she goes to the park? A general polynomial of degree 0 is just a constant, e.g. D. This can make for some pretty tricky integration questions involving trig functions. ƒƒ So P(0.12 < Z ≤ 0.82) = 0.246133... 1 2 º 1 º sin 4x º 2 cosec 4x sin x 4 2 1 + ^2 cos 2 x - 1h 1 + cos 2x º 2 sin x cos x sin 2x cos 2 x cos x º 2 sin x cos x º sin x º cot x 1 + cos 4q b) Use 4q = 2x, so x = 2q and so sin 4q º cot 2q Solve cot 2q = 7 1 fi tan 2q = 7 fi 2q = 8.130°, 188.130°, 368.130°, 548.130° fi q = 4.1°, 94.1°, 184.1°, 274.1° (1 d.p.) x cos 2 x 1 Q6 a) cosec x – cot 2 º sin x – x sin 2 x x cos 2 1 - 2 cos 2 2 1 º - º x x x x x sin 2 2 sin 2 cos 2 2 sin 2 cos 2 A Here we’ve let 2A = x so x = 2 . Find: 11g a) the magnitude of the normal reaction 60° of the wall on the ladder, 35 N b) the frictional force between the wall and the ladder, c) the coefficient of friction between the wall and the ladder. Then R ~ N(m, s2). ƒƒ The conditional probability formula becomes: P(B|A) = P(B) = ƒƒ The product law becomes: P(A Ç B) = P(A)P(B|A) = P(A)P(B) P^ A + Bh P ^ Ah (this is the formula for independent events that you’ve already met). This can get quite confusing — so here’s a handy summary of it all. b) Show that the curve intersects the y-axis when y = 0 and y = 3. Solve these separately and then combine to give v in terms of t. Inverse Functions 5. Given that the body accelerates down the plane at a rate of 0.44 ms–2, find the coefficient of friction between the body and the plane. ƒƒ Here it is in the form it was given on page 139: If y = f(u) and u = g(x) then: d y d y du = × dx du dx ƒƒ Since integration is the opposite of differentiation, you have: Differentiation y y+C ƒƒ So # Integration dy d u × du dx dy d u × du dx d y dy dx = y + C. = 0.170 (3 s.f.) Chapter 12 The Normal Distribution 289 Example 3 The heights of 1000 sunflowers are measured and the mean height is calculated to be 9.8 ft. b) Disprove that the product of an irrational number and an irrational number is always irrational. And don’t worry about the weird m2s–2 units — it’s just because the speed it squared, so the units should be squared too. For a one-tailed test: H1: r > 0 or r < 0 For a two-tailed test: H1: r ≠ 0 Tip: This should all be familiar to you — have a look back at your Year 1 notes if you need a reminder. ƒƒ This can be shown clearly by sketching a graph of y = f(x): 2 y=x –5 y x b) State the domain for f(x) = y = –5 ^ x - 4h , giving your answer in set notation. ÷ 0.06327... = 1 + (4a + 3b)x + (6a2 + 12ab + 6b2)x2 + ... Modelling with Probability Learning Objective: ƒƒ Be able to evaluate assumptions made when modelling the probability of events. q 90° – q ƒƒ You can resolve as usual to find mg the components of the weight using trigonometry. The two-way table below shows the costumes that they are wearing: Werewolf Zombie Ghost Total Boys 4 3 3 10 Girls 2 4 6 12 Total 6 7 9 22 MODELL ING A child is picked at random. a) Find gf(x) and write down its range and largest possible domain. c) f(x) has domain {x : x < 2} and range {f(x) : f(x) < 1}. Find the turning effect of the force upon the bolt. A divergent cobweb diagram would have a similar shape, but each iteration would spiral away from the root rather than towards it. P^L + Sh Using the conditional probability formula: P(L|S) = P^ S h ƒƒ So you need to find P(L Ç S) and P(S) — and the easiest way is by drawing a tree diagram using the information in the question: P(L’) = 1 – P(L) 0.4 L 0.6 L’ 0.7 S 0.3 S’ P(S|L) = 1 – P(S|L) S’ P(S’|L’) = 1 – P(S|L’) 0.2 0.8 Tip: In general, if B depends on A, then A depends on B. Q2 Hint: You should find 4 stationary points. ƒƒ f(2.4715) = 2.47153 – 2.47152 – 9 = –0.0116... Tip: A projectile’s trajectory is just the path that it moves along. second term = - 2 = b) a = 1 and r = 2, 1 first term so the sum of the first 7 terms is: a (1 - r7) 1 (1 - (- 2) 7) = = 43 S7 = (1 - r) (1 - (- 2)) Q8 a = 7.2 and r = 0.38, so: 9 10 .3810h = 11.61 to 2 d.p. / ar k = a^^11 -- rrh h = 7.2^1^1--00.38 h k=0 1 8 a a1 - a - 3 k k - 8 Q9 1.2 = S8 = a (1 r) = = a (0.749...) (1 - r) a1 - a - 13 kk fi a = 1.60 to 2 d.p. Q10 The geometric sequence a, –2a, 4a, ... k = 50.7° (to 3 s.f.) Q2 a) 16 N 18 N 60° 18 sin 60° 8.2 sin 39° Q1 a) b) 16 N Exercise 1.2 — Resultant forces and equilibrium 18 cos 60° q = tan–1 a 16 + 18 sin 60° k = tan–1(3.509...) 18 cos 60° = 74.1° (3 s.f.) above the positive horizontal R = (18 cos 60°) 2 + ^16 + 18 sin 60°h2 = 32.8 N (3 s.f.) Q3 Resolving vertically (≠): F cos 31° + F cos 31° – 14 = 0 2F cos 31° = 14 F cos 31° = 7 fi F = 8.17 N (3 s.f.) Q4 PN 30° RN 10 N 45° 60° 9N Resolving horizontally (Æ): P cos 60° + 10 cos 30° + 9 = R cos 45° 1 Resolving vertically (≠): P sin 60° + 10 sin 30° = R sin 45° 2 Because the resultant acts at 45°, its horizontal and vertical components are equal. Tip: You won’t be given the integrals for sin and cos in your formula booklet, so make sure you remember them. fi a = 24.9° (3 s.f.) Chapter 14 Kinematics 301 Example 3 10 ms–1 MODELL ING 2m A stone is thrown horizontally with speed 10 ms–1 from a height of 2 m above horizontal ground. So the maximum and minimum values of 13 cos (q + 67.4°) are at ±13. Then any components acting in the opposite direction will be negative. a) Find the probability that a randomly selected person took more than 620 seconds. dL = –k(L – R) for some constant k, k > 0. c) Yes, as it is a one-to-one function for the domain {x : x ≥ 4}. Moment The turning effect a force has about a pivot point. 35x 525x 2 (2 – 5x)7 = 128(1 – 2 + 4 – ...) = 128 – 2240x + 16 800x2 – ... 70 N R F 35° 35° 100 N a) Find the magnitude of the normal reaction force, R, on the sledge. adj 8 a) cos q = = - 17 . Tip: Notice that if you differentiated 2e4 – 3x, you’d get –6e4 – 3x, so you need to divide by –3 when you integrate — it’s simple really. l 1 x x2 = 3 - 9 + 27 + ... Is the data appropriate? 214 Chapter 8 Integration ƒƒ The hardest thing about integration by parts is deciding which bit of your dv . Chapter 11 Probability 261 Q7 The Venn diagram below shows the numbers of students studying Maths, English and Art, from a group of 100 students. X - 300 So under H0, Z = ~N (0, 1). Then under H0, X ~ N(300, 452), 45 2 so X ~N a300, 25 k = N^300, 81h. When t = p, the i component cos t = –1, so the object is moving west, not east. The ball rebounds from the wall and lands on the ground. She concludes that the probability 12 that it will rain is 30 = 0.4. Give a reason why this model might be inaccurate. There are lots of answers you could give. ƒƒ Use the substitution method to find A and B: 3 Substituting x = 2 into the identity gives 18 + 6 = 0A + 6B fi 24 = 6B fi B = 4 3 Substituting x = - 2 into the identity gives –18 + 6 = –6A + 0B fi –12 = –6A fi A = 2 Tip: You could also use the ‘equating coefficients’ method to find A and B. ms–1 ƒƒ So the ball was struck with initial speed 23.811... Given that the forces acting at B and C are the same as in part a), will the rod rotate clockwise or anticlockwise about this point? = 17.9 N (3 s.f.) b) RV RH T 20° 1.5g 0.5g Resolving horizontally: RH = T cos 20° = (17.908...) cos 20° = 16.828... 3 =- 1 ^ -^ h5 + C 35 35 8 7 × 1h 3 1 4 & C = 35 = 426 4 dx = 8 # ^2x - 4h dx 1 4 So f(x) = - 35 ^8 - 7xh5 + 35 1 2. 0 76.01... dy 2 1 b) When x = 0, = = 3 and y = ln 36 dx 6 1 So the gradient of the normal = - 1 = –3 a3k y = mx + c fi ln 36 = (–3 × 0) + c fi c = ln 36 So the equation of the normal is y = –3x + ln 36 Q9 Stationary points occur when the gradient is 0. The horizontal component of velocity remains constant, so the particle’s velocity will be (12i – 16j) ms–1. dy Use the answer to part a) to find the points where = 0. F is the frictional force and Fnet is the overall resultant force. ƒƒ Values of the cumulative distribution function (cdf) f(z) (z) = P(Z z) are usually written F(z) — i.e. F(z) = P(Z ≤ z). Chapter 16 Moments 351 Exercise 1.1 Hint: Remember to state the sense of rotation when giving the moment of a force. This

dy d d (y) + y (2x2) + 3y2 = 12x + 0 dx dx fi 2x2 dy dy + y4x + 3y2 = 12x + 0 dx dx dy dy dy dy d d term comes from using the chain rule: (y) = (y) =1 = dx dx dy dx dx dx ff Finally, rearrange to make dy the subject: dx dy 12x - 4xy dy 2 = (2x + 3y2) = 12x – 4xy fi dx 2 x 2 + 3 y 2 dx b) Find the gradient of the curve 2x2y + y3 = 6x2 – 15 at the point (2, 1). Answers 457 Q7 Let R = rainfall in cm. x0 x1 x3 x2 Starting at x0, the next iterations x1, x2 and x3 are getting closer to the point where the two graphs intersect (the root). ff Integrate the expression for velocity with respect to time: r= # v dt = ` # ^8 + 2th dt ji + ` # ^t t = (8t + t2)i + a 4 - 3t 2 kj + C 4 3 - 6 t h dt j j ff When t = 2, r = (10i + 3j). a) (x3 – 14x2 + 6x + 11) ÷ (x + 1) b) (2x3 + 5x2 – 8x – 17) ÷ (x – 2) c) (6x3 + x2 – 11x – 5) ÷ (2x + 1) Q1 Hint: If you're told which method to use make sure you show all your working clearly to prove that you know how to use the method. # cosec x dx = -ln |cosec x + cot x| + C # sec x dx = ln |sec x + tan x| + C # cot x dx = ln |sin x| + C You can check these results easily by using differentiation — differentiate the right-hand side of the results to get the left-hand sides. Establishing what has caused the two clusters in each case and separating the data would lead to better conclusions. P(X ≥ 5) = 1 – P(X < 5) = 1 – P(X ≤ 4) = 1 – 0.8298... Example a) O n the same set of axes, sketch the graphs y = ln x and y = (x – 3)2. tan–1 (0.41666...) = 22.6° (1 d.p.) There is another positive solution in the interval 0 ≤ x ≤ 360° at (180° + 22.6°), so x = 202.6° (1 d.p.) c) cosec x = –2 fi sin x = –0.5 sin–1 (–0.5) = –30° which is not in the interval 0° ≤ x ≤ 360°. + 3g sin 20° = 22.9 N (3 s.f.) Exercise 3.2 Q1 A wooden block of mass 1.5 kg is placed on a R smooth plane which is inclined at 25° to the horizontal, as shown in the diagram. Q4 A curve is defined implicitly by x3 + y2 – 2xy = 0. Let X be the sample mean of the weight of plums in grams. f(x) P(X x) ff Since X is a continuous random variable, P(X = x) = 0, which means that P(X ≤ x) and P(X < x) are the same thing. a - 13 b - 13 P(F < a) = P a Z < 1.8 k = 0.1 and P(F > b) = P a Z > 1.8 k = 0.1 ff Draw another sketch to show this information. s 5m 5m ff So the length of time that the ball is at least 5 m above the ground is: 2.273... Replace f(x) with y. fi m = 0.36 (2 d.p.) Q2 a) N F R 60° 35 N b) Resolving vertically: F + 11g = R Taking moments about top of ladder: (11g cos 60° × 3.5) + (35 sin 60° × 7) = R cos 60° × 7 fi R = 400.826... = 1 + 16x + 112x2 + 448x3 + ... ff Work out m and s2: m = np = 46 and s2 = np(1 – p) = 200 × 0.23 × (1 – 0.23) = 35.42 So approximate X with the random variable Y ~ N(46, 35.42). b) What is the maximum possible magnitude of force that can be applied horizontally to the particle, such that the particle will remain in equilibrium? ff When you're taking moments and resolving forces for a heavy (i.e. not light) beam, you need to remember to include the weight of the beam in your calculations. ! 40.365... Putting this into y = mx + c gives: 3 = 9 ln 3 + c fi c = 3 – 9 ln 3 So the equation of the tangent at (1, 3) is y = (9 ln 3)x + (3 – 9 ln 3) Q5 When x = 25, y = 45 = 1024, so a = 1024. ff Of the people who watched an action film, 6 of them were 20 years old or above. q mg ff Using right-angled triangles, the R angle between the object's weight and its component perpendicular to the plane is equal to q. Projectiles Learning Objectives: ff Be able to resolve a body's velocity into its horizontal and vertical components. dx = cos t dt dy 2 cos t - 2t sin t = = 2 – 2t tan t cos t dx dy = 2cos t – 2t sin t dt When t = p, dy = 2 – 2p(0) = 2 dx When t = p, x = 0, and y = –2p, so the coordinates are (0, –2p). N F = mR fi m = 35.437... That way, even if you get your final answer wrong, you can still get some marks for showing that you understood what the question was asking. You can find the horizontal and vertical components of a force, F, using trigonometry. There is significant evidence at the 5% level to reject H0 in favour of the alternative hypothesis that the weight of plums from the tree has increased. 4 3 1 3 + 3 + 3 3 D # dx ≈ 4 : 2a k+ ln 2 ln 2.5 ln 3 ln 3.5 ln 4 2 ln x = 0.25[4.3281 + 2(3.2741 + 2.7307 + 2.3947) + 2.1640] = 5.82 (2 d.p.) b) It is an over-estimate as the top of each trapezium lies above the curve. ff The population parameter you're testing will always be m, the mean of the population. Differentiating a function divided by a function In maths a quotient is one thing divided by another. + 1.6791...] = 1.484 (4 s.f.) The upper bound is found using: 3 #1 2 (ln x)2 dx ≈ h[y1 + y2 + y3 + y4] = 0.5[0.3288... 1 3x 27x 2 189x 3 Q2 a) ^1 + 3xh4 = 1 + 4 – 32 + 128 – ... For any probability involving less than or less than or equal to (e.g. find x where P(X ≤ x) = p) these questions can be done directly on your calculator. Form a triangle between vector v and a vector p = i, with an angle of 60° between them. Moments ff A moment is the turning effect a force has around a point. F Q3 The diagram shows all the forces acting on a particle in equilibrium. 5 6 3 b) Evaluate # x - x dx, giving an exact answer. The graphs cross twice, so the equation has 2 roots. ff f(x) has degree 3, so q(x) has degree 3 – 1 = 2. dx b) Use your answer to a) to find # ^3x + 4 h4 dx . When suggesting some refinements to the model, you don't have to make up a whole new model — just identify what the changes would be and how you could make them. Finite sequence A sequence that has a 'last term'. ff This means that as long as p isn't too far from ff Instead you can get a good approximation to the probabilities for X using a normal distribution. Limiting equilibrium Describes a body which is at rest in equilibrium, but is on the point of moving. ff Taking moments about A: mg × 1.7 = 42 sin 22° × 5.5 fi m = 86.534... A force of magnitude 5 N acts vertically downwards at point C. Be careful with all these square roots knocking about — it's easy to make a mistake. = 2.86 s (3 s.f.) b) Resolving horizontally, taking right as positive: s = s, u = 80, a = 0, t = 2.857... The standard deviation of the sunflowers' heights is 1.3 ft. 4x 2 - 9 # ff The first step is to write the function as partial fractions as follows: ff Factorise the denominator: 12x + 6 12x + 6 º 4x 2 - 9 ^ 2x + 3h^ 2x - 3h ff Write as an identity with partial fractions: 12x + 6 ^ 2x + 3h^ 2x - 3h º A + B 2x + 3 2x - 3 ff Add the partial fractions and cancel the denominators: A^ 2x - 3h + B^ 2x + 3h ^ 2x + 3h^ 2x - 3h 12x + 6 º A(2x – 3) + B(2x + 3) 12x + 6 ^ 2x + 3h^ 2x - 3h º Tip: Writing expressions as partial fractions is covered in Chapter 2 — but it's recapped here to remind you what they're all about. It gives a range of values for the parameter and is usually written H1. Q9 m(x) = Q8-9 Hint: Sketch a graph of each one first and identify where any asymptotes might be. f (x + h) g (x) - f (x) g (x) + f (x) g (x) - f (x) g (x + h) d c f (x) m = lim g (x + h) g (x) h d x g (x) h"0 g (x) [f (x + h) - f (x)] - f (x) [g (x + h) - g (x)] d c f (x) m = lim g (x + h) g (x) h dx g (x) h"0 ff You might start to recognise the top row here. The lower bound is found using: 2 # x + 2 dx ≈ h[y0 + y1] = 16 2 + 3 @ = 3.146 (4 s.f.) 0 The upper bound is found using: 2 # x + 2 dx ≈ h[y1 + y2] = 16 3 + 4 @ = 3.732 (4 s.f.) 0 b) h = 0.5, so the x-values are 1, 1.5, 2, 2.5, 3 Substitute these into the function to get y-values: y0 = 0, y1 = 0.3288..., y2 = 0.9609..., y3 = 1.6791..., y4 = 2.4138... So the values of x where the graph of dx dy 2 + 3x 2 > 0 and where is increasing are dx 3x - 1 1- 7 1+ 7 x < 3 and x > 3 . Q11 For y = 6 cos2 x – 2 sin (2x) show that dy = –6 sin (2x) – 4 cos (2x). c) The maximum points are at (–p, –1) and (p, –1). Tip: Sometimes there'll already be another k in the differential equation (e.g. if you've formulated the equation yourself). Q7 Hint: To find the largest possible domain, think about what values of x need to be excluded to make the function valid. 1 Sn = 2 n (n + 1) 1 S31 = 2 # 31 # 32 = 496 So Frazer draws 496 dots. × du dx dy ff Writing f(u) instead of y and f'(u) instead of du # gives: du f'(u) dx = f(u) + C dx If you're integrating an expression which contains a function of a function, f(u), try differentiating the function u. You would reject H0 if r ≤ –0.3783. ff Multiply the top and bottom lines of each fraction by whichever factor changes the denominator into the common denominator: 2y × y 2 x × xy^ x + 3h 1×x + x^ x + 3h × y 2 y 2 ^ x + 3h × x y × xy^ x + 3h ff All the denominators are the same — so you can just add the numerators: = 2y 3 + x - x 3 y - 3 x 2 y 2y 3 + x - x 2 y ^ x + 3 h = xy 2 ^ x + 3h xy 2 ^ x + 3h Tip: Always check if there's any more factorising and cancelling that can be done at the end. PROBLE M SOLVING P 2P 20 N Chapter 15 Dynamics 325 Harder equilibrium problems You can use everything from the last few pages to answer questions about all sorts of real-life situations. a) Copy the two-way table below and fill it in using Joe's data. ff Now substitute x = 0 into the identity: when x = 0, 5 = –2D + 5 ff Solving this gives D = 0. i and j are directed horizontally and vertically respectively. The rod is supported by a smooth peg at C, where AC = 2.4 m, in such a way that the rod makes an angle of 28° with the ground. # 10 sec 5x tan 5x dx = 10 a 15 sec 5x k = 2 sec 5x # 1 = 1 -1 2 cosec 3x cot 3x dx 2 a 3 cosec 3x k = - 1 cosec 3x 6 # - cosec 2 ^6x + 1h dx = - a - 1 cot ^6x + 1hk 6 Don't forget the minus that comes from the integration. ff You can now find the speed and direction of motion of the stone at this time. x-1 Find the functions fg(x) and gf(x), writing them as single fractions in their simplest forms. And you can do that using the product law: P(C Ç W) = P(C|W)P(W) = 0.3 × 0.4 = 0.12 ff Now you can draw the Venn diagram: P(W) – P(C Ç W) W 0.28 0.12 C 0.42 0.18 P(C) – P(C Ç W) b) Find the probability that Vikram catches the bus, given that he runs to the bus stop. Derivative The result after differentiating a function. A function is one-to-one if each value in the range corresponds to exactly one value in the domain. Find: a) the stone's horizontal displacement 0.5 seconds after being fired, b) the stone's vertical displacement 0.5 seconds after being fired, c) the straight-line distance from the stone's point of projection to the target. 18 ms–1 40° ff Consider the horizontal and vertical motion separately. c) f(–2) = (2 × (–2)3) – (–2)2 – (7 × –2) = –6 f(–1) = (2 × (–1)3) – (–1)2 – (7 × –1) = 4 There is a sign change (and the function is continuous in this interval) so there is a root in this interval. Find: a) the distance the particle travels before coming instantaneously to rest. Q9 Let y = f(x), where f(x) = cos x. a) The probability that a randomly selected length of elastic is snapped by a force of less than a N is 0.7580. ff This restricts the range of the modulus function to |f(x)| ≥ 0 (or some subset within |f(x)| ≥ 0, e.g. |f(x)| ≥ 1). y = f(x + a) –1 Tip: All these graphs use f(x) = sin x. 11p So x = 12 . Q8 a) Yes, it is a function. Acceleration due to gravity acts vertically downwards, so will only affect the vertical component of the projectile's velocity. d 4x - 1 Using the chain rule, (e) = 4e4x – 1 dx b) Using your answer to a), find the integral #e 4x - 1 dx. ff Find gf(x): gf(x) = g(2x + 1) = (2x + 1)2 ff Now solve gf(x) = 16: (2x + 1)2 = 16 (2x + 1) = 4 or –4 2x = 3 or –5 3 5 ff So x = 2 or x = - 2 Exercise 3.2 Q1 For the following functions, solve the given equations: a) f(x) = 2x + 1, x d R g(x) = 3x – 4, x d R fg(x) = 23 1 b) f(x) = x for {x : x ≠ 0} g(x) = 2x + 5 for {x : x d R } gf(x) = 6 x c) f(x) = x2 for {x : x d R } g(x) = - for {x : x ≠ 3} gf(x) = 4 x 3 2 d) f(x) = x + 1, x d R g(x) = 3x – 2, x d R fg(x) = 50 e) f(x) = 2x + 1, x d R g(x) = x , x ≥ 0 fg(x) = 17 f) f(x) = log10 x, x > 0 g(x) = 3 – x, x d R fg(x) = 0 g) f(x) = 2x for {x : x d R } g(x) = x2 + 2x for {x : x d R } fg(x) = 8 x h) f(x) = + , x ≠ –1 g(x) = 2x – 1, x d R fg(x) = gf(x) x 1 Q2 f : x Æ x2 + b, x d R and g : x Æ b – 3x, x d R (b is a constant). Find the distance of the centre of mass of the plank from A. Example 3 y y = (x – 2) Work out the area enclosed by the curve y = (x – 2)3, the x-axis and the lines x = 2 and x = 3. Q5 A painter of mass 80 kg stands on a horizontal non-uniform 4 m plank, AB, of mass 20 kg. Exercise 1.3 — Expanding (p + qx)n Q1 a) (2 + 4x)3 = 23(1 + 2x)3 = 8(1 + 2x)3 3#2 3#2#1 (1 + 2x)3 = 1 + 3(2x) + 1 # 2 (2x)2 + 1 # 2 # 3 (2x)3 = 1 + 6x + 3(4x2) + 8x3 = 1 + 6x + 12x2 + 8x3 So (2 + 4x)3 = 8(1 + 6x + 12x2 + 8x3) = 8 + 48x + 96x2 + 64x3 4 5 4 5 b) (3 + 4x)5 = 35a1 + 3 x k = 243a1 + 3 x k4 k5 4 5#4 4 2 a1 + 3 x = 1 + 5a 3 x k + 1 # 2 a 3 x k 5#4#3 4 3 + 1 # 2 # 3 a 3 x k + ... N Three forces of magnitude 9 N, 12 N and 13 N act on a particle P in the directions shown in the diagram. – S he has assumed that the card is put back at random, and that the volunteer can't see and track the card before choosing the second one — if they could, they might intentionally try to choose the same one twice, which would affect the probability. Vertex Turning point of a graph — the maximum or minimum point for a quadratic graph. c) Show that g(x) = 0 can be rearranged into the formula x = 3 4x - 1 . adjacent x = , so x = u cos q hypotenuse u ff Vertical component (y): sin q = y= u sin q opposite y = , so y = u sin q hypotenuse u u q x = u cos q Example 1 A ball is thrown with initial speed 9 ms–1 at an angle of 40° above the horizontal. 2 y = (x – 3) b) H ence work out the number of roots of the equation ln x – (x – 3)2 = 0. Q2 A car is modelled as a particle travelling in a plane. A particle of weight 30 N is placed 2 m from the support R. So the object is higher than 11 m above the ground for 2.251... ff The facts about the area under the curve on –z 0 z p.270 still apply but with m = 0 and s = 1 — e.g. points of inflection are at z = 1 and z = –1. The range of f –1(x) is the domain of f(x): {f(x) : f –1(x) > –3. Example 2 Find the particular solution of Tip: There's no 's' term in this differential equation. Tip: The magnitude of the frictional force will only be as big as is necessary to prevent motion. x 100 105 ff So P(X ≤ 105) = 0.894350... If it's too big, there could be multiple roots within the interval. dx 2 d u d v u = x, v = ex fi = 1, = ex dx dx dy dv du =u +v = xex + ex dx dx dx 2 d y d d x = (xex) + (e) = [xex + ex] + [ex] = xex + 2ex dx dx dx 2 2 d y 0 for all x, this means that (x + 2) < 0 fi x < –2 So the curve y = xex is concave when x < –2. 5 47 y = 108 x + 108 fi 108y = 5x + 47 fi 5x – 108y + 47 = 0 108 1 b) Gradient of the normal = - 5 =- 5 a 108 k 108 2 326 y = mx + c fi 3 = b5 × b - 5 ll + c fi c = 3 326 108 So the equation of the normal is y = - 5 x + 3 . 1#2#3 - 2 # -3 = 1 + (–2)(–4x) + (16x2) 2 - 2 # -3 # -4 + (–64x3) + ... y b) 4 y = ½ arctan x x – 4 y 2 1 2 1 – 2 –1 Q6 To find the inverse of the function, first write as y = 1 + sin 2x, then rearrange to make x the subject: 1 sin 2x = y – 1 fi 2x = sin–1(y – 1) fi x = 2 sin–1(y – 1) –1 Now replace x with f (x) and y with x: 1 1 f –1(x) = 2 sin–1(x – 1) = 2 arcsin(x – 1) 4. Integration by parts is another way to deal with integrating a product of two functions — it involves both differentiation and integration. R = 22.3 N (3 s.f.) Q2 a) Resolving horizontally: Fnet = ma 16 cos 20° – 7 = (49 ÷ 9.8) × a a = 8.035... So form two equations: 1 A + B(1) = 17 (at maximum) 2 A + B(–1) = 7 (at minimum) 1 + 2 : 2A = 24 fi A = 12 Sub back into 1 : 12 + B = 17 fi B = 5 So f(q) = 12 + 5 cos q You could also think of this as a transformation of the graph of cos q — a vertical stretch with scale factor 5 followed by a translation up by 12. Tip: The degree of the divisor will always be less than or equal to the degree of the polynomial. = 4a a = 0.227 ms–2 (to 3 s.f.) 346 Chapter 15 Dynamics Exercise 3.3 R Q1 A light, inextensible string passing a over a smooth pulley connects T boxes A and B, of mass 10 kg F A and 8 kg respectively, as shown in the diagram. Tip: Some proofs use P = p1p2...pn + 1 — but the method is the same. dx x ^ 3x - 4 h b) 3 - 2 x 2x 1 # ff Use the formula: # x^3x - 4h dx = ln |x3 – 2x2 – 1| + C x 3 - 2x 2 - 1 Tip: You should get used to spotting when the numerator is the derivative of the denominator — you won't have to differentiate the denominator every time. For example, a general polynomial of degree 2 is Ax2 + Bx + C, where A, B and C are constants to be found. sin 4 x cos x cos x # sin dx = # dx = 4 x ^sin xh4 Evaluate # # cos x^sin xh -4 dx ff Now, f(x) = sin x, so f'(x) = cos x. Q5 Prove that 1 + 2 is irrational. Q2 a) There are 10 'trials', each with a probability of success of 0.65. n is large, so you don't need to use a continuity correction and you can find the probability directly using your calculator. 1 sin (2q + 0.983) = 13 fi 2q + 0.983 = 0.281 (not in correct interval), (p – 0.281), (2p + 0.281), (3p – 0.281), (4p + 0.281). ff If all your points lie exactly on a straight line with a negative gradient (perfect negative correlation), r = –1. You can also assume that a and b do not have any common factors. 65 The line changes from concave to convex at approximately x = 65 ff Use your values for x and m to find s. So: 0 ≤ d < 1.25 Exercise 1.2 — Moments in equilibrium Q1 a) Taking moments about A: Moments clockwise = Moments anticlockwise 1.5y + (2 × 8) = 16 × 5.5 1.5y = 72 y = 48 N Resolving vertically: x + 16 = y + 2 x = 50 – 16 = 34 N b) Resolving vertically: 10 + 6 = 13 + x fi x = 3 N Taking moments about C: 10y + 2x = 6 × 5 10y = 30 – 6 fi y = 2.4 m Q2 a) Mass acts at centre of rod, i.e. 2.5 m from A, 1 m from C and 0.5 m from D. - 4.011... a) Given that this can be represented by the differential equation dN = kN, show that the general solution of this equation is dt N = Aekt, where A and k are positive constants. As with the product rule, there's a rule that lets you differentiate quotients easily — the quotient rule: If y = u v du dv dy v dx - u dx = dx v2 Where u and v are functions of x, i.e. u(x) and v(x). This occurs when s = m + s fi 50 = 46 + s fi s = 4 You could have used any of the other conditions involving m and s, so any answer between 3 and 4 is acceptable. P(S)P(T) = 0.2 × 0.46 = 0.092 ≠ P(S Ç T) so S and T are not independent. Answers 395 9. Usually given as a set of values that y or f(x) can take. Vectors in Three Dimensions 2. 4 Now find an expression for the ball's time of flight. Tip: f(x, y) and g(x, y) don't actually both have to include x and y — one of them could even be a constant. sin x 1 ff Since tan x º cos x , and sec x º cos x , dy 1 sin x = cos x × cos x = sec x tan x dx As with other rules covered in this chapter, the rules for sec x, cosec x and cot x can be used with the chain, product and quotient rules and in combination with all the other functions you've seen so far. b) If the car drops to one half of its initial value in the first year after purchase, how long (to the nearest month) will it take to be worth 5% of its initial value? or lower'. 5 5 Q2 (a + 5x)5 = a5`1 + a x j So the x2 term is: 5#4 5 2 25 250a5 k 2 a5a 1 # 2 k` a x j = 10a5a 2 x 2 k = a x = 250a3x2 a a2 So 250a3 = 2000 fi a3 = 8 fi a = 2 5 7 5 7 Q3 a) (2 – 5x)7 = 27a1 - 2 x k = 128a1 - 2 x k 7 7#6 - 5 2 -5 a1 - 5 2 x k = 1 + 7a 2 x k + 1 # 2 a 2 x k + ... It has range f –1(x) ≥ 0 (the domain of f(x)). c) Repeat parts a) and b) using this new model. 18 0.3170 0.4000 0.4683 0.5425 0.5897 19 0.3077 0.3887 0.4555 0.5285 0.5751 20 0.2992 0.3783 0.4438 0.5155 0.5614 ff So the critical region is r ≤ –0.3783 b) M ary calculates a PMCC of r = –0.28. PROBLE M SOLVING Q10 If f(x) = arccos(x2) for –1 ≤ x ≤ 1, find the equation of the tangent 1 to the graph of y = f(x) when x = and 0 ≤ y ≤ p. ff What happens to the model when the variables get really big/small? ff So the distance from the origin to (a, b, c) is: 2 Tip: A unit vector has a magnitude of 1. Q6 The motion of a particle is modelled by the parametric equations x = t2 – t, y = 3t – t3. dx Q2 x = (2y3 – 5)3, so let x = u3 where u = 2y3 – 5 dx du fi = 3u2 = 3(2y3 – 5)2, = 6y2 du dy dx dx du = × = 3(2y3 – 5)2 × 6y2 = 18y2(2y3 – 5)2 dy du dy dy 1 = dx 18y 2 (2y3 - 5) 2 1 Q3 a) x = 4 + y fi x = u 2 , u = 4 + y 1 1 1 1 dx du fi = 2 u- 2 = 2 , =1 du 4 + y dy 1 dx dx du 1 1 1 = × = 2 ×1= 2x dy du dy 4+y 2 4+y dy fi = 2x dx Use x = 4 + y from the question to get dx/dy in terms of x. Resolving vertically, taking up as positive: u = 19.3, v = 0, a = –9.8, t = t Using v = u + at: 0 = 19.3 – 9.8t fi t = 1.969... y = f(|x|) ff For the graph of y = f(|x|), the negative x-values produce the same result as the corresponding positive x-values. 3e 3t At time t seconds, the body's acceleration is given by a = f 6 p ms–2. a) Find an expression for the rate of change of n with respect to time. ff The volume of water in the tank is V = pr2h = p(20)2D = 400pD. dt dD dt b) If t = 1 day and l = 5, dn = 104 × 5 × 25 × ln 2 = 16640 ln 2 dt = 1.15 × 104 per day (3 s.f.) dV Q9 a) Volume of water V = pr2h, so = pr2. 1 The solutions for tan x = - 2 are x = –26.6° + 180° = 153.4° or x = –26.6° + 360° = 333.4°. b) State the domain and range of f –1(x). The nth term is arn – 1 = 3 × 1.25n –1, The first term is the height of the first doll, so the height of the 8th doll is the 8th term. N T A T 4g ff Resolve horizontally for A (Æ): T – 23.002... n = 3, so n + 1 = 4. Here's the method: ff Be able to integrate ff You'll be given an integral that's made up of two functions of x. ff Now, if you use a calculator to find z for which F(z) = 0.9015, you can form an equation in m. Example Sanaa wants to know the probability that it will rain tomorrow. ff Resolving vertically, taking down as positive: u = 0, v = vy , a = 9.8, t = 0.5 Using v = u + at: vy = 9.8 × 0.5 = 4.9 ms–1 ff The horizontal component of velocity is constant, as there is no acceleration horizontally, so vx = ux = 10 ms–1. Composite function A combination of two or more functions acting on a value or set of values. a) Find the probability that a battery lasts less than 200 hours. Proof Using mathematical arguments to show that a statement is true or false. A B S A B S ff Using the probability formula (and assuming event A has already happened): number of possible outcomes corresponding to B total number of possible outcomes number of outcomes in A + B = number of outcomes in A P(B|A) = ff So if n(A Ç B) = number of outcomes in A Ç B, and n(A) = number of outcomes in A: n^ A + Bh P(B|A) = n ^ Ah ff Now, if you divide the top and bottom of the fraction by n(S) (the total number of outcomes in S), its value doesn't change, but you can write it in a different way: n^ A + Bh = P^B|Ah = n ^ Ah n^ A + Bh n ^ Ah n^ S h n^ S h = P^ A + Bh P^ Ah the formula in the box above And if you rearrange this formula for conditional probability, you get a formula that's known as the product law. dy 1 a) Find the gradient of C, , when t = 2 . du You might have to carry out integration by parts again to find # v dx . Mostly, you'll be resolving an object's weight, because weight always acts vertically downwards, and you'll need to use its components parallel and perpendicular to the slope. # b a f (x) dx . 2 y = x – 2x 2 4 C1 ff C2 is more difficult — it can't be found by a single integration. 4 8 8 4 p 4 h # p 4x tan x dx ≈ 2 [y0 + 2(y1 + y2 + y3) + y4] 2 - 4 p p p p p p = 16 8- p tan ` - 4 j + 2 ` - 2 tan ` - 8 j + 0 + 2 tan ` 8 jj + p tan ` p jB 4 p = 16 [p + 2(0.6506 + 0.6506) + p] = 1.74 (3 s.f.) e) h = (0.3 – 0) ÷ 6 = 0.05, so the x-values are 0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3. 0 .3 #0 e x + 1 dx ≈ 2h [y0 + 2(y1 + y2 + y3 + y4 + y5) + y6] = 0.05 6 e0 + 1 + 2^ e0.05 + 1 + e0.1 + 1 2 + e 0.15 + 1 + e 0.2 + 1 + e 0.25 + 1 h + e 0.3 + 1 @ = 0.025 6 2 + 2^ 2.0513 + 2.1052 + 2.1618 + 2.2214 + 2.2840 h + 2.3499 @ = 0.025[1.4142 + 2(1.4322 + 1.4509 + 1.4703 + 1.4904 + 1.5113) + 1.5329] = 0.441 (3 s.f.) p p p 2p 5p p f) h = 6 , so the x-values are 0, 6 , 3 , 2 , 3 , 6 , p. It has initial speed 20 ms–1 at an angle of 45° above the horizontal. dx Putting this into y = mx + c gives: 1 = 2(0) + c fi c = 1. ff As the rod is in equilibrium, you know that clockwise moments = anticlockwise moments. Calculating with Vectors Learning Objectives: ff Be able to find the magnitude of any vector in three dimensions. x2 = 1 y2 = 3 = 1.732... She asks 20 people MODELLING at a cinema to give a score out of 100 for each type of movie, and calculates the product moment correlation coefficient of the data. b) Using starting value x0 = 3: x1 = ln (10x0) = ln (10 × 3) = 3.401 x2 = 3.527, x3 = 3.563, x4 = 3.573 You could have started with x0 as anything between 3 and 4, in which case you'd get different values for x1 – x4. Q3 Use integration by parts to find the area enclosed by the curve y = x2e–2x, the x-axis and the lines x = 0 and x = 1. Example 6 A uniform rectangular lamina of mass 12 kg is pivoted at one corner at A. Find the coefficient of friction between the peg and the beam at A. Example 1 Prove the following statement: "If x2 is even, then x must be even." ff You can prove this statement by contradiction. ff Step 5: If you're asked for a general solution, leave C (or k) in your answer. c) Solving algebraically: |3x – 6| = |10 – 5x| fi (3x – 6)2 = (10 – 5x)2 9x2 – 36x + 36 = 100 – 100x + 25x2 16x2 – 64x + 64 = 0 fi x2 – 4x + 4 = 0 fi (x – 2)2 = 0 So there is only one solution at x = 2. Find the components of its weight parallel and perpendicular to the sides of the lamina. They'll be really useful when integrating complicated trig functions. Using a calculator Method 2 — Direct ff Use 84 as the lower bound and 106 as the upper bound in the normal cdf function on your calculator. u = 16.5, v = v, a = –9.8, t = 5 v = u + at v = 16.5 + (–9.8 × 5) = –32.5 fi v = (40i – 32.5j) ms–1 Q5 a) s = (15i + 6j), u = u, a = –9.8j, t = 3 1 s = ut + 2 at2 1 (15i + 6j) = 3u + (2 × –9.8j × 32) 3u = (15i + 6j) + 44.1j fi u = (5i + 16.7j) ms–1 b) u = (5i + 16.7j), v = v, a = –9.8j, t = 3 v = u + at v = (5i + 16.7j) + (–9.8j × 3) fi v = (5i – 12.7j) ms–1 c) Velocity immediately following impact with wall is (–2.5i – 12.7j) ms–1. Answers 403 Q5 The value decreases by 15% each year so multiply by 1 – 0.15 = 0.85 to get from one term to the next, so the common ratio r = 0.85. ff First identify u and v for the quotient rule, and differentiate separately: du u = 2x2 − 1 fi = 4x dx dv and v = 3x2 + 1 fi = 6x dx ff Then put everything into the quotient rule: du - dv v u dy (3x 2 + 1) (4x) - (2x 2 - 1) (6x) = dx 2 dx = dx v (3x 2 + 1) 2 Tip: Don't try and simplify straight away or you're more likely to get things mixed up. From the graph you can see there's a root at x = 1 and two others near x = –1 and x = 3. Statistical Tables Critical values for the Product Moment Correlation Coefficient, r The r-values in the table are the minimum values of the Product Moment Correlation Coefficient for which a one- or two-tailed hypothesis test at the given significance level on a data set of size n is statistically significant. cos–1 (–0.57142...) = 124.84990... If the middle 95% of the distribution lies between 6.45 and 20.17, find the value of m. ff The sign has changed, so there is a root between 2 and 3. Product moment correlation coefficient A measure of the strength of the correlation between two variables. ff Start by assuming that the statement is not true, and that 2 can be a written as b with a and b both non-zero integers. In set notation, this is {x : x ≥ 2} È {x : x ≤ –2} Q9 |1 + 2x| ≤ 3 fi -3 ≤ 1 + 2x ≤ 3 fi –4 ≤ 2x ≤ 2 fi –2 ≤ x ≤ 1 Draw the graph of y = |5x + 4| for these values: y 1 e o b) 4 c) y = (x – 1)3 + 4 (= x3 – 3x2 + 3x + 3) y Q3 b) d) c) a) 2 –1 –1 x 4 The transformations can be described as follows: 0 b) A translation of 2 up / by the column vector e o. Iteration sequence The list of results x1, x2, ... The Addition Formulas 7. f(x) ff Draw a sketch to show the information. To allow for this, you might have to use a continuity correction. Solving differential equations ff Solving a differential equation means using it to find an equation Tip: Remember — your equation might be in terms of variables other than x and y. At time t seconds, the body's acceleration is given by a = [(4 – 18t)i + 72tj] ms–2, relative to a fixed origin O. Progression Another word for sequence. –1 (x + 3) is valid for |x| < 3. hyp 17 1 b) sec q = cos q = - 8 . b) the magnitude and direction of the reaction force acting on the beam at A. Including plenty of modelling and problem solving • And a free Online Edition... b) Explain why fg(–1) is undefined. Integration of ex and 1x Learning Objectives: ff Be able to integrate functions containing 1 ex and x terms. y - 1 = 2x log2 (y – 1) = log2 2x log2 (y – 1) = x log2 2 Tip: If you're not given the domain and / or the range of the function you'll need to work it out. = 24.2 N (3 s.f.) b) Resolving vertically: R = mg + 4 sin 25° mg = 24.168... P(B|A) = 0.8 ≠ P(B) = 0.6, so A and B are not independent. Given that tan q = 3, find: a) the particle's speed 2 seconds after being projected, b) the direction of motion of the particle at this time. Uniform Describes a body whose mass is evenly spread throughout the body. Tip: The suvat equations only work for motion in a straight line. ff Be able to find and rearrange iteration formulas and understand that some don't converge to a solution of an equation. Example 2 Tip: So the standard deviation of Y is s = 192. a) Sketch the graphs of f(x) and f –1(x) on the same set of axes. # 8^2x - 4h 4 = 8 × a 1 ^2x - 4h5 + c k 2× 5 8 = ^ - h5 + C 10 2x 4 5 4^2x - 4h = 4 ^2x - 4h5 + C = +C 5 5 b) Use your answer to part a). 4 The derivative of f(x) is given by f'(x) = . a - 13 So, 1.8 = –1.281... Q14 After 0 years, u1 = a and after 1 year, u2 = ar. But in this case it's much easier to just find the next few terms. Learning Objectives: Friction ff Understand the term ff If an object is in contact with a rough surface, then friction can act to try and prevent it moving. m ff Now consider the situation where the ladder is on the point of sliding towards the wall — draw a new diagram showing the forces on the ladder: N F 50 N ff The only change is that the friction force F acts in the opposite direction. Show that there is a root of f(x) = 0 between x = 0.9 and x = 1.0. Q3 f(x) = x3 + ln x – 2, x > 0 Show that there is a root of f(x) = 0 in the interval [1.2, 1.3]. 1-x Find f2(x) and gfg(x). Example 2 Use the trapezium rule to approximate a) n = 2 # 0 4 6x 2 dx to 3 d.p. using: x3 + 2 ff For 2 strips, the width of each strip is h = so the x-values are 0, 2 and 4. × 3 = 10.4 ms–1 (to 3 s.f.) Example 2 Two forces, given by the vectors (12i – 4j) N and (–3i + 16j) N, act on a particle of mass 3 kg, causing it to accelerate from rest. There is a point of inflection at approximately s = 50. Then x2 = (2n)2 = 4n2 = 2(2n2) which is even, which contradicts the assumption that x2 is odd. The thrust in the strut has magnitude 110 N. So the null hypothesis can be rejected at the 5% level, but not at the 1% level. Let X be the sample mean number of hours. ff First, work out the degrees of the quotient and remainder: f(x) has degree 4, so the quotient q(x) has degree 4 − 1 = 3. N ff Resolve parallel to the plane ( ): Fnet = ma T – F – 3g sin 20° = 3 × 0.6 T = 1.8 + 11.050... By the chain rule, 2 2 d y d y du = = eu × 4x = e 2x × 4x = 4xe 2x × dx du dx Chapter 8 Integration 195 b) Find # 4xe 2x 2 dx using your answer to part a). F(z) = 0.9953 for z = 2.60 (2 d.p.) 36 – m fi = 2.6 fi 36 – m = 2.6s 2 s Subtracting equation 1 from equation 2 gives: 36 – 30 – m – (–m) = 2.6s – 1.4s fi 6 = 1.2s fi s = 5 Putting s = 5 into equation 1 gives: m = 30 – 1.4 × 5 = 23 So m = 23 and s = 5 4-m b) P(X < 4) = 0.9332 fi P a Z < s k = 0.9332 F(z) = 0.9332 for z = 1.50 (2 d.p.) 4-m fi s = 1.5 fi 4 – m = 1.5s 1 4.3 - m P(X < 4.3) = 0.9987 fi P a Z < s k = 0.9987 F(z) = 0.9987 for z = 3.01 (2 d.p.) 4.3 - m fi = 3.01 fi 4.3 – m = 3.01s 2 s Subtracting equation 1 from equation 2 gives: 4.3 – 4 – m – (–m) = 3.01s – 1.5s fi 0.3 = 1.51s fi s = 0.198675... We can express C as a logarithm — call it –ln k, where k is a constant. y 1 Exercise 2.2 — Graphs of functions Q1 a) Yes, it is a function. Q2 A uniform ladder of mass 11 kg and length 7 m rests against a rough vertical wall, at an angle of 60° to smooth, horizontal ground, as shown. Chapter 13 Hypothesis Testing 295 ff The significance level is 5%, so a = 0.05. So the largest possible domain for fg(x) is x ≥ –5. The strut exerts a thrust of 16 N at the midpoint of the rod. dt V2 Chapter 8 Integration 221 Example 3 The rate of cooling of a hot liquid is proportional to the difference MODELLING between the temperature of the liquid and the temperature of the room. O One-tailed test A hypothesis test is 'one-tailed' if the alternative hypothesis is specific about whether the parameter is greater or less than the value specified by the null hypothesis. m So it falls short of the target by 7.51 m (3 s.f.) Q4 a) s = (200i – 40j), u = (ai + bj), a = –9.8j, t = 5 1 s = ut + 2 at2 1 (200i – 40j) = 5(ai + bj) + (2 × –9.8j × 52) 5(ai + bj) = (200i + 82.5j) (ai + bj) = (40i + 16.5j) fi a = 40 and b = 16.5 You could also have answered this by considering the horizontal and vertical components separately, and using the suvat equations as usual. Given that the particle is in equilibrium, find the values of P and q. b) Write down an estimate of the value of a to 3 decimal places. Equating the coefficients of x3 gives 2 = A. 10 x f(x) x b) Many-to-one: f(x) b) fg(x) = f(ln (x + 1)) = 1 x 180° –180° c) One-to-one: f(x) x 1 f(x) 2 2 –2 e) Many-to-one: x f(x) 2 –2 Domain: {x : x > 0} (domain of f = range of g = {x : x > 0}) Range: {fg(x) : fg(x) > 0} Q10 fgh(x) = fg(x2 + 1) = f(5(x2 + 1) – 1) = f(5x2 + 4) = 3(5x2 + 4) + 2 = 15x2 + 14 –1 d) Many-to-one: 1 ln ^ x + 1h 2 x 3. 8 ff First, you have to work out the degree of the quotient, which will depend on the degree of the polynomial f(x): deg q(x) = deg f(x) − 1. a) At which two values of y does the line x = 1 cut the curve? The Double Angle Formulas 8. ff You can use trigonometry to resolve forces and find their components in a particular direction. Critical region The set of all values of the test statistic that would cause you to reject the null hypothesis. b) If the initial population is P0, solve your differential equation to find P in terms of P0, k and t. Show that: U 2 sin 2 q a) the body reaches a maximum height of metres 2g above the ground, U sin q b) the body reaches its maximum height g seconds after it is projected. = 1.40 N (to 3 s.f.) 2 2 1.354... cosec x dy = dx −cosec x cot x sec x sec x tan x cot x −cosec2 x If y = Tip: If you can't remember which trig functions give a negative result when you differentiate them, just remember it's all the ones that begin with c — cos, cosec and cot. Chapter 9 Numerical Methods Example 2 The equation x3 − x2 − 9 = 0 has a root close to x = 2.5. What is the result of using xn+1 = x n3 - 9 with x0 = 2.5 to find this root? This means ? Q4 ex + y2 – xy = 5 – 3y is a curve passing through two points (0, a) and (0, b), where a < b. x2 - 4 What is the largest continuous domain which would make m(x) a function? This means that it will take 7 days for the population to halve. y = f(x) 1) S tart by drawing a vertical line from x0 to the curve. x1 = -1 2 (-1) - 5 x2 = –1.772833..., x3 = –1.772001..., x4 = –1.772001... e) Sum of moments = (6 × 2.5) + (10 cos 30° × 5) – (10 sin 30° × 5) = 15 + 43.301... b) He uses up all the paint in the pot near A, and discards the pot. 2 Answers 2 2 Either q = 6, then Q is (4, 5, 13), or q = –2, then Q is (–4, 5, –3). Q2 a) No, as sin x is a many-to-one function over the domain x d R . There might be multiple different substitutions that you could use to get the right answer. 1 0 x ff Step 3: Reflect the negative (dotted) part of the line in the x-axis. Usually written H0. Tension The force in a taut wire or string. 3) Then a vertical line to the curve, etc. Q4 f(x) = a) Find f –1(x), stating the domain and range. cm. a) He stands at the centre of mass of the plank and finds that the reaction forces at C and D are in the ratio 4 : 1. Cobweb diagrams Cobweb diagrams look like they're spiralling in to the root (or away from it). Here are the double angle formulas again: sin 2x º 2 sin x cos x cos 2x º cos2 x – sin2 x 2 tan x tan 2x º 1 tan 2 x You came across two other ways of writing the double angle formula for cos, which come from using the identity cos2 x + sin2 x º 1: cos 2x º 2 cos2 x – 1 cos 2x º 1 – 2 sin2 x Once you've rearranged the original function using one of the double angle formulas, the function you're left with should be easier to integrate using the rules you've seen in this chapter. -1 -1 1 3 -1 ^8 + 2xh 3 = 8 - 3 a1 + 2x k = 1 `1 + x j 3 d) 4 8 2 - 1 #- 4 -1 xj 3 3 ` x j2 ` `1 + 4x j 3 = 1 + a - 1 k + 4 3 4 1#2 - 1 #- 4 #- 7 x 3 + 31 # 23# 3 3 ` 4 j + ... The strut makes an angle of 71° with the wall, and the thrust in the strut has magnitude 95 N. Tip: v2 = u2 + 2as doesn't have a vector equivalent, as you can't really square a vector. = 14.6 kg (3 s.f.) Centres of mass ff The centre of mass (COM) of an object is the point where the object's weight can be considered to act. Show by rearranging that the expressions for y in parts a) and b) are the same. a) X ~ N(m, s2), P(X < 30) = 0.9192 and P(X < 36) = 0.9953 b) X ~ N(m, s2), P(X < 4) = 0.9332 and P(X < 4.3) = 0.9987 c) X ~ N(m, s2), P(X < 20) = 0.7881 and P(X < 14) = 0.0548 d) X ~ N(m, s2), P(X < 696) = 0.9713 and P(X < 592) = 0.2420 e) X ~ N(m, s2), P(X > 33) = 0.1056 and P(X > 21) = 0.9599 f) X ~ N(m, s2), P(X > 66) = 0.3632 and P(X < 48) = 0.3446 The normal distribution in real-life situations PROBLE M SOLVING MODELL ING Now it's time to use everything you've learnt about normal distributions to answer questions in real-life contexts. 2 2 b) x1 = x0 + 5 = 5 + 5 = 5.4, x2 = 5.370, x3 = 5.372, x4 = 5.372 2 Q5 x1 = 2 – ln x0 = 2 – ln 1.5 = 1.595, x2 = 1.533, x3 = 1.573, x4 = 1.547, x5 = 1.563, x6 = 1.553, x7 = 1.560, x8 = 1.555, x9 = 1.558 (all 4 s.f.) The iterative sequence is bouncing up and down but closing in on the correct answer. found with an iteration formula. Integrating using trig identities Integrating using the double angle formulas If you're given a tricky trig function to integrate, you might be able to simplify it using one of the double angle formulas. dy a) Find in terms of t. s after being hit (when it is rising) and 1.558... Chapter 8 Integration 229 Chapter 9 Numerical Methods 1. ff Trials are independent, and the probability of 'success' (p) is constant. Introducing an upper limit on t or N would show at which point the model stops being accurate. Watch out for familiar right-angled triangles such as 5 4 3 13 and here: 2 Q4 A particle is projected at an angle q above the horizontal with speed 18 ms–1. 55x + 7 ^2x - 5h^3x + 1h2 Q4 a) º A ^2x - 5h + B ^3x + 1h + C ^3x + 1h2 55x + 7 º A(3x + 1)2 + B(2x – 5)(3x + 1) + C(2x – 5) Using the substitution method: 17 2 5 275 When x = 2 , 2 + 7 = A b 2 l 289 289 fi 2 = 4 A fi A=2 2 55 1 When x = - 3 , - 3 + 7 = C b - 3 - 5 l - 34 = - 17 C fi C = 2 3 3 Equating coefficients of x2: 0 = 9A + 6B fi 6B = –18 fi B = –3 2 55x + 7 3 2 + So º ^2x - 5h ^3x + 1h ^3x + 1h2 ^2x - 5h^3x + 1h2 1 2 -1 b) (2x – 5)–1 = - 5 a1 - 5 x k 1 2x 4x 2 = - 5 a1 + 5 + 25 + ... N and a vertical component of 7.5 N ff Use Pythagoras' theorem to find the magnitude of the force. It's just a bound. One brother tries to pull the sledge up the slope with a force of magnitude 70 N, while the other tries to pull the sledge down the slope with a force of magnitude F. ln 3 – (3 – 3)2 = 1.098... y –4 –2 0 –2 –4 2 4 c) Rearrange to f(x) = 0 so e2x + x2 – 3 = 0 f(–2) = e(2 × –2) + (–2)2 – 3 = 1.01... (ii) P(R|S Ç T') Q4 X, Y and Z are independent events, with P(X) = 0.84, P(Y) = 0.68 and P(Z) = 0.48. Given that the child is dressed as a werewolf, what is the probability that they're a boy? Chapter 12 The Normal Distribution 271 Example 2 f(x) a) If X ~ N(102, 144), find P(X > 78) to 3 s.f. Tip: You might be able to do this straight away by using 78 as a lower bound and choosing a large upper bound. Once you've got an expression for the gradient, you'll have to use it to do the sort of things you'd normally expect, like finding stationary points of curves and equations of tangents and normals. f(x) is a continuous function, so if f(7) and f(8) have different signs then there is a root in the interval 7 < x < 8: f(7) = (6 × 7) – 72 + 13 = 6. Continuity corrections The binomial distribution is discrete and occurs in situations with a fixed number (n) of independent trials, where the outcome of each trial can be either a success or failure. 23 - m = 1.2901... Find # 8 cosec2 x cot3 x dx. Q4 A body is fired at an angle a to the horizontal with speed 22 kmh–1. Find the car's velocity at this time. b) a) TN 8N 25° 14 N 50° mg TN mg Q2 The diagram shows a particle suspended from two light, inextensible strings. If the domain was not restricted, fg(x) would be undefined in places, e.g. fg(–6) = f(–6 + 5) = f(–1) = - 1 (which is undefined). The significance level is a = 0.05 and it is a one-tailed test. c) Find the position vector of point R, in unit vector form. You would reject H0 if r ≥ 0.7977 or r ≤ –0.7977. Q2 The random variable X follows a binomial distribution. Example 2 Find: a) # 8x 3 - 4 dx x 4 - 2x ff Differentiating: d 4 (x – 2x) = 4x3 – 2 dx and 8x3 – 4 = 2(4x3 – 2) ff The numerator is 2 × the derivative of the denominator, so b) # # 8x 3 - 4 4x 3 - 2 dx = 2 # 4 dx = 2 ln|x4 – 2x| + C x 4 - 2x x 2x 3 sin 3x dx cos 3x + 2 ff Differentiating: d (cos 3x + 2) = –3 sin 3x dx ff The numerator is –1 × the derivative of the denominator, so - 3 sin 3x 3 sin 3x # cos dx = – # dx = –ln|cos 3x + 2| + C cos 3x + 2 3x + 2 ff You can make the answer a lot neater by combining it all into one logarithm — a question might ask you to do this. Q5 Find the range of values of x for which the curve y = xex is concave. Tip: You can multiply the quotient by (x − 5), and then add on the remainder, 6, to check you've got it right. Integration of (ax + b)n Learning Objectives: ff Be able to integrate functions of the form (ax + b)n, n ≠ –1, where a, b and n are constants. t= -8 ! Ignore — t can't be negative ff Check that the j-component of r is greater than zero for t = 1.099... so b = –2.115 (4 s.f.) f) The Newton-Raphson method fails when g'(xn) = 0 because it involves dividing by zero in the formula. Taking moments about C: 8g × 1.5 = 16g × x 1.5 = 2x fi x = 0.75 So the distance of the particle from A is 1 – 0.75 = 0.25 m Q5 a) The painter stands at the plank's COM, so the weight acting at this point is his weight plus the plank's weight, i.e. 80g + 20g = 100g. To avoid having to write "and", "or" and "not" all the time, you can use set notation to describe probabilities: S S A A B P(A and B) = P(A Ç B) S B P(A or B) = P(A È B) A B P(not A) = P(A') So, in this notation, the addition law would be written: P(A È B) = P(A) + P(B) – P(A Ç B) You should also know that: P(A') = 1 – P(A) P(A) = P(A Ç B) + P(A Ç B') For mutually exclusive events, P(A Ç B) = 0 fi P(A È B) = P(A) + P(B) and for independent events, Tip: Mutually exclusive means that the two events can't both happen at the same time (like rolling a 3 and rolling a 4 at the same time on the same dice). ff The graph of the function gives you a rough idea of how many roots there are and where they are. Using the table, the critical value is 0.5494. 1 Q5 The function f(x) = - is undefined when x = 2, x 2 so a = 2. Resolving Splitting a vector up into components. Now integrate the expression for velocity to find an expression for the particle's position vector: r= # v dt = ` # ^4t - 9t 2 + 8 h dt j i + ` # ^36t 2 - 9h dt jj = (2t2 – 3t3 + 8t)i + (12t3 – 9t)j + D ff When t = 1, r = 2i. To find the angle between two vectors: q–p P Q p ff Create a triangle with the vectors as two sides, O and angle q between them. The particle's initial velocity is (7i + 3j) ms–1, and it moves freely under gravity. 7x 3 x2 x = 1 – 12 + 72 – 2592 + ... Example 1 The number of bacteria in a petri dish, b, is increasing over time, t, at a rate directly proportional to the number of bacteria. ff h is the width of each strip — it's equal to ^ b - ah n . Simplifying Expressions Simplifying expressions in this chapter involves a lot of algebraic fractions. ff Be able to find a body's position vector, velocity or acceleration at a particular time given an expression for one of the other vectors. It travels freely under gravity for 4 seconds, before landing on the ground. The coefficient of friction between the ladder and the ground is 0.45. Find the probability that a randomly selected donor from the group takes less time than Edward to replace their red blood cells. Q5 Particles A and B, of mass 8 kg and 9 kg respectively, are connected by a light, inextensible string which passes over a fixed, smooth pulley. 2. If it's more complicated though, like sin (x3), it's worth using the chain rule. fi t = 0.6388... Q1 Find the magnitude of each of the following vectors: JK- 4 NO JK4NO KK OO KK OO a) i + 4j + 8k b) KK2OO c) KK- 5 OO d) 7i + j – 7k K OO KK OO K L 20P L4 P KKJ- 2NOO K O e) KK 4 OO KK OO L- 6P Q2 Find the magnitude of the resultant of each pair of vectors. Location of Roots 2. 2 b) f –1(2) = - = 2 3 2 1 1 1 1 c) f –1 b 2 l = 2 1 = - = 5 6 1 3- 2 x-4 Q5 a) f(x) = + with domain {x : x > –3} has a range x 3 {f(x) : f(x) < 1} — again, sketching the graph will help. Q7 A crystal of a salt is shaped like a prism. For some equations of the form y = f(x), the easiest way to differentiate them is to rearrange them and use implicit differentiation. P(R)P(T) = 0.25 × 0.46 = 0.115 ≠ P(R Ç T) so R and T are not independent. Q2 a) x –4 –2 0 –2 4 2 –4 b) The graphs cross 3 times, so the equation has 3 roots. Resolving vertically, taking up as positive: s = s2, u = 24 sin 70°, a = –9.8, t = 4 1 s = ut + 2 at2 1 s2 = (24 sin 70° × 4) + (2 × –9.8 × 42) = 11.810... For how long is it higher than 11 m above the ground? dx a b x The area of each rectangle is height × width. P^G + Hh 0.24 24 12 b) P(H|G) = = = = 0.7 70 35 P (G) P^A + Bh 0.34 = = 0.5 0.68 P (A) P^A + Ch 0.16 16 = = = 4 b) P(A|C) = 0.44 44 11 P (C) P^B + Clh = 0.49 = 0.875 c) P(C'|B) = P (B) 1 - 0.44 Q4 a) P(B|A) = Q5 a) P(J Ç K) = P(J)P(K|J) = 0.4 × 0.2 = 0.08 P (J + K) 0.08 P(K) = = = 0.125 0.64 P (J|K) P(J È K) = P(J) + P(K) – P(J Ç K) = 0.4 + 0.125 – 0.08 = 0.445 b) P(J È K') + P(J' Ç K') = 1, so P(J' Ç K') = 1 – P(J È K) = 1 – 0.445 = 0.555 Q6 Let F = 'over 6 feet tall' and G = 'can play in goal'. c) Assuming the variance remains unchanged, use a 1% level of significance to test the hypothesis. dx = (x – 4x + 5). x2 + 1 2 domain: x ≥ 0 1 range: f –1(x) ≥ 2 Swap round the domain and range. The reverse process of the product rule. Tip: The º symbol means it's an identity. m Tip: The calculation here is almost identical to the one above, except that the friction term has changed sign. Binomial coefficient The number of orders in which x objects of one type and (n – x) objects of a different type can be arranged. dx 1 Step 2: Rearrange the equation into the form: g (y) dy = f(x) dx. ff But this isn't possible if the statement that x2 is even is true. f(1.0) = sin (2.0) – 1.0 = –0.0907... 1 . b) Show that the function in part a) can be written as –ln (sin x) and differentiate it — you should get the same answer as in part a). So the projectile reaches a maximum height of 5 + 5.102... So (1 + 6x)3(2 – 5x)7 = 128 – 2240x + 16 800x2 + 18x(128 – 2240x) + 108x2(128) + ... = = 34.6 N (3 s.f.) 0.36602... ff Taking moments about A: T sin 45° × (1.2 – 0.4) = 2g × 0.6 T sin 45° = 14.7 fi T = 20.788... He obtains the following data: ff 26 of the 40 people were under 20 years old. a) sin (x + y) = sin x cos y + cos x sin y 7 3 3 1 n+e 4 × o =d4 × 10 10 9+ 7 9 10 + 70 = 40 4 10 b) cos (x – y) = cos x cos y + sin x sin y 7 3 1 3 =e 4 × o+c4 × m 10 10 3 7 + 3 3 70 + 3 10 = = 40 4 10 = 1 40 = sin ^ x + yh 9 10 + 70 18 10 - 2 70 = 37 1 40 d) sec (x – y) = = cos ^ x - yh 3 70 + 3 10 2 70 - 2 10 = 9 p p p p p p p Q5 cos 12 = cos b 4 - 6 l = cos 4 cos 6 + sin 4 sin 6 3+1 3 1 1 1 = e × 2 o+c × m= 2 2 2 2 2 Now rationalise the denominator... = 19 100 N (3 s.f.) b) s = 0.2, u = 0, v = v, a = 0.4 v2 = u2 + 2as v2 = 2 × 0.4 × 0.2 = 0.16 v = 0.4 ms–1 c) Resolving horizontally (Æ) for the truck: Fnet = ma W – F = 2000a 19 055.54... ff Resolving vertically, taking up as positive: s = –y, u = U sin q, a = –g, t=t 1 Using s = ut + 2 at2: g 1 –y = (U sin q × t) + (2 × –g × t2) = (U sin q)t – 2 t2 g 2 2 t – (U sin q)t – y = 0 Using the quadratic formula: t= = U sin q ! g U 2 sin 2 q - c 4 # 2 # - y m g U sin q ! U 2 sin 2 q + 2gy g ff Time should be positive, so t = U sin q + U 2 sin 2 q + 2gy g ff Now, resolving horizontally: s = x, u = U cos q, 1 Using s = ut + 2 at2: x= a = 0, t = U sin q + U 2 sin 2 q + 2gy g 2 2 U 2 sin q cos q + ^ U sin q + 2gy h U cos q g Tip: Since g and y are positive, U 2 sin 2 q + 2gy is greater than U 2 sin 2 q (i.e. U sin q). Mapping An operation that takes one number and transforms it into another. PROBLE M SOLVING Chapter 2 Algebra and Functions 11 2. a) Starting with x0 = 3.1, use the iterative formula xn+1 = 2 + ln xn to find the first 5 iterations, giving your answers to 4 decimal places. 65 = 58 + s fi s = 7 minutes 40 45 50 55 60 65 70 75 You might be asked to explain why a distribution is or isn't suitable so try to spot the conditions from the previous page. Q5 f(x) = 3x , x > –1 x+1 x-4 for {x : x > –3} x+3 a) Find f –1(x), stating the domain and range. Projectile A body, projected into the air, which moves only under the influence of gravity. = 9.8 fi t = 0.261... f –1f(x) = x = ff –1(x) Doing the function and then the inverse... It takes 5 seconds to land on the ground below, level with the foot of the cliff, a horizontal distance of 200 m away. Choosing Probability Distributions 1. Q6 a) The area required is the shaded area below: x = –3 y x -7 y=- 16 – 2x This is found by integrating the curve with respect to x between the limits x = –3 and x = 0. If so, find the inverse, and give its domain and range. Chapter 14: Kinematics 1. dx In other words, it tells you how y changes as x changes. dt dt ff Start off by differentiating the expression for y, with respect to x: dy y = 3e5x fi = 15e5x dx dy ff Write out the chain rule for , using the information available: dt dy dy dx = × dt dx dt dy ff Put in all the things you know to work out : dt dy = 15e5x × 2 = 30e5x dt dy dy ff Now find the value of at x = –1: = 30e–5 dt dt Tip: Sometimes it helps to write out the chain rule first so you know which rate of change is missing and needs to be worked out. If you spot that part of a product is the derivative of the other part of it (which is raised to a power), you can integrate it using this rule (which is just a special case of the 'reverse chain rule' on p.195): # (n + 1)f'(x)[f(x)]n dx = [f(x)]n + 1 + C This function is the derivative... ff The projectile is a particle — i.e. its weight acts from a single point, so its dimensions don't matter. Q1 422 Answers dy 3t 2 - 2t = dx 1 + ln t When t = 1: dy 3-2 = = 1, x = 0 and y = 3. Modelling Modelling questions involve using maths to represent real-life situations. dx dt dx dt When t = 3: dy 21 7 = 6 = 2 , x = 32 = 9 and y = 33 – 6(3) = 9. 2 (cos 5°) × ((0.958...) tan 5° + 0.02) fi U = 6.61 ms–1 (3 s.f.) Q15 The coordinates (3, 1) give you the components of the projectile's displacement — when it passes through this point, its horizontal displacement is 3 m and its vertical displacement is 1 m. Range The set of values output by a mapping or function. Use a suitable normal approximation to model the number of male piglets born. So if a function tells you to do a certain thing to x, the inverse of that function tells you how to get back to the start. There is a change of sign so 7 < a < 8. i.e. cos 45° = sin 45° fi R cos 45° = R sin 45° So, equating the left hand sides of 1 and 2 : P cos 60° + 10 cos 30° + 9 = P sin 60° + 10 sin 30° P cos 60° – P sin 60° = 10 sin 30° - 10 cos 30° – 9 10 sin 30° - 10 cos 30° - 9 fi P= cos 60° - sin 60° - 12.6602... x2 = 1.88066..., x3 = 1.86842..., x4 = 1.86373..., x5 = 1.86193..., so a = 1.86 (3 s.f.) e) Differentiating g(x) gives g'(x) = 3x² – 4. opp 15 8 1 c) tan q = = 8 so cot q = tan q = 15 . d) sec 5x = –1 fi cos 5x = –1 fi 5x = cos–1(–1) = p In this case you're looking for solutions for 5x — the interval you'll need to look in is 0 ≤ 5x ≤ 10p since 0 ≤ x ≤ 2p. The diagram below shows the sample being tested, which is modelled as a triangular prism. So it is 1 m north of O. y = x 2 – 2x Find out where the graphs meet by solving x2 – 2x = 2x: x2 – 2x = 2x fi x2 – 4x = 0 fi x(x – 4) = 0 fi x = 0 or x = 4 They meet at x = 0 and x = 4. Tip: Functions don't always use the letter 'f' — you'll see different letters used for functions over the next few pages. Find the components of the force that act parallel and perpendicular to the plane. 3 3 3 n 3 0 So the value of the root is x = 0.455 to 3 d.p. Q6 a) x2 – 9x – 20 = 0 fi x2 = 9x + 20 fi x = 9x + 20 So an iterative formula is xn+1 = 9xn + 20 b) x1 = 9x0 + 20 =]9 × 10g + 20 = 10.488..., x2 = 10.695..., x3 = 10.782..., x4 = 10.818..., x5 = 10.833..., x6 = 10.839... The last 4 iterations round to 10.8, so the value of the root is x = 10.8 to 3 s.f. c) x2 – 9x – 20 = 0 fi x2 - 5x – 4x – 20 = 0 x 2 - 4x fi 5x = x2 – 4x – 20 fi x = –4 5 2 x 4x So an iterative formula is xn+1 = n 5 n – 4 x 2 - 4x 1 2 -]4 × 1g d) x1 = 0 5 0 - 4 = – 4 = –4.6, 5 x2 = 3.912, x3 = –4.0688..., x4 = 2.5661..., x5 = –4.7358..., x6 = 4.2744..., x7 = –3.7653..., x8 = 1.8479... e) The iterations seem to be bouncing up and down without converging to any particular root. So P(X > 78) = 1 – P(X ≤ 78) = 1 – 0.022750... Q2 Hint: Remember to set your calculator to radians before doing this question. Q2 a) On the same axes, sketch the graphs of y = 2x3 – 7x and y = x2. dx ff First rewrite the equation to make the chain rule easier to use: y = 2 cos2 x + sin (2x) fi y = 2(cos x)2 + sin (2x) ff Then differentiate the parts separately. Carry out a hypothesis test at the 5% significance level to test the claim that maths and science test results are positively correlated. + 13 = 7.6839... y = f(x) + a y 1 a x 0 180° y = f(x) 360° –1 a y y = af(x) ff The graph of af(x) is f(x) y = f(x) 0 reflected in the x-axis. It's actually given as # a b So the trapezium rule says: # a y dx , but this is the same since y = f(x). = 172 N (3 s.f.) b) F T 20° R 12g Resolving horizontally: R = T cos 20° = (171.919...) cos 20° = 161.551... Integration Process for finding the equation of a function, given its derivative — the opposite of differentiation. b) (1 + 6x)4 = 1 + 24x + 216x2 + ... You have to choose the way that makes best use of the information in the question. fi t = 2.857... Here, 'right' is positive, so the i-component of the force is negative, because it acts to the left. Q13 Differentiate y = sec (x 2 + 5). ff OB is a2 + b2 . pn – 1 pn remainder 1 ff In fact, dividing (P + 1) by any prime number gives a remainder of 1. The Trapezium Rule It's not always possible to integrate a function using the methods you learn at A-level, and some functions can't be integrated at all. This comes from the Remainder Theorem, which you met in Year 1. ff If the tangent to f(x) is horizontal at the point xn (i.e. if xn is a stationary Tip: You know from the definition of a stationary point that f'(x) = 0 at that point. f(6.535) = 6.5352 – (5 × 6.535) – 10 = 0.031... Q8 A particle is held in equilibrium on a smooth plane by a light, inextensible string. Find the horizontal and vertical components of its initial velocity. Answers 463 b) The horizontal component of velocity remains constant at 40 ms–1. The trapezium rule between a and b can be approximated by the green trapezium shown. Then P(L < 200) = 0.0228 (3 s.f.) b) P(L > 380) = 0.0548 (3 s.f.) c) Assuming that the lifetimes of the batteries are independent, the probability that both last at least 380 hours = 0.0548 × 0.0548 = 0.00300 (3 s.f.). a) Explain why the normal distribution would be a suitable approximation for the number of people in the sample (X) carrying the disease. If P > 47.04 N, then the particle will move and F will remain constant at 47.04 N. cos x dx can be approximated as p 6 2 # 250 Chapter 9 Numerical Methods b) Without further calculation, state whether this approximation is an under- or over-estimation. Location of Roots Exercise 1.1 — Locating roots by changes of sign Q1 f(2) = 23 – 5 × 2 + 1 = –1 f(3) = 33 – 5 × 3 + 1 = 13 There is a sign change (and the function is continuous in this interval) so there is a root in this interval. b) 1.95 = (2 – 0.05)6 This is the same as (2 – 5x)6 with x = 0.01. k = 38.7° (3 s.f.) Q4 a) Let the distance from A to the COM be x. The container has a hole in it from which water flows out at a rate of 20 cm3s–1. A third force of magnitude P N acts on the particle at 60° above the positive horizontal. Or using the algebraic method, solve: (x + 2)2 = (2x)2 fi x2 + 4x + 4 = 4x2 fi 3x2 – 4x – 4 = 0 fi (3x + 2)(x – 2) = 0 2 So x = 2 and x = - 3 . Now you're going to find out how to do all of that in three dimensions. It's enough to show that the left-hand side of the original equation is greater than the right-hand side for one of the values of x (x1 or x2), and less than the right-hand side for the other. A-Level Year 2 Mathematics Exam Board: AQA Y2 The Complete Course for AQA P G C Numerical Methods CGP 1 b-a y dx ≈ 2 h[y0 + 2(y1 + y2 + ... dx This is another function of a function, so more chain rule: y = sec u and u = 2x2 dy du = sec u tan u = sec (2x2) tan (2x2) and = 4x du dx dy dy du So = × = 4x sec (2x2) tan (2x2) dx du dx Chapter 7 Differentiation 163 c) Find dy if y = ex cot x. dy ^sin x × - sin xh - ^cos x × cos xh - sin 2 x - cos 2 x = = dx sin 2 x ^sin xh2 ff Simplify using a trig identity: sin2 x + cos2 x º 1 seems fitting. fi a = 61.109...° Now resolving horizontally, taking right as positive: s = s, u = 7.5 cos (61.109...°) = 3.623..., a = 0, t = 1.2 1 s = ut + 2 at2 fi s = 3.623... ff Step 4: Rearrange to make dy the subject: dx dy = ex + 3x2 dx dy e x + 3x 2 fi = 2+ dx 3y 2y (3y2 + 2y) Example 2 a) Use implicit differentiation to find dy for 2x2y + y3 = 6x2 – 15. dy b) = 5(25x ln 2) dx dy When x = 1, = 5(32 ln 2) = 160 ln 2 dx Q4 a) Let u = x3, then y = pu, and so dy 3 d u d 3 = (p) × (x) = 3x2(p x ln p) dx du dx b) When x = 2, y = p8 = 6561. Find the functions: a) fg(x) b) gf(x) c) f2(x) 2 for {x : x > 1} and g(x) = x + 4 for {x : x d R }. + yn - 1h + yn @ 2 # x h Tip: This just says 'Add the first and last heights (y0 + yn) and add this to twice all the other heights added up — h then multiply by 2 .' Chapter 9 Numerical Methods 245 Tip: The trapezium rule is in the formula booklet — always look it up. 1 c) Show that one solution of the equation x = x – 2 lies in the interval 2.4 < x < 2.5. Q1 Hint: You might find it useful to rearrange the equation into the form f(x) = 0. Then you know the following probabilities: P(S) = 0.62, P(C) = 0.539 and P(C'|S) = 0.29, and you want to find P(S|C). So the statement must be true. ff 68% of the area under the graph of X is between x = m + s and x = m – s. The area under a normal curve The area under a normal curve shows probabilities. m ≤ x ≤ 3.886... Particular solution A solution to a differential equation where known values have been used to find the constant term. dy dy a) Find in terms of t. ff You're told that the child is a girl, so look at the Girls row of the table — there are 12 girls in total. 1 When t = 2, v = [3 (2)3 + 2(2)2 – 12(2)]i + 14j – 40 = 3 i + 14j ms–1 Q3 a) It's easier to work with the vectors in component 6 form whilst integrating, so use a = 3e3t i + j t 6 dt n j v = # a dt = a # 3e3t dt k i + d # t 1 = a # 3e3t dt k i + a # 6t - 2 dt k j 1 2 = e i + 12t j + C = e3t i + 12 t j + C When t = 0, v = (4i + 6j), so: 4i + 6j = e0 i + 12 0 j + C 4i + 6j = i + C fi C = 3i + 6j 3t So v = (e3t + 3)i + (6 + 12 t)j = e r= b) # v dt = 9 # ^e = 9 # ^e 3t 3t e 3t + 3 o ms–1 6 + 12 t + 3h dtC i + 9 # ^6 + 12 t h dtC j + 3h dtC i + 9 # ^6 + 12t 2 h dtC j 3 = b e + 3t l i + ^6t + 8t 2 hj + D 3 3t 1 When t = 0, r = (2i – 9j), so: e0 2i – 9j = (3 + 0)i + (0 + 0)j + D 5 1 2i – 9j = 3 i + D fi D = 3 i – 9j 3 e 3t 5 So r = (3 + 3t + 3)i + (6t + 8t 2 – 9)j 1 3t + + 5 e 3t 3 pm = f3 6t + 8 t3 - 9 Q4 a) v = ro = [(cos t)i – (sin t)j] ms–1 The object is moving east when the j component is 0, and the i component is positive. + 0.9609... Using the fact that the total area under P(X > x) = 1 – P(X ≤ x) the curve is 1, we get this definition. Remember that this result needs you to have a multiple of n + 1 (not n) — you can check this by differentiating the right-hand side using the chain rule. — call this equation 2 ff Substitute equation 2 into equation 1 : 24.989... Q8 When y = 0: 1 + x2 = 0 + 4x fi x2 – 4x + 1 = 0 Complete the square to solve... cos x ff Although cot x º sin x , you don't need the quotient rule as you know that cot x differentiates to give –cosec2 x. y = 2x + 1 Rearrange to make x the subject. ff Use trigonometry to find its direction, q 7.5 tan q = 32.99... Harriet says, "The probability of a particular card being chosen twice is 0.01." Q3 In a class of primary school children, 56% walked to school one day. Q2 Differentiate the following functions with respect to x: 2x a) y = sin4 (x2) b) y = e sin c) y = tan2 (3x) + sin x d) y = e2 cos (2x) + cos2 (2x) Chapter 7 Differentiation 153 5. Q6 Resolving vertically, taking up as positive: s = 11 – 4 = 7, u = 20 sin 45°, a = –9.8, t = t 1 s = ut + 2 at2 7 = (20 sin 45°)t – 4.9t2 4.9t 2 – (20 sin 45°)t + 7 = 0 Using the quadratic formula: 20 sin 45° ! ^- 20 sin 45°h2 - ^4 # 4.9 # 7h t= 9.8 14.142... So the equation of the tangent is y = (102.4 ln 4)x + (1024 – 2560 ln 4) fi y = 142x – 2520 to 3 s.f. dy Q6 a) = –3(2–3x ln 2) dx dy 3 1 b) When x = 2, y = b = 2–6 = 64 and = - 64 ln 2 dx 1 3 At b 2, 64 l, the gradient of the tangent is - 64 ln 2. Q6 –2 R 0.35 ms T F 10° 5000 N A rock weighing 5000 N is being pulled up a rough inclined plane which is at an angle of 10° to the horizontal, as shown. f(x) x f(x) Not Functions Some values of x are mapped to two different values of f(x). B 1m 3g A uniform rod, AB, of mass 3 kg and length 4 m, is freely hinged to a vertical wall at A. Example 3 A 6 m long uniform beam AB of weight 40 N is supported at A by a vertical reaction R. sin a = 0.4217... Chapter 16 Moments 353 ff You also know that there is no resultant force acting on the rod. If X ~ N(m, s2), then X-m s = Z, where Z ~ N(0, 1) Examples a) If X ~ N(5, 16), find P(X < 7) by transforming X to the standard normal variable Z. 60 N 12g ff Next, resolve forces vertically (≠) to find the normal reaction, R: Fnet = ma R – 12g = 12 × 0 R = 12g = 117.6 N ff The block is moving, so F takes its maximum possible value: F = mR 42 = m × 117.6 m = 0.36 (2 d.p.) Example 3 A small body of weight 20 N is released from rest on a rough plane angled at 15° to the horizontal. Tip: In theory there's nothing wrong here with having a common denominator of all the denominators multiplied together (i.e. xy3(x + 3)2). Q2 Evaluate the following definite integrals, giving exact answers: p 4 # sin 2 x dx b) # cos 2 2x dx a) p 0 0 p #p 2 sin 2 2x dx # sin 2x cos 2x dx d) c) p 0 4 p 2 p 4 # cos 2x sin 2x dx f) #p sin 2 x - cos 2 x dx e) 0 4 Q3 Find the following integrals: # cot 2 x - 4 dx b) # tan 2 x dx a) # 3 cot 2 x dx d) # tan 2 4x dx c) Q4 Find the exact value of # 0 p 4 tan 2 x + cos 2 x - sin 2 x dx. Find an expression for the boat's position vector, r, at time t. Small Angle Approximations 3. The volunteer chooses one, replaces it and chooses another. Learning Objectives: ƒ Understand how to represent vectors in three dimensions, using both unit vectors and column vectors. The painter places paint pots, of mass 2.5 kg, 0.2 m from each end of the plank. Recurrence relation A function that tells you how to work out a term in a sequence from the previous term. Given that cos q = 5 , find the coefficient of friction between the box and the plane. ff When the object starts to move, or is on the point of moving (i.e. friction is limiting), the frictional force takes its maximum value: Fmax = mR Example 1 A horizontal force, P, acts on a particle of mass 12 kg, which is at rest on a rough horizontal plane. a) Use the substitution u = cos x to find 2r #r -12 sin x cos3 x dx. She thinks kidney function and weight are correlated. Points of Inflection 135 2. x2 + y2 + z2 = 22 + 52 + 42 = 45 = 3 5 b) Find the unit vector in the direction of OQ . P(X > 75) ≈ P(Y > 75) = 0.000143 (3 s.f.) If you used a continuity correction in this question, you'd work out P(Y < 75.5) = 0.000108 (3 s.f.) instead. z Tip: You can find the value of a directly using the inverse normal function on some calculators, without having to transform to Z. 4 3x – 4 < 0 for x < 3 (shaded) So there are two equations to solve: 5 1 3x – 4 = 1 fi x = 3 4 (this is valid as it's in the range x ≥ 3) 2 (shaded) –(3x – 4) = 1 fi x = 1 4 (this is also valid as it's in the range x < 3) 5 So the two solutions are x = 3 and x = 1. ff Then take moments about M (moments clockwise = moments anticlockwise): 45d = 30 × 4 d = 120 ÷ 45 = 2.67 m (3 s.f.) Example 2 The diagram below shows a light rod AB, of length 10 m. a) The probability that a randomly selected egg has a mass of at least 60 – m grams is 0.9525. dt dt dV So the overall rate of change of V is = kV – 20, dt for some k > 0. – 0.066807... Then H ~ N(175, s2). 388 Answers p c) cot 2x = 1 fi tan 2x = 1 fi 2x = tan–1(1) = 4 Don't forget to double the interval for the next bit — you're looking for solutions for 2x instead of x. dx dx = x ln x – x + C You can use integration by parts on definite integrals too. The Product Moment Correlation Coefficient291 2. dx Give your answer in the form y = f(x). Common ratio = second term = 15 = 0.6 25 first term d) a = 25 and r = 0.6, so sum of first 10 terms is: a (1 - r10) 25 (1 - 0.610) = S10 = = 62.12 to 2 d.p. (1 - r) (1 - 0.6) 2 2 Q7 a) 1 + x + x = 3 fi x + x – 2 = 0 fi (x – 1)(x + 2) = 0 fi x = 1 or x = –2. ff Be able to sketch the graph of y = |ax + b|. Its radius decreases by 1.6 cm h–1. Differentiation with Parametric Equations 169 10. a) Find the normal reaction force, R, between the plane and rock. Choosing Probability Distributions Exercise 3.1 — Choosing probability distributions Q1 a) Most of the data values are in the middle and the number of values tails off towards the ends. Tip: Here you're just putting y = 5 – x into the original equation to find the values of x at the stationary points. Factorise top and bottom again. Example 1 Find the following: a) # 2 sec x tan x dx Take the constant outside the integral. It means you can use it in all sorts of real-life situations. In this example, the rectangles are above curve, so they calculate an upper bound. Find the body's position vector when it is moving parallel to i. . For example, 2 the vector e - o is a 3 translation of 2 to the right and 3 down. The coefficient of friction between each particle and the slope is 0.6. 200 N T A T B 6g N 8g N 20° Find the acceleration of A. Mutually exclusive Events are mutually exclusive (or just 'exclusive') if they have no outcomes in common, and so can't happen at the same time. Example 2 If the random variable X ~ N(m, 42) and P(X > 19.84) = 0.025, find m. For example, if the equation is for population you might be asked to find the population after a certain number of years, or the number of years it takes to reach a certain population. But you need to think of the situation the 'other way round' — with L conditional on S. For simple functions (like the one in the example above), it's easy to work out what the inverse is just by looking at it. dx 3 3 p p y = mx + c fi 6 = – 2 + c fi c = 6 + 2 3 p So the equation of the normal is y = –x + 6 + 2 . Connected Rates of Change dy A derivative like is also called a 'rate of change' — it's the rate of change dx in y with respect to x. -1 e) r = 2 = - 1 , r = - 1 = 1 < 1, 2 2 2 1 so the sequence converges. x8. Q3 Hint: When the rod is on the point of tilting clockwise about D, the tension in the wire at C will be zero. Interpret this value in context and say whether this result provides sufficient evidence to back the doctor's claim. At the same time, particle B is projected at an angle q above the horizontal, from a point 15 m above ground level, directly below the point of projection of particle A. The centre of mass of a uniform rectangular lamina is at the symmetrical centre of the rectangle. The rod is held horizontally in limiting equilibrium by a light wire attached at B, at an angle of 20° to the rod, as shown. ff Assume that there are a finite number of primes (say n), and label them p1 = 2, p2 = 3, p3 = 5, ... 8 fi t = 0.559... Learning Objective: ff Be able to form differential equations from situations involving connected rates of change. You're Reading a Free Preview Pages 136 to 140 are not shown in this preview. b) Expressing y in terms of cos (2x) and differentiating the result. Find the equations of the tangents to the curve at these points, 14 and hence show that they intersect at a - 25 , - 1 k. So f –1(x) has domain x ≥ 0 and range 0° ≤ f –1(x) < 90°. x3 + 5x2 − 18x − 18 º (Ax2 + Bx + 6)(x − 3) ff Equating the coefficients of x and x gives A = 1 and B − 3A = 5, so B = 8. D PROBLE M SOLVING Chapter 16 Moments 357 2. Resolving vertically for particle B, taking up as positive: s = s, u = 2U sin 60° = 3 U, a = –9.8, t = 2 1 s = ut + 2 at2 1 s = 2 3U + (2 × –9.8 × 22) = 2 3U – 19.6 So, when the particles collide (at a height of 25.4 m above the ground), B has travelled (2 3 U – 19.6) m vertically upwards from a height of 15 m above the ground. Look for the bit that would have been u in the chain rule — here it's 2x2. x4 and x5 both round to 7.7 to 1 d.p., so a = 7.7 (1 d.p.) d) S ketch a diagram to show the convergence of the sequence for x1, x2 and x3. Estimate the standard deviation of the speeds. b) 0.994 is very close to 1, so it suggests strong positive correlation in the sample, i.e. it suggests that longer leaves tend to also be wider. In fact, it's the opposite of a picnic. range 1 –3 x domain Since 1 is always > 0 for the domain x > –3, x+3 1 2 2 a + k + 1 must be > 1. ff Write du and f'(u): dx du = cos x dx and f'(u) = esin x ff Use the formula to write down the result. dy 3s 2 - s - 2s ln s 3s - 1 - 2 ln s b) = = . + 13 = 7.6739... dy b) By finding , evaluate the gradient at each of these points. Peg A fixed support which a body can hang from or rest on. # a (n + 1) (ax + b) n dx = (ax + b)n + 1 + c So a(n + 1) # (ax + b) n dx = (ax + b)n + 1 + c Dividing by a(n + 1) gives the general expression: # (ax + b) 180 Chapter 8 Integration n dx = 1 (ax + b)n + 1 + C a (n + 1) for n ≠ –1, a ≠ 0 Example 2 # ^ 3 - 4x h 2 Find dx , using the general expression for # ^ax + bh n dx . a) Find the acceleration of the puck. 10 ms–1 v = 10 2 + (- 4.9) 2 q = 11.1 ms–1 (3 s.f.) 4.9 ms–1 v 4.9 tan q = 10 Tip: You may see similar questions asked as 'find the horizontal range of the stone.' fi q = 26.1° (3 s.f.) below the horizontal b) F ind the stone's horizontal displacement when it lands on the ground. ff If all your points lie exactly on a straight line with a positive gradient (perfect positive correlation), r = +1. ff Eventually, the frictional force will reach its maximum possible value. Reversing the process of differentiation to get integration we have: d 4x – 1 (e) = 4e4x – 1 fi dx # 4e 4x - 1 dx = e4x – 1 + c fi 4 # e 4x - 1 dx = e4x – 1 + c Take out the factor of 1 fi # e 4x - 1 dx = 4 e4x – 1 + C 4 and divide by it. Then input the probability, mean and standard deviation. ff Once event D has happened, there are 5 possible outcomes corresponding to event C happening and 12 possible remaining outcomes in total. The string is inclined at 45° to the horizontal. Q10 Frazer's series is the natural numbers up to 31. ff Draw a diagram to show all the forces acting on the box: R Q Tip: F always acts parallel to the surface, no matter what angle other forces act at. If a mapping takes a number from the domain to more than one number in the range (or if it isn't mapped to any number in the range), it's not a function. 1 2 n 2 2 2 = =1 1 1 4 11- 4 4 n 1-d ^ 2h 2 2 2 2 2 = ==3 3 3 4 2 2 2× 3 -2 6 Rationalising the denominator: = = 3 3× 3 2 6 So the tangent has a gradient of - 3 and passes 1 p , n. The Product Moment Correlation Coefficient Exercise 1.1 — PMCC hypothesis testing Q1 a) This is a one-tailed test, so the hypotheses are H0: r = 0 and H1: r > 0. It reaches a maximum height of 2.8 m above the ground. ff And if a < 0, the graph is also reflected in the y-axis. Tip: You could also use the substitution u = 2 – x2 to solve this — the differentiation is easier but you get a tougher integral with fractional powers. MODELL ING This section shows you how to choose a suitable probability distribution for a given context and explain why it is suitable for that context. Multiply the total number of boxes by the probability calculated in a): 2138 × P(X < 780) = 2138 × 0.960 ≈ 2052 boxes c) Find the probability that a randomly selected box of cereal contains between 780 g and 790 g of cereal. Chapter 2 Algebra and Functions 13 Q8 State whether or not each of the mapping diagrams below shows a function, and if not, explain why. In each case, find the horizontal and vertical components of the projectile's velocity. Resolving parallel to the plane: T cos 33° T cos 33° = mg sin 12° fi mg = sin 12° Resolving perpendicular to the plane: 15 + T sin 33° = mg cos 12° 2 1 x° x° 10 N Resolving parallel to the plane: 10 sin x = 8 8 x = sin–1 a 10 k = 53.130... So you only need to consider the vertical component. Calculating probabilities ff Before calculating any probabilities, draw a sketch to show the probability you want to find. ff Again, start by transforming the probability you're given for X into a probability for Z. b) Use the iterative formula xn+1 = 2 2xn - 6 starting with x0 = 1 to find the values of x1, x2, x3 and x4, giving your answers to 4 d.p. c) Verify that the value for x4 is a correct approximation to 4 d.p. for the root of the equation 2x - 1 = 4 x . Point of inflection A stationary point on a graph where the gradient doesn't change sign on either side of the point. (2t + 3) 2 + (1 + 6) 2 + (- t - 4) 2 (2t + 3) 2 + (1 + 6) 2 + (- t - 4) 2 = 3 11 fi 4t2 + 12t + 9 + 49 + t2 + 8t + 16 = 99 fi 5t2 + 20t + 74 = 99 fi 5t2 + 20t – 25 = 0 fi t2 + 4t – 5 = 0 fi (t + 5)(t – 1) = 0 fi t = –5 or t = 1 The angle between two vectors Tip: The cosine rule is usually given as: a2 = b2 + c2 – 2bccosA. Top reviews Most recent Top reviews There are 0 reviews and 0 ratings from the United States AQA P CG P G C P G C ...but don't worry — CGP has come to the rescue with this amazing Student Book! • Every single Year 2 Maths topic is in here... Pure, Mechanics and Stats — sorted! • The clearest study notes anywhere... So the possible values are 0 ≤ |5x + 4| ≤ 9 2 –3 Q5 x 3 y 6. Two identical objects are connected by a rigid rod. Find: a) the distance of the centre of mass of the rod from A. dn dn dD So = × = 104l2lt ln 2 per day. 2x + 3 2x - 3 ff Integrate each term separately: 1 1 = 2 × 2 ln|2x + 3| + 4 × 2 ln|2x – 3| + C = ln|2x + 3| + 2 ln|2x – 3| + C ff x > 2 so 2x + 3 > 7 and 2x – 3 > 1 so remove the modulus signs: = ln(2x + 3) + 2 ln(2x – 3) + C = ln(2x + 3)(2x – 3)2 + C Tip: It's a good idea to tidy up your answers using the log laws. The particles are positioned as shown in the diagram above. You do this by substituting in values for x to make bits disappear, and by equating coefficients. Standard normal variable, Z A random variable that follows a normal distribution with mean 0 and variance 1. The rock is held in equilibrium by a horizontal force of magnitude 10 N directed towards the plane and the tension in a string parallel to the plane, as shown in the diagram. M is a point on the line CD, where C has coordinates (–1, 3, –5), J N KK 4 OO M has coordinates (1, 1, –2) and CD = KKK- 4OOO . Tip: The inverse normal function on your calculator might be labelled 'InvNorm' or 'InvN'. It isn't always quite so straightforward — sometimes you need to get rid of some x's

by rearranging the equation to get x in terms of u. x b) e 2 #e x 2 dx = #e 1 2x x d x = 2e 2 + C 1 x If you differentiated this one using the chain rule, you'd get 2 e 2 , 1 so you need to multiply by 2 (divide by 2) to integrate. In Q10-12, it's not actually strictly necessary to rearrange to f(x) = 0. Tip: This version of the product rule is slightly different from the one on page 154 — it's got v(y) instead of v(x). Non-Uniform Acceleration in 2 Dimensions Chapter 16 — Moments Statistics 1. Newton's Laws of Motion 1. b) Using the alternative iterative formula xn + 1 = 5xn + 10 with starting value x0 = 6, find a root to the equation f(x) = 0 to 3 significant figures. Tip: To evaluate integrals like this you have to integrate with respect to u dy (because f'(u) is). Resolving horizontally, taking right as positive: s = s, u = 6, a = 0, t = 2.081... The coach increases the number of training sessions per week, and a random sample of 20 times run since the increase has a mean of 11.2 minutes. There is insufficient evidence at the 1% level to reject H0. When an object of mass 3 kg is placed at B, the rod is on the point of tilting clockwise about D. d2 y To find the nature of the stationary point, calculate dx 2 d2 y f(x) = 9e3x 2 = 3f'(x)e dx d2 y 1 When x = 3 ln 2, = 9eln 2 = 9 × 2 = 18 dx 2 d2 y is positive, so it's a minimum point. one-tailed two-tailed n 4 5 10% 20% 5% 10% 2.5% 5% 1% 2% 0.5% 1% 0.5000 0.6870 0.9000 0.8054 0.9500 0.8783 0.9343 0.9900 0.9587 0.6 7 8 9 10 0.6084 0.5509 0.5067 0.4716 0.4428 0.7293 0.6694 0.6215 0.5822 0.5494 0.8114 0.7545 0.7067 0.6664 0.6319 0.8822 0.8329 0.7887 0.7498 0.7155 0.9172 0.8745 0.8343 0.7977 0.7646 11 12 13 14 15 0.4187 0.3981 0.3802 0.3646 0.3507 0.5214 0.4973 0.4762 0.4575 0.4409 0.6021 0.5760 0.5529 0.5324 0.5140 0.6851 0.6581 0.6339 0.6120 0.5923 0.7348 0.7079 0.6835 0.6614 0.6411 16 17 18 19 20 0.3383 0.3271 0.3170 0.3077 0.2992 0.4259 0.4124 0.4000 0.3887 0.3783 0.4973 0.4821 0.4683 0.4555 0.4438 0.5742 0.5577 0.5425 0.5285 0.5155 0.6226 0.6055 0.5897 0.5751 0.5614 Statistical Tables 369 Answers Chapter 1: Proof 1. Tip: f -1f(x) is a composite function. 2x + 1 Q5 f defines a function f : x Æ f+ for the domain x > 0.01. Tip: The list of results x1, x2, x3... The plank is kept horizontal and in equilibrium by a light wire fixed at its midpoint, at an angle of 30° above the plank, and a light wire fixed at C, 2 m along the plank from A, at an angle of 30° to the wall, as shown. Given that the ladder is inclined at an angle of 70° to the smooth horizontal ground, find the range of possible values for x so that the ladder remains in equilibrium. 35 40 45 speed, mph 50 55 s Q2 A tennis player successfully completes a first-serve 65% of the time. Resolving vertically, taking up as positive: s = s1, u = 24 sin 70°, a = -9.8, t = 3 1 s = ut + at2 1 s1 = (24 sin 70° × 3) + (2 × -9.8 × 32) = 23.557... dy dy du = × = 2 cos x) × (-sin x) dx du dx = -2 sin x cos x b) Using the double angle formula: 1 cos (2x) º 2 cos2 x - 1 fi cos2 x = 2 (cos (2x) + 1) dy 1 = 2 × 2 × (-sin (2x)) = -sin (2x) As the original function was in terms of cos x rather than cos (2x), it would have to be bottomed to rearrange this. E.g. / ˆ2n + 3b 15 = 1 is the sum of the first 21 terms of the series, so find the value of 4 6 8 10 Stick-figure diagrams you're looking for. Tip: To find the sum of a series, then use the method. Differentiating with respect to x using the product rule: This means the volume of water which is being poured into a container is directly proportional to the volume of water (V) in the container. MODELL ING Use a tree diagram to find the probability that a person from the group owns a smartphone, given that their current contract costs over £25 a month. It works for equations that contain a mixture of x and y terms, such as xy2. R F 25° 4N mg A horizontal wire is threaded through a bead of mass m kg. 19.84 - m × 0.025 P(X > 19.84) = P a Z > b ff Draw a sketch to show the information. Write down the probability for Y that would approximate: a) P(X = 50) b) P(X < 300) c) P(X ≥ 99) Q3 An unfair coin is to be tossed 1000 times and the number of heads (X) recorded. c) The particle follows a symmetric curved path, so when it hits the ground, the vertical component of its velocity will have the same magnitude as when it was projected, but in the opposite direction. 1 # + 5 1 = 1 + 6 (-5x) + 6 2 6 (25x2) + ... Differential Equations Differential equations involve differentiation so we have. Now you have: 4 = 3x3 − 3x2 + 10x + 5 º (Ax3 + Bx2 + Cx)(x − 2) + 5 ff For the remaining terms, equate the coefficients on both sides. Q5 2 sin(x + 30°) º 2 sin x cos 30° + 2 cos x sin 30° .sin 1 º 2 sin x cos x 2 1 2 º 3 sin x + cos x p tan 3 - tan a 3 ... tan b 3 - tan 3 b a i tan a tan 3 b + tan a B 1 Q7 tan (A + B) º = A 1 tan A tan B + tan B 1 + tan A tan B 3 tan A + tan B 3 1 fi B 3 4 tan A - 4 - 32 tan B 3 1 3 fi tan B + 32 tan B = 4 - 8 = - 8 35 1 1 32 tan A fi 32 tan B - 8 fi tan B = - 8 × 35 = - 35 Q8 Starting with the addition formulas for sin: sin (x + y) º sin x cos y + cos x sin y sin (x − y) º sin x cos y − cos x sin y Then sin (x + y) + sin (x − y) º sin x cos y + cos x sin y + sin x cos y − cos x sin y º 2 sin x cos y Substitute A = x + y and B = x − y Then A + B = 2x and A − B = 2y, A-B A+B so x = a 2 k and y = a 2 k. + yn - 1h + This may seem like a lot of information, but it's simple if you follow this step by step method: To approximate the integral # a b f (x) dx - ff Split the interval up into a number of equal sized strips, n. b) Given that D is decreasing at a rate of 2 cm s–1 when D = 40 cm, find k. dt dx Q2 The rate of change of x with respect to t is . (x - 212) - 4 fi x = - 2 3 fi x = 2 3 fi x = 2 So x = 2 + 3 and b = 2 - 3. Pressing '=' repeatedly will give you the following terms. Tip: Taking moments about A means you don't need to worry about the frictional and reaction forces acting there. 150 ff It's not always the part inside the brackets. Tip: Z is symmetrical about the mean 0 so P(Z < -z) = P(Z > z) (see page 273). Tip: It's usually easiest to list the variables you know from the question, and then use the one you want to find, and pick the suvat equation that has them in it. dt dt dy dx By using the chain rule, = × dx dt dt dy dt dx So: tan 1 = 3 − 1 − 3 t − 4t3 + 3t2 dx so 4t3 = 3t2 – 2t, dt dt dy 3t 2 - 2t 3t - 2 so = dx 4t3 4t 2 Q1 a) Answers 421 dy dy dx 2t - 4 d) = sin t, = 4 – 2t, so sin t dx dt dt dy dt 2e2t, so = t dt dx dt dy 2 b) When t = 1, = e. p-value The probability under H0 of the test statistic taking a value at least as extreme as the observed value in a hypothesis test. OQ = 2i + 5j + 4k z Q(2, 5, 4) y Plug the coordinates, which are the i, j and k coefficients in OQ , into the formula. The method above produces two different types of diagrams — cobweb diagrams and staircase diagrams. Give your answer to 2 d.p. Q4 A spherical satellite, radius r m, expands as it enters the atmosphere. So replace a with 2k for some integer k: 2b2 = (2k)2 = 4k2 fi b2 = 2k2 ff Like before, this tells you that b must be even (since b2 is even). Again, if you find it easier you can use the triangle method here. (9ii a) The power series given by 2% each year, so the price after 1 year is 102% of £90 000. 90 000 × 1.02 = 91 800 So after 1 year it is £ 81 600. Applying differentiation to real-life problems ff Some questions involve taking real-life problems and using differential equations to model them. 1 2 ff Find the total area under the curve is 1 by taking the same, so a wider curve needs to have a lower height. The B means the union of two sets (everything that is in either set — i.e. the outcomes corresponding to either event A or B or both happening), and Ç means the intersection (only the things that are in both — i.e. the outcomes corresponding to A and B both happening). Q3 A curve is defined implicitly by x2y + y3 = x + 4y. Many variables can be modelled by normal distributions, which can be very useful in real-life situations. # 0 p 2 px sin ` 2 jb 0 8- 8x cos ` 2 jB0 - # 0 p 2 - 8 cos ` x p x p = - 8 8 x cos ` 2 jB02 + 16 8sin ` 4 j - 0 os (0) B p 1 + 1 = - 8 2 + 16 ; E 2 2 4p + 16 = 2 2 216 Chapter 8 Integration x 2 j # 2 dx = 2 ff 2 2 0 sin Exercise 9.1 Q1 Use integration by parts to find: # xex dx b) # x e-x dx c) # x - 3 dx d) # x(ex + 1) dx a) Q2 Use integration by parts to find: # 0 x sin x dx) # 2x cos x dx) # x ln x dx) # - 1 # 3x cosh 2 x 1 dx) # p2 2x(1 + sin x) dx 1 2 p Q3 Use integration by parts to find the exact values of the following: p 1 2 # 0 12x e2x dx b) # 1 x12 ln x dx # 0 3 18x sin 3x dx 4 Q6 Find # ' Q7 Find # (x + 1)(x + 1) Q8 Find # ln x dx Q9 Find # ln x dx. Tip: Don't forget to include the constant of integration. Lots of things can be modelled as moving only under the influence of gravity, the only acceleration the projectile will experience will be acceleration due to gravity (g = 9.8 ms–2). Also 4 + 2 ≥ 0 for x ≥ - 2 1 4x + 2 < 0 for x < - 2 (shaded) So there are two equations to solve: 1 4x + 2 = 10 fi x = 2 1 (this is valid as it's in the range x ≥ - 2) 5 2 (shaded) –(4x + 2) = 10 fi x = –3 1 (this is also valid as it's in the range x < - 2) So the two solutions are x = 2 and x = –3. ff For any inverse f −1(x): ff Be able to draw and interpret graphs of functions and their inverses. ff The mean is in the middle, so m ≈ 58 minutes. dy 3 - 3t 2 Q6 a) = dx 1 - dy When t = 2, = –3, x = 2 and y = –2. Find the distance of the particle from its point of projection 1.8 seconds after it is projected. Modulus graphs y = |f(x)| ff For the graph of y = |f(x)|, any negative values of f(x) are made positive by reflecting them in the x-axis. This means you have the following results: P(B|A) = P(B|A') = P(B). ff Next, turn the question into a probability for X. In general terms: |f(x)| = f(x) when f(x) ≥ 0 f(x) = 0 Tip: The modulus is sometimes called the absolute value. = -10(-1 + 1)[-1 ÷ (1 + 1) ÷ ...] Tip: You'll need to use the identities from that fits into the equation: 4f = 4 - 4 .... Tip: Scatter graph B should now show the points as the first dart. Tip: Scatter graph B should now show the points as the first dart. Tip: As x becomes large, the curve gets close to the x-axis and time spent playing outside on the y-axis — then the clusters could be children and adults. ff When you have a trig function multiplied by itself, like cos2 x, it's often Chapter 7 Differentiation easiest to turn the trig function into a single value. F = 0.98 sin 20° = 0.335 N (to 3 s.f.) ff Resolving perpendicular to the plane ( ): Tip: The force F acts parallel to the plane already, so you don't need to resolve it into components. Chapter 2: Algebra and Functions 1. Q9 B PROBLE M SOLVING C A 368 Chapter 16 Moments 40° 12 N A uniform beam AB of weight 12 N and length 5 m rests on a smooth surface at A and a rough peg at C, 4 m from A. Tip: Remember, continuous variables can take any value within a certain range. From your calculator, P(Z ≤ z) = 0.95 for z = 1.645. The magnitude of the normal reaction force exerted by the plane on the particle is 25 N. ff The unit vector in the direction of the z-axis is k, so three-dimensional vectors can be written like this: f KJ K ONO xi + yj + zk or KKK y OO KzO L P Three-dimensional vectors are used to describe things in three-dimensional space, e.g. an aeroplane moving through the sky. dx 2 2 d y = 0 at both stationary points, dx 2 d2 y so we can't tell anything about them from this. Given that you need to find the region happen to x = 1 and x = −1. = 2in sec x + tan x + cosec x + cot x 1 2 cosec 2x dx It's already the case, so you need to integrate the following to find the coefficient of x. a) x0 = 3.5 y 4 y = f(x) 3 2 1 b) x0 = 3 0 1 2 3 4 5 6 7 x y y = f(x) 5 4 3 2 1 b) x0 = 3 0 1 2 3 4 5 6 7 x y y = f(x) 5 4 3 2 1 Q1c) Hint: Take care with the scale on this diagram. a Then + is also a positive rational number, b 1 and b is smaller than x, which contradicts the assumption that x is the smallest number of the form eax + b where a and b are constants. It is thrown towards a target on the ground, a horizontal distance of 20 m from the window. However, you assumed at the start that a and b had no common factors, so you have contradicted your initial assumption. Exercise 4.1 Q1 Do the functions shown in the diagrams below have inverses? Find the speed of the particle after 3 seconds, given that it starts from rest. So the root is between 1.8 and 2.0 f(1.9) = ln (1.9) – 21.9 + 3 = -0.090... u = 0, v = v, a = 1.607..., t = 7 v = u + at v = 0 + (1.607... 3b2 = (3k)2 fi 3b2 = 9k2 fi b2 = 3k2 fi b2 is a multiple of 3. Calculating with 2D vectors should already be familiar to you, and 3D vector calculations are pretty similar. 1 1 1 d) y = fi y = x2 fi y +2x x 2 1 Set f-1(x) = x + 2 f(x) has domain {x : x # 2} and range {f(x) : f(x) # 0}, so f-1(x) has domain {x : x # 0} and range {f -1(x) : f -1(x) # 2}. You could solve this as an equation instead: Find the value of x such that |x| = 4 and rounding down. ff Be able to find the relationship between any two three-dimensional points using vectors. e) Some possible answers are: - The model suggests that the number of signatures eventually reaches the entire population of 30 000 (if you round to the nearest whole number). Obviously, in that case you can't use ln k as the constant of integration, so pick another letter — you might see examples that use A instead. The lifetimes of individual batteries can be modelled by a normal distribution with a mean of 300 hours and a standard deviation of 50 hours. Product Rule 154 6. f -1(x) = Rearrange to make x the subject. Sense The direction of a rotation (clockwise or anticlockwise). b) The athletes with personal bests in the top 20% of times are selected for a special training programme. ff Resolving vertically (#): Remember to use g = 9.8 ms–2 unless you're told otherwise. Its cross section is an equilateral triangle with sides x mm and the height of the crystal is 20 mm. Then, P(M ≤ c) = 0.1. c - 60 60 - c So P a Z # 3 k = 0.1 P a Z # 3 k = 0.9 60 - c F(Z) = 0.9 for z = 1.28 fi 3 = 1.28 fi 60 = 3 × 1.28 = 56.16 = 56 g to the nearest gram So the maximum mass is 56 grams. dx Q6 The curve C is defined by the parametric equations x = 3t2 – 4t + 3t2 + 4t3 . Sketch your answer for the interval –90° ≤ x ≤ 90°. Putting this back into the identity gives: A+B A-B sin A + sin B º 2 sin a 2 k cos a 2 k Answers 391 Q9 a) sin x cos y + cos x sin y = 3 cos x cos y fi sin x cos y + cos x sin y 3 cos x cos y fi tan x + tan y = 3 cos x cos y b) tan (x + y) = tan x + tan y ... Comparing the equation you have to solve to the one in part a) you can see that y = 4 . In these cases, no matter how close to the root you have x0, the iteration sequence diverges — the numbers get further and further apart. b) The normals intersect when: 22 −7x − 9 = 12x − 31 fi 22 = 19x fi x = 19 264 325 65 And so 5y = 19 − 31 = − 19 fi y = − 19 22 65 So they intersect at 19 , - 19 k. ff Now you know how long the stone is in the air, you can find its horizontal displacement in this time. Q5 u m s–1 MODELL ING MODELL ING 6m 15 m A football is kicked from a point on flat, horizontal ground towards a vertical wall, a horizontal distance of 15 m away. b) No, as it is a many-to-one map, and many-to-one functions do not have inverse functions. Tip: Rearrange the equation to get ln(x + 3) − 5x = 0 f(0.23) = ln(0.23 + 3) − (5 × 0.23) = 0.022... has first term a and common ratio r = −2, so 12 / a (− 2) k k is the sum of the first 13 terms of the sequence. You're Reading a Free Preview Pages 52 to 54 are not shown in this preview. When x = 0, y = 9(0)2 + 0 4 2 16 4 When x = − 9 , y = 9 9 − 9 4 16 So the stationary points are at (0, 0) and (− 9 , − 9 1. Formulate a differential equation to show that the rate of change of profit dP/dt. b) The root lies in this interval, so a solution to 1 d.p. is x = 0.7. Q9 f(0.245) = e0.245 − 0.2453 − (5 × 0.245) = 0.037... Tip: You could also say to y1 = m(x − x1) to get the gradient. b) Putting x = 0 into the equation gives: 3y − y2 = 0 fi y(3 − y) = 0 fi y = 0 and y = 3 ^2 0h + 2 y 2 At (0, 0), = − 3 dx 2e0h − 3. The y-intercept is 0 (as it goes through (0, 0)). At a certain time, the car reaches its maximum velocity when the acceleration falls to zero. F≤ mR where R is the coefficient of friction and R is the normal reaction of the surface on the object. All functions are much easier to deal with when they're in their simplest forms, so the first thing to do with algebraic fractions is to simplify them as much as possible. c) Find the value V approaches as t gets very large. (Where m is the coefficient of friction and R is the normal reaction between the object and the surface.) ff Friction opposes motion, so it will always act in the opposite direction to the direction an object is moving in. 4N ff By the alternate angles theorem, the angle 10° between the direction of the force and the 20° 20° slope of the plane is 20° + 10° = 30°. ff Solve the differential equation, and state the value of k from part b), to find the general solution: dD - 3200 = fi D2 dD = −3200 dt fi D2 fi # D2 dD = # -3200 dt 1 fi D3 = −3200t + C ff When t = 0, D = 60. Many-to-one functions Tip: There are also mappings known as 'one-to-many' and 'many-to-many', but neither of these types can be functions. Find the equation of the normal at the point t = −1. For a root, a, of a function f(x), the iterations will converge if the gradient of f(x) at a is between –1 and 1 (i.e. |f'(a)| < 1) for a suitable choice of x0. c) The maximum points are at (135°, –2) and (315°, –2). The method used is very similar to the hypothesis testing you did in Year 1. O 2m Tip: Modelling the plank as a light rod means that you don't have to worry about its weight. Find: a) the values of a and b, b) the velocity of the golf ball when it hits the ground. B Q3 Hint: The question doesn't mention i and j components or column vectors so you can answer it using either. Find the possible values of a, in surd form, and hence the launch speed of the rocket. Always start by identifying u(x) and v(y), and do it in steps if you need to. So as the width of dx approaches 0, the sum of areas is indistinguishable from the actual area. Let's try an example: 3 ff So the diagram looks like this: 42 N F R 22° A mg ff Resolving horizontally: R = 42 cos 22° = 38.941... Given that the beam is in equilibrium, find the range of possible values of the coefficient of friction between the beam and the peg. Exercise 3.2 — Friction and inclined planes Q1 a) Resolving parallel to the plane (): Fnet = ma 1.5g sin 25° = 1.5a a = 9.8 sin 25° = 4.141... p) for cosec x = −2 fi sin x = −0.5 fi (sin−1(−0.5) = − 6 There are two negative solutions in the interval p 0 ≤ x ≤ 2p at b p + 6) and b 2p − 6 l, so 7p 11p x = 6 and 6 . The Newton-Raphson Method 5.1 11 p p 1 Q5 a) arcsin 2 = 6 , so tan barcsin 2 l = tan 6 = . ff Here, f(x) = 2x4 − 5, so f'(x) = 8x3. Tip: Remember to use the unrounded z-values in your calculations. Q2 a) Show that the equation x3 − 2x2 − 5 = 0 can be rewritten as 5 x = 2 + 2. 7.5 p = tan-1 b 32.99... dx p6 (−1) + 2p5 (0) p6 b) The gradient of the normal when p = is 3 , 1 and y = − 3 , x = 0. You can use these facts to check if a normal distribution is suitable for the situation you're modelling — see page 288. ff First, define a random variable to represent the mass of cereal in a box. V = xi + yj, such that 2 and y dt dt x ff The particle is stationary at t = 0. 170 Chapter 7 Differentiation Finding tangents and normals Once you've found the gradient of a parametric curve at a particular point, you can use this to find the equation of the tangent or normal to the curve at that point. Use the given number of intervals in each case. Find: a) the value of t for which the particle's velocity is (−3i + 2j) ms−1, b) the value of t for which the direction of motion of the particle is 45° above i. Look back at your Year 1 notes if you want to recap weight and tension. ff m = np = 800 × 0.4 = 320 and s2 = np(1 − p) = 800 × 0.4 × 0.6 = 192 ff So use the approximation Y ~ N(320, 192) b) Find an upper bound for P(320 < X < 350). ff Use the formula for limiting friction to solve problems for a body in equilibrium. Some integrations look really tricky, but with a clever substitution they can be made a lot simpler. f -1(x) = log2 (x − 1) The range of f(x) is f(x) ≥ 2, so f −1(x) has domain {x : x ≥ 2}. Tip: You're told the weight, so use the formula W = mg to find the mass. b) Find the Cartesian coordinates of the point at which the path of the particle crosses the x-axis. 1 b) Find the coordinates of the point at which the normal to the path at t = 2 cuts the x-axis. ff Use Newton's second law to find the magnitude of the frictional force acting on the box. 9 9 #3 31x dx = 13 #3 1x dx = 13 (ln x @ 93 # 9 3 ff Now put in the limits and use log laws to simplify: 1 1 9 = 3 (ln|9| − ln|3|) = 3 (ln (3)) = 1 3 ln 3 184 Chapter 8 Integration a ln a − ln b = ln b Tip: Integrate linear transformations of functions of the 1 form) by considering the result of differentiating ln |ax + b|. Find the particle's speed, v, and direction of motion at this point. AB = So: AB = 3 11 . 298 Chapter 14 Kinematics Example 2 At a particular point on its trajectory, a particle has velocity v, with horizontal component 12 ms–1 and vertical component –5 ms–1. ƒ ff Now use Pythagoras on the purple triangle. x Q4 For parts a) and b), give your answer as exact values. Remember to write the brackets in your answer and simplify. Next ff Describes a body which is not moving. x x Q13 y = (4 − 2)3, so let f(y) = u sd where u = 4x − 3u2 = 3(4 − 2)2, = 4 du dx dy d2 y dy 3 y = 4x2 + 2 where the chain rule again to find : dx 2 dy d y x x 3 dx 2 dx x2 dy d y 3 x dx d y dx d2 y 2 1 fi u = 2 u 2 (4 − 2) = 8x − 3, = 4 du dx dy dx 2 3 −3 = 32 x 4 d2 y 3 3 The curve is convex when 2 > 0 fi 32 x > 0 fi x 3 x 2 > 8 ff So the curve is concave when 2 < 0 fi 32 x 4 < 0 dx 3 3 fi 32 x < 4 fi x < 8 So the curve is convex for x > 8 and concave for x < 8. ff sin x dx 2 ff Rearranging the cos double angle formula: cos 2x º 1 − 2 sin2 x gives sin2 x º 2 (1 − cos 2x). ff If two vectors are parallel, and also have a point in common, then they must lie on the same straight line — their points are collinear. Null hypothesis A statement which gives a specific value to the parameter in a hypothesis test. Q9 A particle is moving with a velocity of v = [mi + nj] ms–1. 1 = 9.8 ft = 2.251... This will occur when cos (Ct + D) = 1, i.e. when Ct + D = 0. A force of 20 N is applied at an angle of 60° to the other end of the spanner, as shown. Then V – N(m, 52). 282 Chapter 12 The Normal Distribution Q3 The volume of vinegar contained in bottles is modelled by a normal distribution with a standard deviation of 5 ml. A cut of mass 1.5 kg sits on the plane and is at distance... a) Give the 2-d... Divergent sequences/series A sequence/series that does not have a limit. b) The rod is now pivoted at point D. 23 13 y = 12 x + 6 fi 12 x − 3 fi 12y = 6 + 8 fi 12x − 13 – 12y + 6 = 0 Tip: The curve for a cut of the point (2, 6). 1 1 Q7 a) u = e, 2 × (x + 4) + du dx 1 1 fi = − dx 2 − 1 2 du 2 × (4 × 4) + 2 dx dx 2 2 The path of the particle passing through M. 3 1 6 = x − 8 × 24 = 8 x − 24 + du v x du 1 = 4 x 2 = ... ln u dx + x 1 = 6 × 9 a) 3 ... 3 × 108 4 2 dy = 3 9 2 47 5 y = mx + c fi 3 = b × 5 108 1 + c fi c = 108 5 47 So the equation of the tangent is y = 108 x + 108 Write this in the form ax + by + c = 0 where a, b and c are integers, as all coordinates must be multiplied by 108 and by 108 to get... Division by x. ff Now multiply all of these together: p1 p2 p3... Use of partial fractions You can integrate algebraic fractions where the denominator can be written as a product of linear factors by splitting them up into partial fractions. Mappings and functions Mappings A mapping is an operation that takes one number and transform it into another. So you can solve problems like this just as before, by resolving forces and taking moments. Verify your answer is correct to this level of accuracy. Significant result The observed value of a test statistic is significant if, under H0, it has a probability lower than the significance level. ff The null hypothesis here is always that r is zero — i.e. that there's no correlation between the two variables. dy (− 6) − e0) b) As (y − 2(0, −4), m = 1. Tip: The rate of increase in number of bacteria after t days (b) ... dx2 = 2.x ... ∴ 5 2x dx ... dx = 0 ... (1 + x) = 3 × 6 − 2 ... + ln ... − 1 Differentiating ... Q1 Differentiate with respect to: (x − 7) 4 x+5 a) y = ... as t = k(x + 3)x sec(x + 3) ... d) Write the following functions: x3 (x + 3) 2x 2 c) f(x) = x + 2 b) f(x) = x2 -7 x2 d) f(x) = x ... tan x , giving your answer in terms of cot x and cosec x. ff The rope is parallel to the plane. Find the magnitude and direction of u. + yn − 1) + yn], where h = n a f (xn) The Newton-Raphson formula: xn + 1 = xn − f (xn) f ′ (xn) Trapezium rule: Fantastic in-depth Student Books for A-Level Science! ff Binomial Series n n (a + b)n = an + e o an − 1b + e o an − 2 b2 + ... b) Evaluate the gradient of the curve at each of these points. ff As the distribution is symmetrical (i.e. not skewed), this means that mean = median = mode. Q5 f(x) = ln (2x + x3) Show that f(x) = 0 has a solution in the interval 0.4 < x < 0.5. 3 e -x b) Show that f(x) = 0 can be rewritten in the form x = 2 − e, 0.564 (3 s.f.) Tip: The direct method is much quicker here but for some questions you might need to split the probability up — see page 275. To find the y-intercept, put x = 0 into the equation. f(x) = p = 2 sin 60 = fi 2 × 3 = 2 × 2 So the y-intercept is at (0, 3). 12 h 1 fi 1 − 12 x = 2x + 2 ... (b) On the graph of the tangent at x = 25 is 1 b) (1024 ln 4) = 102.4 ln 4 Putting this into y = mx + c gives: 1024 = 2560 ln 4 + c fi c = 1024 – 2560 ln 4. The method looks a bit tricky, but follow through the examples on the next page to see how it works. + x + ... 1 sin ^180° − 45°h The CAST diagram shows that sin 135° is the same size as sin 45°, and also lies in a positive quadrant for sin: f) cosec 135° = cosec (180° − 45°) = 135° S 45° A sin + ve 45° 180° − 45° T C 1 1 = m 2 1 g) cot 330° = cot (360° − 30°) = tan 330° The CAST diagram shows that tan 330° is the same size as tan 30°, but lies in a negative quadrant for tan: S A 30° 1 So cot 330° = − sin 45° = 1 So cosec 135° = sin 45° = 2 2 ... The CAST diagram shows that cos p + 4 1 lies in a negative quadrant for cos: S A 4 cos − ve cos 4 4 0 T C 1 1 5p So sec 4 = = − 2 − cos p cos 4 2 p 5p 1 p 3 + − cos 2 + cos p = 2 2 3 − 1 = − 2 ... sin 3 = 2 6 sec so ... The CAST diagram shows that p ... The CAST diagram shows that sin p + 3 lies in a negative quadrant for sin: S A − p 3 ... 3 cot 6 + tan p 0 6 1 1 1 1 Q4 a) = = = 1 sec 60° 1 + a 1 k 1 + cos 60° = 12 k 1 0) cot 315° = cot (360° − 45°) = 1 tan (360° − 45°) = ... The CAST diagram shows that tan 315° is the same size as tan 45°, but lies in a negative quadrant for tan: ... 1 + tan 60° = 1 + 3 3 1 e) cot 150° = cot (180° − 30°) = ... 3 7 + 3 cot 150° 7 + 3 ... Exercise 4.3 — Simplifying expressions and solving equations Q1 a) cos x × sec x = cos x × 1 cos x = 1 ... Restricting the domain to {x : x ≥ −1} would make h(x) a function. 156 Chapter 7 Differentiation d2y ff Now differentiate again to find : dx d 2 d x 2 First part: dx d Second part: (3x2 ln x) — use the product rule again: dx u = 3x2 and v = ln x dv 1 fi = 6x and = x dx dx du dv dx d2 y ... Putting these together: = 2x + 3x + 6x ln x = x(5 + 6 ln x) dx 2 Q5 a) dy Q5 So 0 = either when x = 0 or when 5 + 6 ln x = 0 fi 6 ln x = −5 fi ln x = −6 fi x = e. Exercise 5.1 Q1 Differentiate y = x(x + 2) with respect to: a) x ff Be able to integrate other trig functions by considering the derivatives of cosec x, sec x and cot x. ff In the direction: dx 9 = 2 fi dt x = 9 fi dt x = 9 fi dt Tip: See p.222–224 for a recap on solving differential equations. ff Putting this into the Newton-Raphson formula gives: 2 + f(xn) = xn − 6xn x n 1 3 xn + 1 = xn − 1 f (xn) 6 − 2xn ff Starting with x0 = −1, this gives: x1 = −1 ... Tip: Notice how much quicker this Newton-Raphson formula converges to the root ... in part c) it took 5 iterations to find the root to 4 d.p. ff Further iterations give: 6 (−1) ... Finding the exact value of the integral: # 1.5 0 246 y = Chapter 9 Numerical Methods x 2 + 2x dx = 2 6y0 + 2 ^y1 + y2h + = 2 [0 + 2(1.118... ff Horizontal component, Fx: cos q = adjacent F = Fx, so Fx = F cos q hypotenuse F Fy q ff Vertical component, Fy: opposite Fy sin q = = F , so Fy = F sin q hypotenuse Fx Example 1 ff Draw a vector triangle and use trigonometry to calculate the horizontal and vertical components. ff Friction always acts in the opposite direction to the motion (or potential motion). So you need −x in the middle of the bracket to get another −x on the RHS (from the y2 term). From differentiating, 1 + 2y dy/dx ... When you multiply out the RHS you get the LHS (the terms in x2 cancel), so this is the right quadratic. dx You need to differentiate each term of the equation with respect to x. ff This is a two-tailed test, so you need to check whether the p-value (the probability of the test statistic being at least as extreme as the value observed) is less than 2 = 0.005. Intersection (of events A and B) The set of outcomes corresponding to both event A and event B happening. ff Resolving vertically, taking down as positive: s = 0, a = 9.8, t = t Using s = ut + at2: 1 2 ... 43.4 N Tip: ... a) As you can see from the graph... b) Using the chain rule ... 64x3 6^6 - 1h (1 + 6x)6 = 1 + 6(6x) + ... As h tends to zero, the blue bit at the bottom becomes g(x)g(x), or (g(x))2. dx f (x) m = g (x) fi f (x) - f (x) g (x) dx g (x) (g (x)) 2 Example 1 Find the domain of each function ... A function Y is a map which maps every number in the domain to one number in the range. The daily rainfall is less than 4 cm on only 10.2% of days, and it's greater than 7 cm on 64.8% of days. ff Do the same to find the equation of the tangent at (p, 1). As a result of the impact with the wall, the horizontal component of the ball's velocity is reversed and halved in magnitude. So b = a fi 3b2 = a2 fi a2 is a multiple of 3. The probability that neither X nor Y occurs. −8 It's easy to see what the domains and ranges are from the graph — f(x) has domain x ≥ 0 and range f(x) ≥ −8, and f −1(x) has domain x ≥ −8 and range f −1(x) ≥ 0. = 6.4 cm (1 d.p.) 1 A = 2 r2q = 2 × 82 × 0.802... Q4 Hint: Remember, taut means that the rope is tight and straight, and has tension in it. The rod is held horizontal in equilibrium by a light vertical strut attached to the rod 1 m from A, as shown. The expected value is 7.5 ms−1 at an angle of θ above the horizontal, so: yes |f(x)| ff Step 2: Where the graph goes below the x-axis (i.e. for negative values), draw it as a reflection in the x-axis. Tip: The common denominator should be the lowest common multiple (LCM) of all the denominators. Equating constant terms: 2 = −2A fi A = −1 Equating x coefficients: 0 = A + B fi B = −1 fi B = 1 ff Replace A and B in the original identity: -1 1 2 1 1 º x + · 0 · − x x 2 x (x + 2) x 2 ff Integrate each term separately: 4 fn = x 2 ... A3 Tip: The question asks for the answer as a single logarithm so make sure you fully simplify your answer. sin a ms−1 Q5 v 8 ms−1 q 6 ms−1 speed = 62 + 82 = 10 ms−1 8 q = tan−1a 6 k = 53.1° (3 s.f.) above the horizontal Q6 17 ms−1 q 12 ms−1 −2.5 ms−1 q speed = 17.2 ms−1 (3 s.f.) q = tan−1a 17 k = −8.37° (3 s.f.) i.e. 8.37° below the horizontal Exercise 1.2 — The constant acceleration equations Q1 a) Resolving vertically, taking up as positive: u = 15 sin 50°, v = 0, a = −9.8, v2 = u2 + 2as fi 0 = (15 sin 50°)2 − 19.6s s = (15 sin 50°)2 ÷ 19.6 = 6.74 m (3 s.f.) Q2 a) Resolving horizontally, taking right as positive: s = x, u = 12 cos 37°, a = 0, t = 4.791... So the expansion is valid for |x| < 5. ff Given that g(x) = −f(x) ... Q7 Hint: ... b) Describe the shape of the ... The modulus function makes a number positive by removing any negative sign. The modulus of a number is its size — it doesn't matter if it's positive or negative. Chapter 9 Numerical Methods 243 y = ÷6x + 13 y= ÷x Tip: Here the values of x1, x2 and x3 have been rounded to 2 d.p. to make it easier to label them on the graph. questions about the model. t x and y represent the displacement of the particle from the origin in the i and j directions respectively, at time t seconds. c) The lines intersect when e4 −4 8 6 (x − 4) = 6e (4 − x) fi e (x − 4) = 36(4 − x) fi 4 e x + 36x = 4e + 144 fi x(e8 + 36) = 4e8 + 144 fi x = e8 + 36 Q12 When a = 2: 2y2 + 4y − 24 = 4 fi 2y2 + 4y − 28 = 0 fi y2 + 2y − 15 = 0 fi (y + 5)(y − 3) = 0 fi y = −5 or y = 3 so 2y = 14 [(x − 2)4] 32 1 1 1 (x − 2) 3 = 4 [(3 − 2)4] − [(2 − 2)4] = 1 − 4 − [0 − 4] 1 − 0 = 4 Tip: There's no need to write down the integral for indefinite integrals you can just cancel out when you do the subtraction. 344 Chapter 15 Dynamics b) When B hits the ground, A carries on moving along the plane. PROBLE M SOLVING Q5 Show that the Newton-Raphson method will fail to find a root of the equation 2x3 − 15x2 + 109 = 0 if x0 = 1 is used as the starting value. Choose probability distribution An exam question might ask you to choose a suitable probability distribution or explain why one is appropriate. The lower bound is found using: 3 # 1 2 fi (2x + 3) = 0.3288... Example The curve C is defined by the following parametric equations: x = sin t, y = 2t cos t. ff The rate of change, , is proportional to the population, so b dt dt ff We're looking for an equation, not a proportional relation, so rewrite it: So db = kb for some constant k, k > 0. ff Be able to find missing distances and forces in calculations involving rigid bodies in equilibrium. 16 ms−1 123° Q2 A particle is moving with speed 8 ms−1 at an angle of 35° to the horizontal. Q5 Hint: The overall rate of change of V is the difference between the rate at which water is flowing in and the rate at which it's flowing out. (1 + x)−3 = 1 − 3x + 6x2 − ... #e x dx = ex + C Example 1 Integrate the function 6x2 − 4x + 3ex with respect to x. Tip: For a graph of y = f(|x|), first draw the graph of y = f(x) for positive values of x, then reflect this in the y-axis to form the rest of the graph. ff Resolving vertically: R(#) = 6.5 sin 69° + 8 cos 61° − 10 sin 47° = 2.633... 15° 29° Calculate the magnitude of T, given that this force is only just large enough to stop the sledge sliding down the slope. 326 Chapter 15 Dynamics Resolving an Object's Weight ff You want to find the components of an object's weight parallel and perpendicular to the plane. Then the required ... Differentiate the denominator: This is the d 3 (x − 2x2 − 1)) = 3x2 − 4x = x(3x − 4) numerator. or t = −1.844... = 2.99 N (3 s.f.) Using trigonometry: 2.263... ff Find the x-coordinates of these points, leaving your answers as exact values. This table contains an extract of the data that you'll be given in the exam — the formula booklet contains critical values for n up to 100 (the table below has all the values you'll need to answer the questions in this book). Some examples might be natural immunity to a disease, immigration/ emigration of a population or seasonal variation in weather. p ... The domain is x d R , x ≠ bp + ... fi (when n is an integer). X is to be approximated by the normal variable Y. c) Give one possible limitation of this model. ff Start by defining the random variable, and stating how it is distributed. You might want to look back at your notes on 2D vectors first, to refresh your memory. dx 1 Step 3: Now integrate both sides: # (y) dy = # f(x) dx. Answers 425 p Q13 a) We want to find the ... probability, P(X = x) ... Use the formula with the horizontal component of ... the probability that the particle 0.8 seconds after it is projected. ff The string fixing Q to the ground is cut. 76.0 e o +e o e o = 0.03 ... Now use the formula du dx du and f'(u): dx du dx # 6x 5 6 e 6x (f'(u) du f'(u) dx = f(u) + C to write down the answer. dt dt Replace f(x) with y to get an equation for y in terms of x. She wants her investment to double so u11 = ar10 = 2a fi r10 = 2 fi |r| = 10 2 = 1.071773. p h # ... the trapezium rule ... [ln(2 + sin 0) + 2(ln(2 + sin 6) + ln(2 + sin 3) + ln(2 + sin 2) + ln(2 + sin 3) ...] = 12 [0.6931 + 2(0.9163 + 1.0529 + 1.0986 + 1.0529) + 0.6931] = 0.691... Q2 ... With a minimum ... MODELL ING Q3 Hint: You might see brackets describing intervals — x takes values in the interval [a, b] means a ≤ x ≤ b and x is in the interval (a, b) means a < x < b. Q8b) Hint: Write this as an inequality ... Chapter 6: The Binomial Expansion 1. The range of f −1(x) is the domain of f(x): f −1(x) > −1. Given a number of whole numbers ... Proof by Contradiction Exercise 1.1 — Proof Q1 Suppose that there is a number x that is the largest multiple of 3, so it can be written as x = 3k for some integer k. A biologist counts the bacteria in the colony when its diameter is 2 mm and estimates that there are approximately 208 bacteria. ff Be able to solve ... For example, the modulus of 8 is 8, and the modulus of −8 is also 8. MODELL ING ff Integrate the expression for acceleration with respect to time to find an expression for velocity: ... Using Pythagoras' theorem: magnitude = 24 2 + 22 = 24.1 ms−2 (3 s.f.) Q3 ... ff The expressions ... b) Find the magnitude and direction of the reaction force acting on the rod at A. b) Find the number of blood donors that this model would predict to take less time than Edward to replace their blood cells. b) (i) a = v0 = (6ti + 2j) ms−2 (ii) When t = 4, a = 6(4)i + 2j = (24i + 2j) ms−2 Using Pythagoras' theorem: ... At the maximum velocity, a = 0, so: tc2 + 4t − 12 = 0 fi (t + 6)(t − 2) = 0 fi t = −6 and t = 2 so t ≥ 0 ... When D = 2, n = 208: 208 = 2k ... So the expression has 4 terms and 15 terms ... Therefore u15 is the last term that's less than 4, so there are 15 terms that are less than 4. d) n is not a large (6), so a normal approximation would not be suitable. ff Write down the suvat equation ... The particle's displacement from the point of projection 0.8 seconds after it is projected. Q4 Hint: Remember that you need to work in radians when differentiating and integrating trig functions. Tip: The function is continuous because it's just a cubic curve. ... Tip: ... y = e3x 2 ... 2 e ... 1 + ... Tip: You may see ... ff An equivalent answer to b) would be ... Use the general formula for integrating ax + b instead. ff Show that the equation f(x) = 0 has a root in the interval (0.6, 0.7). The lengths of these sides are: |a| = 5 2 + ... Using the cosine rule: b2 = a2 + c2 − 2ac cos A = 2bc 2 ... 2 570 Chapter 10 Vectors - 40 m ... = 146.9° (1 d.p.) ... 2 570 ... 3 fi ... A Exercise 2.1 Unless specified, give each answer in its simplest form. ... The expansion is valid for |x| < 4. Set x = −1: −1 − 14 = 6 ... B = −10. So f −1(x) has domain {x : x < 1} and range {f −1(x) : f −1(x) ... Modelling the ladder as a uniform rod and the cat as a particle, find the coefficient of friction between the ladder and the ground. dt dt ... By introducing a constant of proportionality, k, this becomes: dP = −kP dt ff The minus sign shows that the population is decreasing. The display shows that the ... b) Resolving vertically, taking up as positive: s = −5, u = 10, v = vy, a = −9.8 v2 = u2 + 2as vy2 = 100 + 2 × −9.8 × −5 = 198 vy = −14.071... 19.3 Now you can find its initial speed using Pythagoras' theorem: speed = 38.6 2 + 19.3 2 = 43.2 ms−1 (3 s.f.) The stone ... The significance level is α = 0.05, and it is a one-tailed test. Finding the critical region is quite a lot like doing a hypothesis test. 2.24 m = 2 (3 s.f.) ff Using Newton ... (10i − 5j) + (−6i + 2j) = (4i − 3j) N [Rnew] = 4 2 + ... So the acceleration decreases by 1.24 ms−2 (3 s.f.) Answers Q4 a) 7i + j (6i − 2j) ... = (5i + 5j) + x = (7 + 2) ... Find the resultant force ... 2 4 ... The impulse the wind ... ff The model states that the rate of decrease in the rabbit population dP dt (i.e.) is proportional to P. 252 Chapter 10 Vectors PROBLE M SOLVIN G Exercise 1.1 Q1 It is the point (4, −5, 1) and S is the point (−3, 0, −1). In each case, state the quotient and remainder. A+Bk aA− Bk 2 cos 2 PROBLE M SOLVING Chapter 8 Integration 203 7. Sketching Iterations Exercise 3.1 — Cobweb and staircase diagrams Q1 a) y = x 5 4 y = f(x) 3 2 1 x2 x1 x0 A convergent cobweb diagram. ff 18 people went to see a comedy film. The coefficient of friction between the sledge and the slope is 0.1. A force, T, acts on the sledge at an angle of 15° to the slope, as shown. ff The relationships between displacement (or position), velocity and acceleration are shown below: POSITION (r) Differentiate Integrate VELOCITY (v) Differentiate Integrate ACCELERATION (a) ff Use calculus to find a body's position vector. Q12 Find the set of values of x for which the curve is increasing. ... Exercise 9.2 — Finding tangents and normals dy dx = 2t, 3t2 − 6. ... function f(x) = e + 3x is a rational ... ff Verify ... Q6 The function f(x) = e... Tip: ... Q9 ... Q10 f'(x) = g'(x)eg(x) = esc x sec x tan x g'(x) − cosec x cot x Q11 a) f'(x) = −cot x cosec x ... Here you could also rearrange by saying ln (1/sin x) = ln 1 − ln sin x = −ln (sin x). Finding enclosed areas To find the area between a curve, a line and the x-axis, you'll either need to add or subtract integrals to find the area you're after — it's always best to first draw a graph. In this example, the velocity is constant because the rod is about to slide towards the wall, and when it is on the point of sliding ... Using the iterative formula ... 6.51941 ... 6.53047... 6.53087... The last 4 iterations all round to 6.53, so the value of the root is x = 6.53 to 3 s.f. To verify this, check it's between the upper and lower bounds: f(6.525) = 6.5252 − (5 × 6.525) − 10 = −0.049... ff Find the ... Exercise 1.2 — Sketching graphs to find approximate roots Q1 a) 4 2 1 2 ... The graphs cross twice, so the equation has 2 roots. The projectile is fired from the origin with speed 3 ms−1, at an angle to the horizontal, so ... Remember — the reaction force is in the hinge, not the wall, so that's why R can act to the left. Find, as vectors in i + j notation: a) the initial velocity of the ball, b) its velocity when it hits the wall. = 1 + 6(3x) + 15(9x2) + ... 8N x° 10 N Q4 Hint: Use the fact that cos 45° = sin 45°. Quotient: x2 − 15x + 21 Remainder: −10 b) 2x3 + 5x2 − 8x − 17 º (Ax2 + Bx + C)(x − 2) + D ... Set x = 2: 16 + 20 − 16 − 17 = D, so D = 3. The range of h is R, but squaring the result means that g(x) will always be positive. dy f l (x) 3x2 − 6x + 3 = 3 − 2... 2x4 + 3x2 − 6x + 13 ... ff The gradient ... ff First, sketch the graphs of y = x and y = f(x) ... 1 1 ... The root lies between 0.6 and 0.7 ... ff Draw a sketch ... So A and B are independent. Using Pythagoras, the ... Show that C passes through (1, 3) and (1, −2). Correlation A linear relationship between two variables showing that they change together to some extent. The coefficient of friction between the surface is 0.4. a) Draw a diagram to show the forces acting on the block. So f(|−2|) = f(2). Mappings and Functions 3. So the paper thickness forms a geometric progression: After 1 fold, thickness = 0.01 × 2 = 2 cm. 27.2 N (3 s.f.) b) T ... Fnet = ma 110 cos 22° − F = mg sin ... So F = 6.721. ff Remember, the unit vectors in 2D ... Glossary A B Absolute value Another name for the modulus. There is significant evidence at the 5% level to reject H0 in favour of the alternative hypothesis that the mean time spent studying has gone up. ... Bearing = 270° + q = 355° (nearest degree) ... ff Find the position vector of the particle at time t ... Q1 Sketch the following graphs and state the range of each: ... y = |x + 3| b) y = |5 − x| c) y = |3x − 1| d) y = |x| − 9 ... e) y = 2|x| + 5 Q2 For each of the following functions, sketch the graph of y = |f(x)|: a) f(x) = 2x − 5 b) f(x) = 4x ... Q3 Match up each graph (1−4) with its correct equation (a−d): ... ff Use the ... Since −1.818 < −1.645, the result lies in the critical region, so there is significant evidence at the 5% level to reject H0 in favour of the alternative hypothesis that the mean score this year is lower. Q6 a) Prove by exhaustion that if x2 is a multiple of 3, then x must be a multiple of 3. b) The range of g(x) is between the maximum and minimum values, which are at 5. Tip: In set notation, the answer would be: {x : x < −1} È {x : x > 2}. ff The graph of y = e2 + 3x − 2 e2 ... The force of friction ... the velocity ... 1 ... 13 ... The acceleration ... find the speed of the block 3 seconds after it is released. ff Find the magnitude of the acceleration: |a| = 3 2 + 4 2 = 5 ms−2 ff Now list the variables s=s u=0 a=5 t=4 s = ut + at2 1 s = 2 × 5 × 4 2 = 40 m Tip: You could get the same answer either way. 11.050... or t = 0.634... That means you'll have to split the projectile's motion into its horizontal and vertical components, then use the constant acceleration equations to find out more about the projectile's motion. 10° F 10g ff Resolving vertically: R + Q sin 10° = 10g Q sin 10° = 10g − R ... Q cos 10° = F Q cos 10° = 0.7R ... Tip: You can assume that the box is about to slide horizontally in the direction of Q. b) The period of y = cos x is 180°, and the period of x is 4 × 180° = 720°. ... 'Ladder' questions You may also be asked to consider cases where a rigid body rests at an angle against the ground and a wall — a common example of a situation like this is a ladder. 3 A giant meal cube from space is cooling after entering the Earth's atmosphere. 87 ff Next, turn the question into a probability for X. ... ff So the root of the divisor: 2x3 + 5x2 ... So the forces ... 4.03 m (3 s.f.) Q5 a) FW 22 N 75 N a 9g FG Resolving horizontally: FG = 22 N Resolving vertically: 75 + FW = 9g fi FW = 13.2 N Taking moments about base of ladder: 9g cos a × 2.4 = 22 × 4 sin a fi 21.168 cos a = 88 sin a fi tan a = 0.2405... ... a = 13.5° (3 s.f.) ... 143.32 cos a = 105.6 sin a tan a = 1.4045... ff If you know that A has already happened, then the only outcomes you're interested in are the ones corresponding to A. When a body is freely hinged, the reaction force is the force in the hinge that keeps the body in equilibrium. The reaction force is ... the resultant ... variables: u = √ x ... Exercise 10.1 ... Exercise 9.2 ... 175 P(H > 170) = 0.8 fi P a Z > k = 0.8 fi P(Z) = 0.8 for z = 0.842 (3 s.f.) 5 So s = 0.842 fi s = 5 ÷ 0.842 = 5.94 cm (3 s.f.) b) Lower bound = 175 − 4 = 171 and upper bound = 175 + 4 = 179. Using the table, the critical value is 0.3783. We've detected that JavaScript is disabled in this browser. r = −0.28 is outside the critical region so there is insufficient evidence to reject H0 at the 5% significance level. ff Population questions come up quite often — the population might be increasing or decreasing, and you have to find and solve differential equations to show it. H0: r = 0 The alternative hypothesis (H1) is what you're going to conclude if you reject the null hypothesis — there are two kinds of alternative hypothesis: ff A one-tailed alternative hypothesis specifies whether the parameter you're investigating is greater than or less than the value you used in H0. Integrate 3A Sec2 ... ff The larger the magnitude of the force, and the greater the distance between the force and the pivot, the greater the moment. Using Pythagoras' theorem, the magnitude ... So x = −1.8494... ff Describe ... Tip: ... It helps to number the equations, so you can refer to them later. ff The values 'tails off' either side of the mean — values become less likely as you move further away from the mean. Q2 ... Chapter 2 Algebra and Functions 17 Types of function One-to-one functions Tip: Sketching a graph is a good way to help you identify the type of function. So the gradient of the normal is 5. c) Show that the equation f(x) = 0 has a root between 1.8 and 2.2 and find this root to 1 decimal place. 2.5x − 6.3 + ... ff Know the shape and properties of the normal distribution. Tip: ... ff ... So, the ... Chapter 6 ... = −7. Of course — that's what we do here ... • All the practice questions you need... Oh, yes... Carry out a hypothesis test at the 10% significance level to investigate whether the evidence suggests that test scores and hours spent studying are positively correlated. ... Q6 Identify f(x) and g(x): f(x) = 2(1 + x2) and ... ff Show that ... Exercise 11.1 MODELL ING Q1 The number of fleas (N) on a cat is increasing over time, t, at a rate directly proportional to the number of fleas. In exam questions you'll usually be told the value of x that a root lies between, so you can find its speed using Pythagoras' theorem. ... Resolving vertically: Vertical component = 9 sin 40° = 5.79 ms−1 (3 s.f.) ff Given the horizontal and vertical components of a projectile's velocity, you can find its speed using Pythagoras' theorem ... The ball is 23 ms−1, at an angle to the horizontal. = 4.51 m (3 s.f.) ff As in the previous example, split the acceleration into two components, RH and RV , and draw a diagram. ... the tangent at this point ... The gradient of the tangent ... The probability distribution B(n, p) A discrete probability distribution which models the probability of x successes in n independent trials when the probability of success in each trial is p. ... Practical ... the Newton-Raphson formula ... 2.114975... = 2.115 (3 d.p.) ... Exercise ... k unit vector The standard unit vector used in 3D to represent movement along the z-axis. MODELL ING () Tip: An initial velocity 2q of d ms−1 means 2q ms−1 and the horizontal component is 3q ms−1 ... fi 7 fi 2 ... fi 2 ... the iteration ... 1.7984... cot x = 2.414 fi tan x = 0.41666... Q4 a) Let the mean weight in grams of all the pigeons in the city centre this year be m. If P(Z > z) = 0.15, find the value of z. Percentage error The difference between the actual value and its approximation, as a percentage of the real value. 1 The function y = ln (cosec x) ... asymptote), but this isn't in the range where the function is increasing, so it doesn't affect the answer. The range of f −1 is the domain of f(x): f −1(x) > −2. Q11 a) If f(x) = ln (cosec x). m 20 − 12.490... Set notation In Year 1, you saw some of the laws of probability, such as the addition law: P(A or B) = P(A) + P(B) − P(A and B). du = cos (2x + 1) dx dv v = 2x + 1 fi = 2 dx ff Now you have all you need to complete the equation: dx dy dx = ... Exercise 4.2 ... d) Give a limitation of the model and suggest a possible improvement that could be made to address it. ... dx ... Q1 Split up that '+2' into two lots of '+1' so it starts to resemble the identities... following integrals: a) # sin x cos x dx ff Rearranging the sin double angle formula: sin 2x º 2 sin x cos x gives sin x cos x º 2 sin 2x. 25.7 cm2 (1 d.p.) ... Example 2 A ... the angle in radians: 4p 80 × 2p 80° = 180° = 9 radians ... Q4 a) ... 3.60 cm 4.8 cm2 ... 5.24 cm (3 s.f.) ... a 9 4 5 ... 160p 2 r 2 q 160p fig r 2 q r = 8 cm 16p ... Q9 Area A ... ff ... the area of each trapezium is A = Then an approximation for # a b f y1 y2 yn − 1 ... The data collected is continuous and the distribution of the heights is symmetrical about the mean. a) Find OQ , giving your answer in reduced surd form. So when the ± is a −, t is negative. ff This isn't exactly how the question wants the answer, so it needs a little more rearranging: 6x2(2x − 1)2 + (2x − 1)3 = 6x2 (2x − 1)(3x + 1) ... when you integrate a definite integral, you put the function in square brackets ... Tip: The force in the rod is called the thrust. Example 2 The ... Using Pythagoras' theorem: v = 12 2 + 5h2 = 13 ms−1 To find q: 5 q = tan−1a 12 k = 22.6° (3 s.f.) below the horizontal Exercise 1.1 Q1 Each of the following diagrams shows the speed of a projectile and the angle its velocity makes with the horizontal. For each value of x in the domain (which goes along the horizontal x-axis) you can plot the corresponding value of f(x) in the range (up the vertical y-axis). f(x) 2 f(x) = x range x domain Drawing graphs can make it easier to identify functions, as shown below. Replace f(x) with y: y = 3x + 4 y-4 Replace with f −1(x) and x: f −1(x) = 3 . Tip: This should be quite familiar from Year 1, where you used these formulas for motion in one dimension. a) How long after being fired does the bullet hit the sea? ... ff Write the answer ... X ~ B(600, 0.51) b) X ~ B(100, 0.98) c) X ~ B(200, 0.85) d) X ~ B(6, 0.5) Q2 The normal random variable Y ~ N(m, s2) is to be used to approximate these binomial distributions. ... ff Explain how the diagram shows that there is just one solution ... Exercise 2.2 ... A scientist is testing how many ... 3 R ... The horizontal component of velocity will remain constant throughout the motion. Similarly, if |x| > 5, you'd end up with x > 5 and x < −5. This can be written as: dD ? 19 N 14 N 2N C) 1.5 m d) 10 N 3m 30° N 3m 30° e) 8N f) 2 N 24 N 13 m 32° 25° X N 4m 45° X 7N 15 N 3m 50° 18 N 8m 28° 105° 10 m X 6N 1N XN Q3 2m A 1 m 3m 38 N P 3m 150° 24 N Chapter 16 Moments 13 N The diagram shows the light rod AB. dA dt dH dA = × = −32pr J cm−1 dt dA dr

dH When r = 12.3, = –1236.53 J cm–1 (2 d.p.) dr dH = –0.5 mm h–1 = –0.05 cm h–1 (length is decreasing). Partial Fractions Exercise 7.1 — Expressing in partial fractions 3x + 3 A + B / (x - 1) (x - 4) (x - 1) (x - 4) & (x -31x)+(x3- 4) / A (x(x -41))+(xB-(x4) 1) & 3x + 3 / A (x - 4) + B (x - 1) Substitution: x = 4 fi 15 = 3B fi B = 5 x = 1 fi 6 = –3A fi A = –2 3x + 3 2 5 This gives: /- - + (x - 1) (x - 4) (x 1) (x 4) 5x - 1 A B Q2 / x + x (2x + 1) (2x + 1) fi 5x – 1 º A(2x + 1) + Bx Equating coefficients: x terms: 5 = 2A + B constants: –1 = A A = –1, putting this into the first equation gives: 5 = –2 + B fi B = 7 5x - 1 1 7 This gives: /-x + x (2x + 1) (2x + 1) 3x - 2 3x - 2 A + B / / Q3 x 2 + x - 12 (x + 4) (x - 3) (x + 4) (x - 3) & (x +34x)-(x2- 3) / A (x(x +34))+(xB-(x3+) 4) & 3x - 2 / A (x - 3) + B (x + 4) Equating coefficients: x terms: 3 = A + B constants: –2 = –3A + 4B Solving simultaneously gives: A = 2, B = 1 3x - 2 2 + 1 This gives: 2 + / x + x 12 (x + 4) (x - 3) 2 2 A + B Q4 / / x 2 - 16 (x + 4) (x - 4) (x + 4) (x - 4) fi 2 º A(x - 4) + B(x + 4) 1 Substitution: x = 4 fi 2 = 8B fi B = 4 1 x = –4 fi 2 = –8A fi A = - 4 2 1 1 + This gives: 2 /4 (x + 4) 4 (x - 4) x 16 Don't worry if you get fractions for your coefficients — just put the numerator on the top of your partial fraction and the denominator on the bottom. Enter the starting value, and then type the iteration formula replacing xn with 'ANS' — each time you press enter you'll get another iteration. ff A curve can be defined by two parametric equations, often with the parameter t: y = f(t) and x = g(t) dy ff To find the gradient, dx , you could convert the equations into Cartesian form (see pages 93-94), but this isn't always possible or convenient. dy f l(x) 1 b) = = + dx 1 x f (x) The coefficient of x is 1 so it's a 1 on top of the fraction. Q19 2 - Q2 Hint: Both denominators are a multiple of x. T Q8 Hint: Think carefully about which direction the frictional force will act in. Chapter 9 Numerical Methods Example Show that one root of the equation x3 – x2 – 9 = 0 is x = 2.472 correct to 3 d.p. Tip: Although 2.4725 would actually be rounded up to 2.473, everything below it would be rounded down to 2.472. ff The following results are in the formula booklet, but it will help a lot if you understand where they come from. MODELL ING Q6 A group of people were asked about their mobile phones. = 26.1 m (3 s.f.) 462 Answers c) Resolving vertically, taking up as positive: s = 2, u = 19 sin 28°, v = vy , a = –9.8 v2 = u2 + 2as vy2 = (19 sin 28°)2 + (2 × –9.8 × 2) = 40.365... Alternatively, you can 4 use q = cos–1a 5 k, as long as you don't round it. 1 ^ 4 - x 2h2 1 dx. ff This is the stone's displacement from its starting point when it reaches its highest point. A third force of magnitude 3 N acts at an angle of q above i, where 180° ≤ q ≤ 360°. ff In context, this suggests that the more hours of exercise a person does, the less time it takes them to cycle the route (i.e. they can cycle quicker). an upper and lower bound of the area beneath a curve. It is also fairly symmetrical. Example 1 Tip: You won't always be told what substitution to use, so you might have to take an educated guess. Putting this into the Newton-Raphson formula gives: x 2 ln x - 5 xn + 1 = xn - n + n xn (1 2 ln xn) 2 2 ln 2 - 5 = Starting with x0 = 2, x1 = 2 2.466709... The important thing to remember is that: the starting condition occurs when t = 0 ff Once you've solved the differential equation you can use it to answer Tip: This might seem pretty obvious, but it's really important. So 'at least as extreme as the observed value' means '–1.7888... A random sample of 25 pigeons gives a mean of 314 g. In these questions, you'll need to consider the normal reaction of the ground, the normal reaction of the wall, and any frictional forces which may be acting. Show that there is no overall turning effect about this point. It may be more accurate to assume that there are some people who will never sign the petition, no matter how much time elapses, and allow for this in the model. So write x3 – 1 = (x – 1)(ax2 + bx + c) then expand the brackets and compare the coefficients: 1 = a –1 = (–1) × c fi c = 1 0 = b – a fi b=1 x2 + x + 1 2x + 7 Chapter 2 Algebra and Functions 3 Tip: Take your time with messy expressions and work things out in separate steps. ff Z is a continuous random variable, so P(Z > –0.42) is just the same as P(Z ≥ –0.42). ff The alternative hypothesis, H1, will either be Tip: X is a random variable representing the mean of any sample. Adjusting the function so that p varies with time may make the model more accurate. If the derivative of u is also part of the expression, you might be able to use the formula above. Here the sign shows you'll need to get the x2 on its own and square root both sides. Normally, the way to approach these problems is to resolve forces on each particle separately and form a pair of simultaneous equations, although sometimes it will be easier to treat the whole system as one object. Q4 Prove that there is no smallest positive rational number. Q6 Divide 3x 4 + 7x3 – 22x 2 – 8x by x – 2, and hence solve the equation 3x 4 + 7x 3 – 22x2 – 8x = 0. Two-tailed test A hypothesis test is 'two-tailed' if the alternative hypothesis specifies only that the parameter doesn't equal the value specified by the null hypothesis. ff Friction will only act if there is a force acting on the object which could cause it to move along the surface. b) The period of the graph will be the same as for the graph of y = cosec x, i.e. 360°. Q8 Find dy 1 if y = cos x . ff The downward force of the rod's weight (2g = 19.6 N) is greater than the upward vertical component of the tension in the string (T sin 45° = 14.7 N), so the reaction will act upwards to balance this out: A T RH 45° 0.6 m C 0.4 m 2g Chapter 16 Moments B 0.4 m 2g RV 358 C B ff Resolving horizontally: RH = T cos 45° = (20.788...) cos 45° RH = 14.7 N Tip: The rod is in equilibrium, so: 'forces left' = 'forces right' and 'forces up' = 'forces down'. Q4 Q8 a) The path cuts the y-axis when x = 0, so s3 ln s = 0. So the probability of B, given that A happens, is just the probability of B. When this happens, just put the multiple in front of the ln. Answers 459 Q5 Let the mean number of hours that Kyle has spent studying each week this year be m. 6 = 1 + 8x + 3(16x2) + (–4)(–64x3) + ... tan–1 (–0.41666...) = –22.6° (1 d.p.) There are two negative solutions in the interval 0 ≤ x ≤ 360° at (–22.6° + 180°) and (–22.6° + 360°), so x = 157.4° and 337.4° (1 d.p.) f) 4 sec 2x = –7 fi cos 2x = –0.57142... a 8 c) x1 = x0 + b = - - 3 = –4.6 5 x2 = –4.739, x3 = –4.688, x4 = –4.706, x5 = –4.700, x6 = –4.702 So x = –4.70 to 2 d.p. 1 Q4 a) 2x – 1 = 4 x fi 2x – 1 = 22 x 2 1 1 fi 2x – 1 × 2–2 = x 2 fi 2x – 3 = x 2 x–3 2 2x – 6 fi (2) = x fi x = 2 b) Using the iterative formula given: x1 = 0.0625, x2 = 0.0170, x3 = 0.0160, x4 = 0.0160 448 Answers 3 - 7x b) x + 7x – 3 = 0 fi x + 5x + 2x – 3 = 0 - - 4 fi 2x = 3 – 5x – x4 fi x = 3 5x x 2 c) x4 + 7x - 3 = 0 fi x4 = 3 – 7x 3 - 7x fi x2 = 3 - 7x fi x = x 5 Q2 a) x3 – 2x2 – 5 = 0 fi x3 = 2x2 + 5 fi x = 2 + 2 x 5 b) x1 = 2 + 2 = 3.25 2 x2 = 2.473..., x3 = 2.817..., x4 = 2.629..., x5 = 2.722..., 4 c) f(0.01595) = 20.01595 – 1 – 4 0.01595 = 0.00038... R2 = (20 + 12.99...)2 + (7.5)2 R R = 32.99... Using your calculator, this gives a value for z of 2.576. 22 Chapter 2 Algebra and Functions 4. Example 2 Find the following integrals: 5 a) # x dx 5 is a constant coefficient — you can take it outside the 1 integral so that you're just integrating x . Tip: Because the ball is thrown from 1.5 m above the ground, it only has to travel 3.5 m to reach a height of 5 m above ground level. Example 1 X ~ N(85, 25). x=a You saw on the previous page that you can find upper and lower bounds for the area under a curve by adding up the area of rectangles. b) The domain shown is {x : –1 ≤ x ≤ 4}. Tip: If you were asked to solve the equation f(x) = 0, you would set each bracket equal to 0 and find the solutions — x = 3 and x = –4 ± 10. the resultant force and solve problems for objects in equilibrium. ff Like other iterative methods, if you choose a start point too far away from the root, the sequence might diverge. Q6 The masses of the eggs laid by the hens on farmer Elizabeth's farm are assumed to follow a normal distribution with mean 60 g and standard deviation 3 g. = 21.6 N (3 s.f.) Using trigonometry: 13.475 q = tan–1 a 16.828... b) Find the position vector of point B, if OA = 3i + j + 2k. Tip: You're summing the areas of the rectangles where the left hand corner meets the curve. Exercise 1.3 — Converting to the Z distribution Q1 a) Standard deviation = 100 = 10 and mean = 106. Binomial expansion A method of expanding functions of the form (q + px)n. Doing it this way means you don't have to put x back in at the last step — just put the values of u into the integration for u. In this example, x is always at least 0, so f(x) must always be at least 20 + 1 = 2. State whether the function has an inverse. 0 c) x0 = 1.75 y 1 2 3 4 5 6 7 x y = f(x) 5 4 3 2 1 d) x0 = 4 0 2 1 x 3 y 5 4 3 2 y = f(x) 1 240 Chapter 9 Numerical Methods 0 1 2 3 4 5 6 7 x 4. The beam is on the point of slipping. 0.8 2 b) r = 0.8 = 0.8, |r | = | 0.8| = 0.8 < 1, so the sequence converges. Q4 a) Show that the function f(x) = x2 – 5x – 2 has a root that lies between x = 5 and x = 6. This can happen, for example, if the equation contains a number of different powers of y, or terms where x is multiplied by y. a) F find the value of q and hence the initial speed of the stone. Q2 f(x) = sin 2x – x (x is in radians). Q3 Hint: Try sketching the graphs from the given functions to see what shape they should be, then compare them in terms of where they cross the x and y axes. So you want to find P(780 < X < 790). dy dv du =u +v = (x3 × sec2 x) + (tan x × 3x2) dx dx dx ff Finally, rearrange to make it look nicer: dy = x3 sec2 x + 3x2 tan x dx You might have to differentiate functions using a mixture of the product rule and the chain rule (as well as the rules for e, ln and trig functions). × a a = 3.454... ff Be able to sketch graphs when y = f(x) has been affected by a combination of these transformations: y = f(x + a), y = f(x) + a, y = af(x), and y = f(ax). If x is not a multiple of 3, then there are two cases to consider: x = 3k + 1 and x = 3k + 2 for some integer k. fi P(T ≤ b) = 1 – 0.063 = 0.9370 P(X ≤ x) = 0.9370 for x = 45.18 days Don't forget to say 'days' in your answer — you need to answer the question in the context in which it was asked. 2 - 6th dt C i + 9 # ^4t + 5h dt C j 1 = (3 t3 – 3t2)i + (2t2 + 5t)j + C When t = 0, r = 0i + 0j, so: 1 0i + 0j = (3 (0)3 – 3(0)2)i + (2(0)2 + 5(0))j + C fi C = 0i + 0j 1 So r = [(3 t3 – 3t2)i + (2t2 + 5t)j] m Equating j-components at position vector (bi + 12j) m: 2t 2 + 5t = 12 2t 2 + 5t – 12 = 0 (2t – 3)(t + 4) = 0 fi t = 1.5 and t = –4 The time can't be negative, so the particle passes through the point (bi + 12j) m at time t = 1.5 s. Binomial Expansion and Partial Fractions Exercise 3.1 — Finding binomial expansions using partial fractions Q1 a) 5 – 12x º A(4 + 3x) + B(1 + 6x) Using the substitution method: When x = – 4 , 5 + 16 = –7B fi B = –3. Use the substitution u = x2 – 2 to find # 4x(x2 – 2)4 dx. = U cos 5° × t 0.958... 19.84 - m = 1.96 fi 19.84 – m = 7.84 fi m = 12 4 Tip: So X ~ N(12, 4²). The Newton-Raphson method The Newton-Raphson method works by finding the tangent to a function at a point x0, and using its x-intercept for the next iteration, x1. tan A - tan B º + 1 tan A tan B cos (A - B) - cos (A + B) Q2 a) cos A sin B ^cos A cos B + sin A sin Bh - ^cos A cos B - sin A sin Bh º cos A sin B 2 sin A 2 sin A sin B º cos A sin B º cos A º 2 tan A 1 b) 2 [cos (A – B) – cos (A + B)] 1 º 2 [(cos A cos B + sin A sin B) – (cos A cos B – sin A sin B)] 1 º 2 (2 sin A sin B) º sin A sin B c) sin (x + 90°) º sin x cos 90° + cos x sin 90° º sin x (0) + cos x (1) º cos x Q1 tan (A – B) º p p Q3 4 sin x cos 3 - 4 cos x sin 3 º cos x fi 2 sin x - 2 3 cos x = cos x fi 2 sin x = (1 + 2 3) cos x 1+2 3 sin x fi cos x = = tan x 2 fi x = –1.99 and 1.15 (2 d.p.) p p tan 6 - tan 4 p p p Q4 a) tan b - 12 l = tan b 6 - 4 l º p p 1 + tan 6 tan 4 1 1 3 1- 3 º 1 º 3+1 1+ 3 Now rationalise the denominator... At time t = 4 , the particle p has position vector r = : 4 i + jD m, relative to the origin. 1 sin (x + 0.245) = 17 fi x + 0.245 = 0.245 and (p – 0.245) = 2.897 fi x = 0 and 2.65 (3 s.f.) Q6 a) 8 cos x + 15 sin x º R cos (x – a) º R cos x cos a + R sin x sin a fi R cos a = 8, R sin a = 15 15 tan a = 8 fi a = 1.08 (3 s.f.) 2 R = 8 + 15 2 = 289 = 17 fi 8 cos x + 15 sin x º 17 cos (x – 1.08). Sometimes you might need to expand out some brackets before you notice it. Q4 10 N 4° The diagram shows a force of magnitude 10 N acting on an object on an inclined plane. These mappings are called functions. The integration by parts formula for definite integrals can be written like this: #a b u dv dx = 6uv @ba dx #a b v du dx dx Example 3 Find the exact value of # 0 p 2 x 4x sin ` 2 j dx . Q5 A local activist is trying to get lots of signatures on his petition, and has just launched a new online campaign. Method 2: Finding the critical region ff This is a one-tailed test and the critical region will be at the lower end of the distribution. You then show that this would mean that something impossible would have to be true, which means that the initial assumption has to be wrong, so the original statement must be true. then the moment is measured in newton-metres, Nm. Example 1 A 2 m long plank is attached to a ship at one end, O. q = tan–1 a 69.681... Q6 A particle moves with velocity u. p 8 1.466 0 x p 4 3p 8 p 2 1 2.718 y b) (i) Using the trapezium rule with 2 intervals, estimate # 0 p 2 e sinx dx to 2 d.p. (ii) Repeat the calculation using 4 intervals. The three graphs on the next page all show normal distributions with the same mean (m), but different variances (s 2). So (1 + 3x)4(1 + x)–3 = 1 – 3x + 6x2 + [12x(1 – 3x)] + 54x2 + ... -^sin 2 x + cos 2 xh dy -1 = = dx sin 2 x sin 2 x sin x 1 ff Linking this back to the question, tan x º cos x and cot x º tan x , cos x so y = sin x = cot x. Give your answer in metres to 1 decimal place. Q3 Hint: Start by finding the acceleration of the truck and the car. a) What is the first non-zero value of t for which the object is moving in an easterly direction? + 1.732...) + 2.291...] 1 = 4 [7.991...] = 2.00 (3 s.f.) The approximation that the trapezium rule gives will either be an overestimate (too big) or an underestimate (too small). Exercise 4.2 — Evaluating cosec, sec and cot 3 –2 – 3 – – 2 2 –1 1 Q1 a) cosec 80° = sin 80° = 1.02 1 b) sec 75° = cos 75° = 3.86 1 c) cot 30° = tan 30° = 1.73 1 d) sec (–70)° = = 2.92 cos ^ - 70°h 1 e) 3 - cot 250° = 3 – tan 250° = 2.64 2 f) 2 cosec 25° = sin 25° = 4.73 1 Q2 a) sec 3 = cos 3 = –1.01 1 b) cot 0.6 = tan 0.6 = 1.46 1 c) cosec 1.8 = sin 1.8 = 1.03 1 d) sec(–1) = = 1.85 cos ^ -1h p 1 e) cosec 8 = p = 2.61 sin 8 p 1 f) 8 + cot 8 = 8 + p = 10.4 tan 8 1 1 g) = 0.487 p = + 1 1 + sec 10 1 p cos 10 1 1 h) = 0.136 p = + 1 6 + cot 5 6 p tan 5 1 1 Q3 a) sec 60° = cos 60° = 1 = 2 a2k 1 1 b) cosec 30° = sin 30° = 1 = 2 a2k 1 1 c) cot 45° = tan 45° = 1 = 1 2 3 p 2 1 1 d) cosec 3 = = = 3 p = 3 3 sin 3 c m 2 1 1 e) sec(–180°) = = = –1 cos ^ - 180°h cos 180° The graph of y = cos x is symmetrical about the y-axis, so cos (–x) = cos x. x2 x3 x = 1 + 8 – 128 + 1024 – ... Example 4 The random variable X ~ N(m, s2). ff Now think about P + 1 — if you divide P + 1 by p1, you get: (P + 1) ÷ p1 = (p1 p2 p3 ... Differentiation of ex, ln x and ax Exercise 3.1 — Differentiating ex Q1 a) y = ef(x), where f(x) = 3x, so f'(x) = 3 dy = f'(x)ef(x) = 3 × e3x = 3e3x dx dy b) = f'(x)ef(x) = 2 × e2x – 5 = 2e2x – 5 dx dy c) = f'(x)ef(x) = 1 × ex + 7 = ex + 7 dx dy d) = f'(x)ef(x) = 3 × e3x + 9 = 3e3x + 9 dx dy e) = f'(x)ef(x) = (–2) × e7 – 2x = –2e7 – 2x dx dy 3 3 f) = f'(x)ef(x) = 3x2 × e x = 3x2e x dx Q2 a) f'(x) = g'(x)eg(x) = (3x2 + 3)e x b) f'(x) = g'(x)eg(x) = (3x2 – 3)e x 3 + 3x 3 - 3x - 5 2x 2 + x c) f(x) = e , so: 2 f'(x) = g'(x)eg(x) = (4x + 1) × e 2x + x = (4x + 1)ex(2x + 1) Q3 a) ex differentiates to ex and e–x differentiates to –e–x so 1 1 f'(x) = 2 × (ex – (–e–x)) = 2 (ex + e–x) 2 b) f(x) = e x + 7x + 12 2 f'(x) = g'(x)eg(x) = (2x + 7)e x + 7x + 12 4 2 4 2 d d ^ 2x h = 2x c) ^e x + 3x h = ^4x 3 + 6xh e x + 3x and 2e 4e dx dx x 4 + 3x 2 3 2x So f'(x) = (4x + 6x)e + 4e dy f(x) 2x 2x Q4 = f'(x)e = 2 × e = 2e dx dy At x = 0, = 2 × e2 × 0 = 2 × 1 = 2 dx y = mx + c fi 1 = (2 × 0) + c fi c = 1 So the equation of the tangent is y = 2x + 1. = 21.8 N (to 3 s.f.) Resolving vertically: T sin 50° = mg T sin 50° 21.78... Q4 Divide 10x3 + 7x2 – 5x + 21 by 2x + 1, stating the quotient and remainder. Given that the resistive forces acting on A remain the same as in part a), and that A does not hit the pulley, how long does it take A to stop after B hits the ground? The shape of a normal distribution ff For example, this histogram shows the distribution of the weights of some hedgehogs. b h f (x) dx . Hypothesis test A method of testing a hypothesis using observed sample data. b) Calculate the magnitude of the resultant of the three forces. 3 ff Using symmetry, P ` Z < - s j = 0.2 From the calculator, F(z) = 0.2 for z = –0.84162... + yn - yn–1 This formula sums the areas of the rectangles which meet f(x) with their left hand corner. It's just like adding zero, and it can get you from algebraic mess to perfectly formed equations. d) Some possible answers are: – The model could be adjusted to use a discretevalued function, since the number of mice is a discrete variable. Q5c) Hint: You just need to sketch the graph of the equation you found in part b). Then you can deal with the two components separately using the suvat equations below. # 4 cos ^ 2x + 7h dx = 2 ln|k sin(2x + 7)|. Find the critical value for a two-tailed significance level of a = 1% and a sample size of n = 9. p –1 Q4 a) sin (–1) = - 2 . a) Find d. Tip: The box is on the point of sliding (i.e. in limiting equilibrium), so F = mR. Q3a) Hint: Evaluate the left-hand and right-hand sides of the equation separately to show they're the same. When you're writing the exact integral, you use d instead of d, and you replace / with # , which are both just ways of writing 'S' (for 'sum'). When modelling real situations, consider which of these assumptions would not be valid. Learning Objectives: ff Be able to differentiate and integrate vectors with respect to time. – The number of germs is a discrete variable (you can't have half of a germ) but the model is a continuous function. Here's a step-by-step guide to using the formula: Tip: You can use this formula with divisors of higher degrees — the degree of the quotient is deg f(x) – deg d(x), and the degree of the remainder has to be less than the degree of the divisor. If your calculator can only do the standard normal distribution (see p.273) you'll have to read p.274 first, then look at Example 1. Q6 a) y = 2 cosec 2x is the graph of y = cosec x stretched 1 horizontally by a factor of 2 and stretched vertically by a factor of 2. ff The horizontal component of the tension in the string acts on the rod to the left, so the horizontal component of the reaction will act to the right to balance this out. Because the gradients are the same, these lines are parallel and can never intersect. x Q6 a) Differentiate y = cos (2x) with respect to x. ff The crucial thing is to write down everything in steps. This minimum occurs when cos2(x – 1.08) = 0 so p cos(x – 1.08) = 0 so x – 1.08 = 2 fi x = 2.65 (3 s.f.). 0 Give your answer to 1 decimal place. KJK1 ONO K O a = KK3 OO , |a| = 1 2 + 3 2 + 2 2 = 14 KK2OO L P KJK3 ONO K O b = KK2OO , |b| = 3 2 + 2 2 + 1 2 = 14 KK1 OO L P JK 2 NO KK OO b – a = KK - 1OO , |b – a| = 2 2 + ^- 1h2 + ^- 1h2 = 6 KK - OO 1 L P Using the cosine rule, 2 2 2 ^ 14 h + ^ 14 h - ^ 6 h 22 11 = 28 = 14 2 # 14 # 14 11 q = cos–1 b 14 l = 38.2132... – 3g sin 30° = 0 Tip: You've already done all the hard work in part a), so in b), you only need to repeat the very last step and resolve parallel to the plane, with the frictional force acting in the opposite direction. ff Let u = x2 and let dv du = sin x. The maximum and minimum points have the same x-coordinates as on the graph of y = sec x, but the y-coordinates have all been increased by 2. Tip: Taking moments about A means you don't have to worry about the reaction force when finding T. ff So rewrite the integration: p p p #0 4 sin 2x cos 2x dx = #0 4 12 sin 4x dx = : 12 a - 14 cos 4x kD 04 = - 18 6cos 0p4 = - 1 `8cos 4p B - 6cos = 1 ^cos 0 - cos ph 4 8 8 1 2 1 = ^1 - ^ - 1hh = = 4 8 8 c) # Tip: Don't forget to 'double the angle' when using these formulas. Equating the coefficients of x3 gives 6 = 2A, so A = 3. This is an example of a divergent staircase diagram: y y = f(x) 2) T hen a horizontal line to y = x. s = –0.02, u = U sin 5°, a = –9.8, t = U cos 5° 1 2 s = ut + 2 at 0.958... The x ≤ 7 means that the domain must be the integers 1-7. Example Find dy if y = sin2 (2x + 1) dx ff Start by setting up the first stage of differentiation with the chain rule, remembering to rewrite the sin2 part to make differentiating easier: y = sin2 (2x + 1) = [sin (2x + 1)]2 fi y = u2, u = sin (2x + 1) dy is easy, so start with that: du dy = 2u = 2 sin (2x + 1) du du ff To find you're going to need the chain rule again, so just set it up dx with u in terms of v instead of y in terms of u. c) Show that the equation 2x3 – x2 – 7x = 0 has a root between x = –2 and x = –1. Putting this into the Newton-Raphson formula gives: 2f (xn) = xn - 5xn 6 xn + 1 = xn 10xn fl(xn) b) Differentiating g(x) gives g'(x) = 3e3x – 8x. fi 23 – m = 2.5802... dx dy dx 1 dy Q6 a) = t , = 6t – 3t2, so = 6t2 – 3t3 dt dx dt dy b) When t = –1, = 9. Try to cancel any common factors before multiplying. So x = 19.5°, 160.5°, 153.4°, 333.4° (1 d.p.) are all solutions. Tip: Check that this is correct by seeing if f –1f(x) = x: f –1f(x) = f –1(x + 7) = (x + 7) – 7 = x So it's correct. = 5.77 m (3 s.f.) opp Q4 a) tan q = , so draw a right-angled triangle with 3 adj as the 'opposite' side and 1 as the 'adjacent' side. This method isn't as slow as it looks because you can use a scientific calculator to get the terms of the series quickly: press '2 =' then '× 1.5 =' to get the second term. The ball's initial velocity is u ms–1. ff The square root looks like the most awkward part of the integral, so try the substitution u = 2x - 3 . Q1 a) (i) v = ro = [(3t2 – 3)i + 2tj] ms–1 # v dt = 9 # ^t b) When the particle's direction of motion is 45° above i, the horizontal and vertical components of the particle's velocity will be equal. Tip: The 'line of action' of the force is just the direction that the force acts in. e 8 + 36 Q11 Hint: Remember, 1 is the same as e–x. Pretty much any time you see the word 'rough', you know you're going to be dealing with it. c) How many solutions are there to the equation f(x) = f –1(x)? y y y = x2 – 2x y = x2 – 2x C3 2 0 C4 4 x 0 y = 2x C3 = # 4 0 2x dx = 6x 04 C4 = = 4 – 0 = 16 2 # 2 4 x ^ x 2 - 2xh dx = : x - x 2D 3 3 4 2 3 3 = a 4 - 4 2 k - a 2 - 2 2 k = 20 3 3 3 20 28 So the area C2 = C3 – C4 = 16 - 3 = 3 ff Now the total area is given by adding the two areas: 4 28 32 C = C1 + C2 = 3 + 3 = 3 Chapter 8 Integration 4 y = 2x 2 206 2 Exercise 7.1 Q1 Find the shaded area in the following diagrams: y 2 a) b) y = 3x + 4 y 3 y=x +4 y = 16 2 x x 1 y c) d) y = —2 y x 4 x y = –5 2 y = –1 – (x – 3) 4 x e) f) y Q1 Hint: If it doesn't look like you've been given enough information, look for any hints on the graph that may tell you what line to use. 6 Answers 405 5 3x 3 = 1 + 2 + 8 (9x2) + a - 16 k(–27x3) + ... R P F W Resolving vertically: R = W P Resolving horizontally: P = F, so using F = mR, m = W 5 a) m = 20 = 0.25 8 b) m = 15 = 0.53 (to 2 d.p.) Q3 a) R 50 F 12g Resolving vertically: R = 12g Find the frictional force, F: F ≤ mR F ≤ 0.5 × 12g F ≤ 58.8 N 50 N isn't big enough to overcome friction — so the particle won't move. In a question, you might be told which rules to use, but it's not guaranteed, so make sure you get used to spotting when the different rules are needed. ff Subsequent iterations give: x2 = 2.8376..., x3 = 3.7214..., x4 = 6.5221... 0 ff n = 3, a = 0 and b = 1.5, so the width of each strip is h = 1.5 - 0 = 0.5. 3 ff This gives x-values of x0 = 0, x1 = 0.5, x2 = 1 and x3 = 1.5. ff Calculate the value of y for each of x0, x1, x2, x3: Tip: The graph shows that the estimate here is less than the actual value of the integral — there's a gap between the curve and the top of each strip. b) Calculate the magnitude of the normal reaction of the plane on the particle. 91x 5827x 2 = 1 + 6 + 72 + ... Example 1 The times taken by runners to finish a 10 km race are normally distributed. Note that the 3x2 – 9x has been factorised first. Q = 61.063... + 13 = 7.6482... It acts perpendicular to the surface. The ladder has mass 4.5 kg and length 5x m. 2 1 p p y = mx + c fi 0 = b 2 # 4 l + c fi c = – 8 So the equation of the normal is 1 p y = 2 x – 8 (or 4x – 8y – p = 0) dy 1 dx 1 Q14 a) = 2 cos (2y), = 2 cos (2y) = 2 sec (2y) dx dy 3 p dy 1 At the point c 2 , 6 m, = p =1 dx 2 cos 3 3 3 p p y = mx + c fi 6 = 2 + c fi c = 6 – 2 3 p So the equation of the tangent is y = x + 6 – 2 . Using the rules on p.270, you know that 97.5% of values are less than 2 × s above the mean (and 2 × 8 + 766 = 782). ff subtracting the mean (m), and then ff dividing by the standard deviation (s). ff Substitute in the limits of integration to find the area. a) Verify that a normal approximation is appropriate, and specify the distribution of Y. Q5 a) Resolving vertically, taking down as positive: s = 40, u = 0, a = 9.8, t = t 1 s = ut + 2 at2 40 = 4.9t2 t2 = 8.163... X - 42 So under H0, Z = ~ N (0, 1) . ...of this function. y i) y = |x – 3| y = |2x + 3| –1.5 3 x The shaded region shows the values of x that satisfy the inequality. The critical region is given by a P(Z > z) = 2 = 0.005 or a P(Z < –z) = 2 = 0.005. Non-Uniform Acceleration in 2 Dimensions The constant acceleration equations can only be used in situations where the acceleration in a particular direction isn't changing — e.g. vertical motion due to a constant gravitational force with no resistance. y = 2 cosec 2x y 2 –2 90° 180° 270° x 360° b) The minimum points are at (45°, 2) and (225°, 2). Answers 2. Q3 a) The stone is thrown from a point 2.5 m above the ground, so when it is 6 m above the ground, it is 6 – 2.5 = 3.5 m above its point of projection. The plank is in limiting equilibrium. m Bodies supported along their lengths Rather than leaning against a wall, a rigid body may be held in equilibrium by resting on supports at points along its length. Q1 15 ms–1 50° A projectile is launched from a point on horizontal ground with speed 15 ms–1 at an angle of 50° above the horizontal. Resultant forces and equilibrium In Year 1, you learned about resultant forces and what it means for an object to be in equilibrium: ff The resultant force is the sum of all the forces acting on an object. b) y = x3 fi 3 y = x So f–1(x) = 3 x f(x) has both domain x < 0 and range f(x) < 0, so f–1(x) also has domain x < 0 and range f–1(x) < 0. ff Be able to use integration by parts to integrate functions, including ln x. The question will tell you whether the ground and wall are rough or smooth — this lets you know whether you need to take friction into account in your calculations. dh dh dV = × dt dV dt 3 dV 1 1 = × = 2 × –0.3 = cm s–1 dt dV 10pr 2 pr dh dh - 1 b) r = 6, so = 120p cm s–1. = 3.44 N (3 s.f.) d) F = mR 3.435... fi root is between 2.1 and 2.15. ff Be able to choose upper and lower bounds to show that a root is accurate to a certain number of decimal places. b) Find the horizontal distance from the bottom of the cliff to the point that the bullet hits the sea. There are different ways you can do this. f(b) Tip: The area of a trapezium is given by the formula h ^a + bh. ff Resolving vertically, taking up as positive: s = 3.5, u = 30 sin 25°, a = –9.8, MODELL t=t ING 1 Using s = ut + 2 at2: 1 3.5 = (30 sin 25° × t) + (2 × –9.8 × t2) 4.9t2 – (12.67...)t + 3.5 = 0 Using the quadratic formula: 12.67... Remember cos q = cos(360° – q). Using Pythagoras' theorem: speed = ^- bh2 + b 2 = 5 ms–1 fi 2b 2 = 5 2b2 = 52 5 2 52 5 b2 = 2 fi b = ± =± 2 2 You know you want the positive root to give a north-west 5 2 5 2 direction, so initial velocity u = - 2 i + 2 j At t = 10, the particle is moving south, so final velocity v = – 2 j. State the domain and the range of the inverse. 14 - m So s = 1.8494... 3 x dx = # cos x dx - # cos x^sin xh 2 dx ff If f(x) = sin x, then cos x(sin x)2 = f'(x) × [f(x)]2. Then H0: m = 300 and H1: m > 300. Substituting this and 2 into 1 : T cos 15° + 0.1(–T sin 15° + 20g cos 29°) = 20g sin 29° 20g sin 29° - 2g cos 29° 77.880... Arc The curved edge of a sector of a circle. At time t seconds, the acceleration of B is given by aB = (6ti + 6tj) ms–2. Example 4 Sketch the graph of the inverse of the function f(x) = x2 – 8 with domain x ≥ 0. 20.2 310 Chapter 14 Kinematics Example 3 An ice hockey puck is modelled as a particle moving with MODELLING constant acceleration, a, across a horizontal surface. This crosses the x-axis at y = 0, so 0 = x – 8 fi x = 8 and so the coordinates are (8, 0). 4^ 4 - 1h 4^4 - 1h^4 - 2h 3 (1 – x)4 = 1 + 4(–x) + 1 # 2 (–x)2 + 1 # 2 # 3 (–x) 4^4 - 1h^4 - 2h^4 - 3h + (–x)4 1#2#3#4 = 1 – 4x + 6x2 – 4x3 + x4 6^6 - 1h (1 + 3x)6 = 1 + 6(3x) + 1 # 2 (3x)2 + ... f(x) = 2(2x – 5)–1 – 3(3x + 1)–1 + 2(3x + 1)–2 2 ≈ 2b - 1 - 2x - 4x l – 3(1 – 3x + 9x2) 5 25 125 + 2(1 – 6x + 27x2) 2 2 4 x 8 x =- –3+2– + 9x – 12x – – 27x2 + 54x2 5 25 125 2 = - 7 – 79x – 3367x 5 125 25 Chapter 7: Differentiation 1. Collinear points Three or more points are collinear if they all lie on the same straight line. Q4 A 3D printer is being used to make the plastic toy sketched below. Q3 Hint: See p.277 for a reminder of how to find the mean of a normal distribution. ff This could also be written as: 2x 3 - 7x 2 - 16x + 11 6 = 2x2 + 3x – 1 + x-5 x 5 Exercise 1.4 Q1 Use the formula f(x) º q(x)d(x) + r(x) to divide the following expressions. ff Let x be the distance of the centre of mass of the rod from A. ff Transform the probability you're given for X into a probability for Z. Fnet = ma ff Newton's third law: For two bodies, A and B, in contact with each other, the force exerted by A on B is equal in magnitude but opposite in direction to the force exerted by B on A. Q4 3.5 N 80° A B 71° 95 N B 1.8g A non-uniform rod, AB, of mass 1.8 kg and length 7 m, is freely hinged to a vertical wall at A. So the critical region is Z < –2.575 or Z > 2.575. one-tailed 9 0.4716 0.5822 0.6664 0.7498 0.7977 10 0.4428 0.5494 0.6319 0.7155 0.7646 11 0.4187 0.5214 0.6021 0.6851 0.7348 ff Write your conclusion — you will either reject the null hypothesis H0 or have insufficient evidence to do so. ff Degree — the highest power of x in the polynomial. Convex curve A curve with a positive second derivative — i.e. f''(x) > 0 for all x. Divergence A sequence diverges if the terms get further and further apart. y0 In this example, the rectangles are below the curve, so they calculate a lower bound. ff P(C Ç W') = 0.42 and P(W') = 1 – 0.4 = 0.6 P^C + Wlh 0.42 ff So, P(C|W') = = 0.6 = 0.7 P^ Wlh ff So P(Vikram catches the bus, given that he runs to the bus stop) = 0.7 Exercise 1.1 Q1 Given that P(A) = 0.3, P(B) = 0.5 and P(A È B) = 0.7, find: a) P(A') b) P(A Ç B) c) P(A Ç B') Q2 The events X and Y are independent, where P(X) = 0.8 and P(Y) = 0.15. First identify u and v and differentiate them separately: du u = sin x fi = cos x dx dv and v = 2x + 1 fi =2 dx ff Then just put the correct bits into the quotient rule. N ≤ K ≤ 26.73... ff The maximum possible magnitude of the frictional force depends on the coefficient of friction, m, between the surface and the object. Answers Exercise 8.2 — Applying the R addition formulas Q1 a) 5 cos q – 12 sin q º R cos (q + a) º R cos q cos a – R sin q sin a fi R cos a = 5, R sin a = 12 12 tan a = 5 fi a = 67.4° (1 d.p.) R = 5 2 + 12 2 = 169 = 13 fi 5 cos q – 12 sin q º 13 cos (q + 67.4°) b) 13 cos (q + 67.4°) = 4, in the interval 67.4° ≤ (q + 67.4°) ≤ 427.4° 4 cos (q + 67.4°) = 13 4 fi q + 67.4° = cos–1 13 = 72.1° and (360° – 72.1°) = 287.9° fi q = (72.1° – 67.4°) and (287.9° – 67.4°) = 4.7° and 220.5° (1 d.p.) c) The maximum and minimum values of cos q are at ±1. Cartesian coordinates are given in the form (x, y) in 2D (or (x, y, z) in 3D). Example 2 A restaurant has several vegetarian meal options on its menu. MODELLING Show that counters placed at coordinates (1, 0, 3), (3, 1, 2) and (7, 3, 0) lie on a straight line. Q7 Find f'(x) if f(x) = e–2x cosec (4x). The ground is rough, so there is a frictional force between it and the ladder. Quotient: 2x2 + 9x + 10 Remainder: 3 c) 6x3 + x2 – 11x – 5 º (Ax2 + Bx + C)(2x + 1) + D 1 6 1 11 Set x = - 2 : - 8 + 4 + 2 – 5 = D, so D = 0. 1 , x > 3. 76.01... Then H0: m = 32 and H1: m > 32. Transformations of graphs The four transformations The transformations you've met before are translations (a vertical or horizontal shift), stretches (either vertical or horizontal) and reflections in the x- or y- axis. Tip: The alternative hypothesis is r < 0, so you need to make the critical value from the table negative, since it's negative correlation you're expecting to see. dx There's an example of this in part c). 3 Q3 f : x Æ + , x > – 2 and g : x Æ 2x, x d R . ^ x < 1, n ! R h 2! r! Geometric Series Sn = a (1 - r n) 1-r S3 = Hypothesis Test for the Mean of a Normal Distribution a for | r | < 1 1-r If X ~ N(m, s2), then: 2 X-m ~ N(0, 1) X ~ N bm, sn l and s n Arithmetic Series 1 1 Sn = 2 n (a + l) = 2 n 62a + (n - 1) d @ The Binomial Distribution n n-x If X ~ B(n, p), then P(X = x) = d n p x _1 - pi x Probability P(A È B) = P(A) + P(B) − P(A Ç B) P(A Ç B) = P(A)P(B|A) Mean of X = np Variance of X = np(1 – p) Kinematics Unbeatable companions to the AQA and OCR courses — from day one until the final exams! cgpbooks.co.uk • • • Motion in a straight line v = u + at 1 s = ut + 2 at 2 1 s = 2 (u + v)t 1 s = vt – 2 at 2 v 2 = u 2 + 2as Motion in two dimensions v = u + at 1 s = ut + 2 at 2 1 s = 2 (u + v)t 1 s = vt – 2 at 2 0517 - 16015 AQA P CG P G C P G C ...but don't worry — CGP has come to the rescue with this amazing Student Book! • Every single Year 2 Maths topic is in here... Pure, Mechanics and Stats — sorted! • The clearest study notes anywhere... b) A third force, of (–6i + 2j) N, begins to act on the particle. So the stone is at least 6 m above the ground for 0.719 s (3 s.f.) b) Resolving vertically, taking up as positive: s = –2.5, u = 9, a = –9.8, t = t 1 s = ut + 2 at2 –2.5 = 9t – 4.9t2 fi 4.9t2 – 9t – 2.5 = 0 Using the quadratic formula: 9 ! ^- 9h2 - ^4 # 4.9 # - 2.5h t= 9.8 fi t = 2.081... 5 5 #3 2 8^2x - 4h4 dx = 54 6^2x - 322 2 5 5 = 4 a:a2 a 5 k - 4 k D - :a2 a 3 k - 4 k Dk 5 2 2 = 4 ^6^1 - 6^- 5 = 4 ^ - 6- = 8 5 5 Q3 Answers # 0 1 ^6x + 1h 3 dx = : - 1 ^ + h-2 1 6x 1 D 6×-2 0 = - 1 6^6x + 10 12 = - 1 ^6^7 - 6^1 = - 1 a 1 - 1 k = 4 49 12 12 49 Q4 You've been given that f'(x) = (8 – 7x)4 and you need to find f(x), so integrate with respect to x. Q2 The personal best times taken by athletes at a sports club to run 400 m are known to follow a normal distribution with a mean of 51 seconds and a standard deviation of 2.1 seconds. ff Be able to solve integration problems with functions of the form (ax + b)n. Tip: Remember cos2 x is another way of writing (cos x)2. The other method that you need to know is proof by contradiction. ff So the sequence diverges. q = tan–1 b 0.3410... Taking moments about D: 3g × 1.5 = 9g × 9x = 4.5 fi x = 0.5 So the distance from B to the midpoint is 1.5 + 0.5 = 2 m, and so the length of the rod is 2×2=4m Q4 a) TB TC 1m A 1.5 m B C 2.5 m 8g Taking moments about C: 8g × 1.5 = 4TB TB = 3g = 29.4 N Resolving vertically: TC + TB = 8g TC = 78.4 – 29.4 = 49 N b) When the beam is about to tilt about C, TB = 0. ff The mass of a uniform beam is spread evenly along the length of the beam, and so the centre of mass is at its midpoint: Midpoint, COM l l mg Tip: Up until now, all the beams have been light, so you haven't needed to worry about their weights. Trajectory The path followed by a projectile. PROBLE M SOLVING Q8 A sledge of mass 20 kg is at rest on a rough slope inclined at 29° to the horizontal. PROBLE M SOLVING Q8 A block of mass 5 kg is at rest on a rough plane inclined at 35° to the horizontal. Find: a) the magnitude of the tension, T, in the string, b) the mass, m, of the particle in kg. dx 2 1 1 1 The graph of y = 10 x5 – 3 x3 + 2 x + 4 has stationary 56 64 points at b - 1, 15 l and b1, 15 l, and both stationary points are points of inflection. Cot The reciprocal of the tangent function, sometimes written as cotangent. ÷ 39.832... = 0.341 N (to 3 s.f.) 13 N ff Use the components to form a right-angled triangle. Q2 a) Range: f(x) ≥ –2 f(x) x –1 h(x) is undefined when x is less than –1. ff This is the reason you probably won't be asked to both rearrange and use a formula to find a root without prompting. A wire with tension of magnitude 86 N is fixed to B, at an angle of 10° to the plank, keeping the plank horizontal. p Q4 Find f'(x) if f(x) = sin b x 2 + 3 l. b) y = |4x – 1| y y = |2x + 3| 8 y = |x| x –8 3 The shaded region shows the values of x that satisfy the inequality. t = 1.8 ff Resolving vertically, taking up as positive: s = sy, u = 18 sin 40°, a = –9.8, t = 1.8 1 Using s = ut + 2 at2: 1 sy = (18 sin 40° × 1.8) + (2 × –9.8 × 1.82) = 4.9503... 1 ff n is large, and p is close to 2 , so a normal approximation is suitable. du f'(u) dx f(u) # 4xe 2x 2 2 dx = e 2x + C Example 2 Find the following integrals: a) Tip: You might need to take a constant outside of the integral to get it in du the form # f'(u) dx. T – (0.1 × 5000 cos 10°) – 5000 sin 10° = 178.57... So the critical region is Z > 1.645. After 2 folds, thickness = 0.01 × 22 cm After n folds, thickness = 0.01 × 2n cm Distance to the moon: 384 000 km = 3.84 × 105 km = (3.84 × 105 × 1000) m = (3.84 × 105 × 1000 × 100) cm = 3.84 × 1010 cm Therefore the paper reaches the moon when: 0.01 × 2n = 3.84 × 1010 2n = 3.84 × 1012. 3 1 1 1 At x = 2 , y = 2e2x – 2 e3 – 4x = 2e1 – 2 e1 = 2 e. N R q 9.842... b) So solve for 17 cos (x – 1.08) = 5 in the interval –1.08 ≤ (x – 1.08) ≤ 5.20. p a) Find y when x = 2 and when x = p, 0 ≤ y ≤ p. ff Now just neaten it up: dy (2x + 1) cos x - 2 sin x = dx (2 x + 1) 2 Example 2 2x 2 - 1 Find the gradient of the tangent to the curve with equation y = 3x 2 + 1 at the point (1, 0.25). Example 3 Tip: You have to use the laws of logs in this example — take a look back at your Year 1 notes if you need a reminder. 336 Chapter 15 Dynamics ff Resolving forces horizontally (Æ), Fnet = 7 cos 25° Using Newton's second law to find the acceleration, a: Fnet = ma 18 7 cos 25° = g a 6.344... tan ` 2 - 4 j 0 Put in the limits. Tip: The body is accelerating down the plane, so the frictional force acts up the slope. When t = 3.5, vA = [(3(3.5)2 – 2(3.5) – 4)i + (3(3.5)2 – 4(3.5) + 3)j] = (25.75i + 25.75j) ms–1 Both components are positive, so the particle is moving along a path 45° above i when t = 3.5 s. The ball passes through a target 3 seconds after being fired, and then passes through a second target 4 seconds after being fired. = 32.99... ff However, you need to be careful with your conclusion. N 90.432... The beam is held in equilibrium by a light, inextensible wire with tension T attached at M. = 6.39 m (3 s.f.) Example 4 A particle is projected from a point y m above flat horizontal ground. N R = (5.069...) 2 + (-1.192...) 2 = 5.21 N (to 3 s.f.) –1 a - 1.192... Q6 The diagram shows a 2 kg block held at rest on a smooth inclined plane by a horizontal force of 14.7 N. When you're using i and j vectors to describe projectile motion, you can either consider horizontal and vertical motion separately, as before, or you can deal with both components in one go using the suvat equations in vector form: v = u + at s = 12 (u + v)t 1 s = ut + 2 at2 s = vt – 12 at2 Tip: Remember, i and j are a pair of perpendicular vectors, each of magnitude one unit. H Hypothesis A statement or claim that you want to test. The system is released from rest and A begins to move down the slope and B begins to move up the slope. — finding the critical value(s) of the test statistic and seeing if your observed value lies in the critical region. ff Resolving vertically: RV + T sin 45° = 2g RV = 4.9 N ff Now use Pythagoras' theorem to find R, the magnitude of the reaction at A: |R| = RH2 + RV2 = 14.7 2 + 4.9 2 R RV = 4.9 N RH = 14.7 N = 15.5 N (3 s.f.) Example 2 6m A B 70° 16 N 0.1g 2N A non-uniform rod, AB, of length 6 m and mass 0.1 kg, is freely hinged on a vertical wall. Throughout the question the units used for length are metres, and the units for time are seconds, dx so must be in units dt –1 of m s . One of the players is selected at random. dx Q1 a) 412 Answers 4 y2 d) x = y = 4y–1 + y dy dx = –4y–2 + 1 fi = dx dy 2 1 = y 4 y2 - 4 1- 2 y dy 4 At (5, 4), y = 4 so = 3. Example 1 The population of rabbits in a park is decreasing as winter approaches. Sketching graphs to find approximate roots Sometimes it's easier to find the number of roots and roughly where they are if you sketch the graphs first. = 98 – 10a a = 4.385... f(x) So you want to find P(X < 780). = 5.27 m (3 s.f.) b) Resolving vertically, taking down as positive: s = 16, u = 3 sin 7°, v = vy , a = 9.8 v2 = u2 + 2as vy2 = (3 sin 7°)2 + (2 × 9.8 × 16) = 313.733... Tip: There are a few steps to part c) — first you have to find the general solution, then sub in the values given to find the particular solution, then use this solution to answer the question. X - 32 So under H0, Z = ~N (0, 1). x2 goes off the scale of the graph so you can't label it. Sketching graphs was covered in Year 1 if you need a reminder. 2h 7y0 + 2y1 + y2 A = 12 6 2 + 2 3 + 4 @ = 1 61.4142 + 3.4641 + 2 @ = 3.44 (3 s.f.) 2 b) h = (3 – 1) ÷ 4 = 0.5, so the x-values are 1, 1.5, 2, 2.5, 3. ff Start by transforming the probability you're given for X into a probability for Z. dx 2 a) x = t , y = t3 – t b) x = t3 + t, y = 2t2 + 1 c) x = t4, y = t3 – t2 d) x = cos t, y = 4t – t2 Q2 The curve C is defined by the parametric equations x = t 2, y = e2t. Use the fact that the cos graph repeats itself every 2p. 4 3 x The graph shows that there are two solutions to 4 3x – 4 ≥ 0 for x ≥ 3 |3x – 4| = 1. 1 x y= 2 x The graphs intersect once, so there is one real root of the x equation cos–1 x = 2 . Show that tan q = 0.75. N 15 ff Vertical component = 15 sin 30° = 7.5 N upwards ff Horizontal component = 15 cos 30° 30° 15 cos 30° = 13.0 N (to 3 s.f.) to the right 320 Chapter 15 Dynamics 15 sin 30° A particle is acted on by a force of 15 N at 30° above the positive horizontal. dt t 1 b) Given that the volume of the cube is initially 15 cm3, find the length of time, to 3 s.f., for it to reach a volume of 18 cm3. Quotient Rule Learning Objectives: ff Be able to use the quotient rule for differentiation and understand when it's needed. From the method above, this means that –b ≤ x – a ≤ b, so adding a to each bit of the inequality gives a – b ≤ x ≤ a + b. arctan x - 1 1 - x2 1 1 + x2 Most implicit differentiation questions aren't really that different from any other differentiation question. Is there anything about the way that the experiment is being carried out that could affect the outcome? p Find the equation of the normal to the particle's path at q = 2 . ff You need two sets of branches — one for event M and one for event N. dy d) First part: y = ef(x), f(x) = 2 cos (2x) fi = f'(x)ef(x) dx f(x) = 2 cos (2x) fi f'(x) = –4 sin (2x) dy So = –4 sin (2x)e2 cos (2x) dx Second part: y = cos2 (2x) = (cos (2x))2, so let y = u2 where u = cos (2x) dy = 2u = 2 cos (2x) du du For , set up the chain rule again: dx du u = cos (2x) fi –2 sin (2x) dx dy dy du fi = × = –4 sin (2x) cos (2x) dx du dx Putting it all together: dy = –4 sin (2x) e2 cos (2x) – 4 sin (2x) cos (2x) dx For 5. N If the ladder is on the point of slipping towards the wall: N C F Let l be the length of the beam. 2 4.9a U cos 5° k = (0.958... dx dv x ff Let u = 4x and = sin ` 2 j du x = 4x fi du dv x x =4 = sin ` 2 j fi v = –2cos ` 2 j dx dx ff Substitute everything into the formula and complete the integration: Tip: Go back to p.186 if you want a reminder about integrating trig functions. Combine into one fraction ff Once everything's over the common denominator you can just add (or subtract) the numerators. ff Resolving perpendicular to the spanner, Fy = 20 sin 60°. So 3x4 – 8x3 – 6x – 4 º (3x3 + x2 + 3x + 3)(x – 3) + 5 or (3x4 – 8x3 – 6x – 4) ÷ (x – 3) = 3x3 + x2 + 3x + 3 remainder 5 Q3 a) x2– 15x + 21 r –10 x + 1 x3– 14x2 + 6x + 11) - (x3 + x2) –15x2+ 6x – (–15x2– 15x) 21x+ 11 – (21x+ 21) –10 Quotient: x2 – 15x + 21, remainder: –10 b) x2+ 7x – 6 r 5 x + 3 x3+ 10x2 + 15x – 13 – (x3+ 3x2) 7x2 + 15x – (7x2 + 21x) –6x– 13 – (–6x– 18) 5 Quotient: x2 + 7x – 6, remainder: 5 c) 2x2+ 9x + 10 r 3 x – 2 2x3+ 5x2– 8x – 17 – (2x3– 4x2) 9x2– 8x – (9x2– 18x) 10x– 17 – (10x– 20) 3 Quotient: 2x2 + 9x + 10, remainder: 3 d) 3x2– 15x – 3 r 24 x + 5 3x3+ 0x2 – 78x + 9 – (3x3+ 15x2) –15x2 – 78x – (–15x2 – 75x) –3x+ 9 – (–3x– 15) 24 Quotient: 3x2 – 15x – 3, remainder: 24 e) x3 + x2 + x + 1)))) x – 1 x4 + 0x3 + 0x2 + 0x – 1 – (x4 – x3) x3 + 0x2 – (x3 – x2) x2 + 0x – (x2 – x) x – 1 – (x– 1) 0 Quotient: x3 + x2 + x + 1, remainder: 0 f) 4x2+ 3x + 5 r 25 2x – 3 8x3– 6x2 + x – 10 – (8x3– 12x2) 6x2 + x – (6x2 – 9x) 10x+ 10 – (10x– 15) 25 Quotient: 4x2 + 3x + 5, remainder: 25 Q4 5x2 + x – 3 r 24 2x + 1 10x3+ 7x2 – 5x + 21 – (10x3+ 5x2) 2x2 – 5x – (2x2 + x) –6x+ 21 – (–6x– 3) 24 Quotient: 5x2 + x – 3, remainder: 24) Q5 8x3 + 12x2 + 18x + 27 r 81 2x – 3 16x4 + 0x3 + 0x2 + 0x + 0 – (16x4– 24x3) 24x3 + 0x2 – (24x3 – 36x2) 36x2 + 0x – (36x2 – 54x) 54x + 0 – (54x– 81) 81 Quotient: 8x3 + 12x2 + 18x + 27, remainder: 81 Q6 3x3+ 13x2 + 4x)) x – 2 3x4 + 7x3 – 22x2 – 8x – (3x4– 6x3) 13x3 – 22x2 – (13x3 – 26x2) 4x2 – 8x – (4x2 – 8x) 0 So 3x4 + 7x3 – 22x2 – 8x = (3x3 + 13x2 + 4x)(x – 2) = x(3x2 + 13x + 4)(x – 2) = x(3x + 1)(x + 4)(x – 2) 1 and the solutions are: x = 0, x = - 3 , x = –4 and x = 2 You could have taken out a factor of x at the start, although it wouldn't affect the method too much. So: 6 # 2x 2 - 5x + 2 dx = 2 ln ^ x - 2h - 2 ln ^2x - 1h + C 2 = 2 ln a x 2 k + C = ln :a x 2 k D + C 2x - 1 2x - 1 c) Using part b) 2 5 5 #3 2x 2 - 65x + 2 dx = :ln :a 2xx--21 k DD3 2 - 2 = ln :a 5 2 k D - ln :a 3 2 k D 10 - 6 1-1 = ln 1 - ln 1 = ln 25 9 25 9 Q4 First express the fraction as an identity: + f (x) A+ B = 3x 5 g (x) x^ x + 10h /x x + 10 fi 3x + 5 º A(x + 10) + Bx Now use the equating coefficients method: 1 Equating constant terms: 10A = 5 fi A = 2 1 Equating x coefficients: A+B=3 fi 2 +B=3 5 fi B= 2 3x + 5 1 + 5 So / x^ x + 10h 2x 2^ x + 10h Now the integration can be expressed: 2 2 2 5 1 5 #1 gf((xx)) dx = #1 x^3xx++10 d x = # 2x + dx h 1 2^ x + 10h 2 = : 1 ln x + 5 ln x + 10 D 2 2 1 1 5 = : ln 2 + ln 2 + 10 D - : 1 ln 1 + 5 ln 1 + 10 D 2 2 2 1 5 5 = : ln 2 + ln 12 D - :0 + ln 11 D 2 2 2 1 5 5 = ln 2 + ln 12 - ln 11 = 1 aln 2 + 5 ln 12 k 2 2 2 2 11 5 1 12 = aln 2 + ln a k k = 0.564 (3 d.p.) 2 11 Q5 Begin by writing the function as partial fractions. This method gives us a general result for integrating all functions of the form (ax + b)n. The system is released from rest and the boxes begin to accelerate 10g at a rate of 0.5 ms–2. The upper bound is the upper limit of the values which will be rounded down to the correct answer. Q5 A uniform ladder of mass 9 kg and length 4.8 m rests in limiting equilibrium with one end on rough, horizontal ground and the other end against a rough, vertical wall. a ff Therefore 2 cannot be written as a fraction b , so it is irrational. Cartesian equation An equation relating the perpendicular axes x and y in 2D (or x, y and z in 3D). ff You'll need to take out a constant to get the integral you want. The coefficient of friction between the ground and the ladder is 0.3, and the ladder makes an angle of 65° with the ground. Normal Approximation to a Binomial Distribution 3. — equation 1 Tip: Try not to round the numbers in any of the intermediate steps of a calculation, otherwise your final answer might not be correct. e) Describe the behaviour of this sequence. 1 3x - 7 , so let y = u 3 where u = 3x – 7 dy 1 2 du 1 fi = 3 u- 3 = =3 2 , du 3^3 3x - 7 h dx dy dy du 1 1 = × = 2 × 3 = 2 dx du dx ^3 3x - 7 h 3^3 3x - 7 h dy 1 1 3 When x = 5, = 3 2 = 4 , y = 15 7 = 2 dx ^ (3 # 5) - 7 h -1 Gradient of normal = 1 = –4 4 y = mx + c fi 2 = (–4 × 5) + c fi c = 22 So the equation of the normal is y = 22 – 4x. Find the vectors JK , KL and LJ . Sometimes you might need to use the following trick to find the probability that X takes a value greater than (or greater than or equal to) x. y = f(ax) Tip: Translations can also be given as column vectors. = 2.39 m (3 s.f.) ^4.791...h2 + ^2.385...h2 = 5.35 m (3 s.f.) c) Q3 a) Resolving vertically, taking up as positive: s = 0, u = 8 sin 59°, a = –9.8, t = t 1 s = ut + 2 at2 0 = (8 sin 59°)t – 4.9t2 0 = t(8 sin 59° – 4.9t) fi t = 0 or (8 sin 59° – 4.9t) = 0 fi t = 8 sin 59° ÷ 4.9 = 1.399... Therefore, by exhaustion, x2 cannot be a multiple of 3, which contradicts the initial assumption. There is a sign change between the upper and lower bounds (and the function is continuous in this interval), so the root to 3 d.p. is x = 3.577. At time t seconds, its position in the plane is given by r = [(t 2 – 1)i + (2 + 5t)j] m relative to a fixed origin O. KK OO KK 4 OO 9 L P L P Q3 Triangle JKL has vertices at the points J (4, 0, –3), K (–1, 3, 0) and L (2, 2, 7). ff Be able to interpret 1 y For a > 0: ff f(x + a) is f(x) translated a left, Learning Objectives: 360° – a 360° 180° – a 180° transformed graphs, including finding coordinates of points. It's fairly simple to use, but can get tricky when put together with other rules. d2 y Q5 When y = xex is concave, is negative. ff Or you can write this the other way around, by swapping A and B: P(A Ç B) = P(B)P(A|B) Chapter 11 Probability 259 Example 1 For events A and B: P(A) = 0.6, P(B) = 0.5, P(A Ç B) = 0.3, P(B'|A) = 0.5. a) Find P(A|B). 1.2 m O ff Resolving vertically, taking up as positive: s = –1.2, u = q, a = –9.8, t = 4 1 Using s = ut + 2 at2: –1.2 = 4q + (–4.9 × 42) fi 4q = 77.2 fi q = 19.3 ff So the stone's initial velocity is e 38.6 o ms–1. y 2 y=x y = 2x x 3 y=x y = 4x x Q2 For each part, find the area enclosed by the curve and line: a) y = x2 + 4 and y = x + 4 b) y = 3x2 + 11x + 6 and y = 9x + 6 c) y = x2 + 2x – 3 and y = 4x Q3 Find the shaded area shown to the right: 1 y 0.5 0 –1 2 y = cos x Q4 The area between the graphs of y = x2 and y = ax is 36, where a is a constant and a > 0. f(z) Area = 0.025 0 19.84 – 4 z ff Remember, your calculator finds the z-value for P(Z ≤ z). fi mR = N fi N = 0.3 × 60g = 176.4 N Let x be the girl's distance up the ladder from the base. What's new in Year 2 is conditional probability, which is the probability of an event happening, given that another event happens. Find the possible values of t. b) Find the horizontal component of the force. c) vB = #a B dt = a # 6t dt k i + a # 6t dt k j = 3t2i + 3t2j + C When t = 1, vB = (4i – j), so: 4i – j = 3(1)2i + 3(1)2j + C 4i – j = 3i + 3j + C fi C = i – 4j So vB = [(3t2 + 1)i + (3t2 – 4)j] ms–1 rB = #v B dt = 9 # ^3t 2 + 1h dt C i + 9 # ^3t 2 - 4h dt C j = (t + t)i + (t3 – 4t)j + D 3 Answers 465 When t = 1, rB = (2i + 3j), so: 2i + 3j = (13 + 1)i + (13 – 4(1))j + D 2i + 3j = 2i – 3j + D fi D = 6j So rB = [(t3 + t)i + (t3 – 4t + 6)j] m d) rB – rA = [(t3 + t)i + (t3 – 4t + 6)j] – [(t3 – 2t2 – 4t + 3)i + (t3 – 2t2 + 3t – 7)j] = [(t2 + 5t – 3)i + (2t2 – 7t + 13)j] m e) When t = 4, rB – rA = (42 + 5(4) – 3)i + (2(4)2 – 7(4) + 13)j = (33i + 17j) m Using Pythagoras' theorem: distance = 33 2 + 17 2 = 37.1 m (3 s.f.) dy 1 dx = - 3 Q7 a) = t , and , dt dt t4 3 dy dy dx 1 - 3 = = ' 4 =- t ' so t 3 dx dt dt t 1 1 dy - 1 When t = 2 , = = - 24 dx 3 × 23 1 dr dx + dy 3 b) r = xi + yj fi v = = i j=- 4i+ t j dt dt dt t 2 12 1 dv d 2 x + d y fi a= = 2i j= 5 i- 2j dt t dt dt 2 t 1 So when t = 2 , 12 1 a = 1 5 i - 1 2 j = 384i – 4j (as required) b l b l 2 2 Q8 Use double angle identities to change the components of v into expressions that are easier to integrate: cos 2t º 1 – 2 sin2 t fi 2 sin2 t º 1 – cos 2t sin 2t º 2 sin t cos t fi 4 sin 2t cos 2t º 2 sin 4t fi v = (1 – cos 2t)i + (2 sin 4t)j r= # v dt = 9 # ^1 - cos 2th dt C i + 9 #]2 sin 4tg dt C j 1 1 = bt - 2 sin 2t li – b 2 cos 4t l j + C p p When t = 4 , r = 4 i + j, so: p l b1 p bp - 1 l 4 i + j = 4 2 sin 2 i - 2 cos p j + C p 1 1 1 1 p b - l 4i + j = 4 2 i + 2j + C fi C = 2i + 2j 1 1 1 1 r = bt - 2 sin 2t + 2 li – b 2 cos 4t + 2 l j 1 1 = 2 (2t – sin 2t + 1)i – 2 (cos 4t – 1)j 1 1 1 So when t = 0, r = 2 (0 – 0 + 1)i – 2 (1 – 1)j = 2 i, 1 so distance from the origin = 2 m. Chapter 16 Moments 367 Q6 Robert holds an 8 m uniform ladder in place against a smooth vertical wall by applying a horizontal force of K N to it, 1 m from the base of the ladder. fi v = 0.431 ms–1 (3 s.f.) Chapter 16: Moments + B 9g Resolving vertically (Ø) for B: Fnet = ma 9g – T = 9 × 0.4 fi T = 84.6 N Resolving vertically (≠) for A: R + 60 sin 40°= 8g fi R = 39.832... = 1 + 5(–5x) + 10(25x2)+ ... = 44.160... Resolving vertically, taking up as positive: s = 3.5, u = 9, a = –9.8, t = t 1 s = ut + 2 at2 3.5 = 9t – 4.9t2 fi 4.9t2 – 9t + 3.5 = 0 Using the quadratic formula: 2 9 ! ^- 9h - ^4 # 4.9 # 3.5h t= 9. Q3 a) Sketch the graphs of y = 2x – 3 and y = ln x on the same axes. fi dV = 400p dD ff So using the chain rule: dV dV dD 3200 - 1280000p = × = 400p × = dt dD dt D2 D2 1 ff When t = 10, 3 D3 = 72000 – 3200(10) = 40000 fi D = 3 3 × 40000 = 49.324... Q8 a) Form a triangle between the two vectors (call them a and b) with angle q between them. In a random sample of 12 weeks the sample mean, x , was found to be £22.50. dx x sin ` 2 j will give a cos function whether you integrate or differentiate du it, so the only way to get a simpler # v dx is to make u = 4x. k 2 3 x 2 3 x2 = 3 - 3 - 9 - ... ff Let m = mean time (in minutes) since the increase in training sessions. This means this result is not significant at this level. The mean is unknown, so just leave it as m for now. Since any value between them would be rounded to 2.472 to 3 d.p. this answer must be correct. ff Whatever the values of m and s 2, the graph of a normal distribution always looks like the curve below. Identity An equation that is true for all values of a variable, usually denoted by the 'º' sign. Q6 a) h = (4 – 2) ÷ 4 = 0.5, so the x-values are 2, 2.5, 3, 3.5, 4. ff Because of how you defined it, P is a multiple of every prime number. Repeating the process iteratively to find x2, x3, etc. See p.137138 for more on this. etc. 208 Chapter 8 Integration ff Now you've got a much simpler expression to integrate with respect to u: # 2u4 du = 2 5 5u + C ff And finally, substitute u = x2 – 2 back in: 2 = 5 (x2 – 2)5 + C That first example worked out nicely, because the x's cancelled out when you substituted in the expressions for u and dx. x (x - 2) ff Write as an identity with partial fractions: A (x - 2) + Bx 2 A B º x + - º x (x - 2) x (x 2) x 2 fi 2 º A(x – 2) + Bx ff Use the equating coefficients method to find A and B: Tip: When the constant term only contains one of the unknowns, it'll often be easier to compare coefficients than to substitute. Resolving Forces 2. 3 b) To find cos c - 2 m, first find the angle a 3 for which cos a = 2 : Exercise 4.1 — Graphs of cosec, sec and cot Q1 a) y 1 - 3 –2 – – –1 2 2 3 6 ÷3 3 p So cos 6 = 2 : 1 Now use the CAST diagram to find the negative solutions that lie in the domain 0 ≤ x ≤ p: 2 S A cos - ve 6 6 6 6 cos –ve T C 3 2 0 2 3 2 2 x b) The minimum points are at (–2p, 1), (0, 1) and (2p, 1). ff This method might fail to find a particular root. 1 s = ut + 2 at2 s = 6 × 2.081... So the equation of the tangent is 1 + 6 ln 2 a 3 ln 2 k y= – 64 x 64 fi 64y = 1 + 6 ln 2 – (3 ln 2)x. Area of cross-section Length of prism 1 V = a 2 × 7x × 4x k × 4x fi V = 56x3 cm3 ff Differentiate this expression for the volume with respect to x: dV = 168x2 dx ff You know that dx = 0.05 cm min–1. A horizontal force of magnitude 50 N is applied to the particle. 0 -1 d) A translation of 1 left / by the column vector e o 0 and a stretch vertically by a scale factor of 2. A non-uniform wooden plank of mass M kg rests horizontally on supports at A and B, as shown. The Addition Formulas Exercise 6.1 — Finding exact values Q1 a) cos 72° cos 12° + sin 72° sin 12° 1 = cos (72° – 12°) = cos 60° = 2 b) cos 13° cos 17° – sin 13° sin 17° 3 = cos (13° + 17°) = cos 30° = 2 tan 12° + tan 18° c) = tan (12° + 18°) 1 - tan 12° tan 18° 1 = tan 30° = 3 tan 500° - tan 140° d) = tan (500° – 140°) 1 + tan 500° tan 140° = tan 360° = 0 e) sin 35° cos 10° + cos 35° sin 10° = sin (35° + 10°) 1 = sin 45° = 2 f) sin 69° cos 9° – cos 69° sin 9° = sin (69° – 9°) 3 = sin 60° = 2 2p 2p 2p p p p Q2 a) sin 3 cos 2 - cos 3 sin 2 = sin a 3 - 2 k p 1 = sin 6 = 2 b) cos 4p cos 3p + sin 4p sin 3p = cos (4p – 3p) = cos p = –1 5p 5p tan 12 + tan 4 b 5p + 54p l c) 5p 5p = tan 12 1 - tan 12 tan 4 p 5p p = tan 3 = tan b 2p - 3 l = –tan 3 = – 3 Q3 a) sin (5x – 2x) = sin 3x b) cos (4x + 6x) = cos 10x c) tan (7x + 3x) = tan 10x d) 5 sin (2x + 3x) = 5 sin 5x e) 8 cos (7x – 5x) = 8 cos 2x Q4 Before answering a)-d), calculate cos x and sin y: 9 9 7 3 sin x = 4 fi sin2 x = 16 fi cos2 x = 1 – 16 = 16 7 fi cos x = 4 x is acute, meaning cos x must be positive, so take the positive square root. Integration by Parts 214 10. So the root is between –1.5 and –2 f(–1.7) = e(2 × –1.7) + (–1.7)2 – 3 = –0.07... Reaction Forces and Friction Chapter 11 — Probability 298 313 320 331 336 349 358 258 268 Reference Chapter 12 — The Normal Distribution 1. Find the maximum possible tension in the string given that the toy remains stationary when: a) a = 0° b) a = 35° T 8N Q5 12 N A box of mass 2 kg is at rest on a rough plane inclined at an angle of q to the horizontal. 290 Chapter 12 The Normal Distribution Chapter 13 Hypothesis Testing 1. Q12 y = 420 Answers 7. f) f(x) has domain {x : 3 ≤ x ≤ 4}, which will give a range of {f(x) : ln 9 ≤ f(x) ≤ ln 16}. P ≥ 0 as you can't have a negative population. - 3 Q4 The curve y = f(x) goes through the point a1, 35 k and 4 f'(x) = (8 – 7x) . x – 2 ≥ 0 for x ≥ 2 x – 2 < 0 for x < 2 (shaded) So there are two equations to solve: 1 x–2=6 fi x=8 (this is valid as it's in the range x ≥ 2) 2 (shaded) –(x – 2) = 6 fi x = –4 (this is also valid as it's in the range x < 2) So the two solutions are x = 8 and x = –4. fi t = U cos 5° Resolving vertically, taking up as positive: 0.958... Q3 h(t) = 14 + 5 sin t + 5 cos t º 14 + R cos (t – a) º 14 + R cos t cos a + R sin t sin a fi R cos a = 5, R sin a = 5 p tan a = 1 fi a = 4 R = 5 2 + 5 2 = 50 = 5 2 p fi h(t) º 14 + 5 2 cos bt - 4 l The maximum and minimum of cos are 1 and –1, so the maximum of h(t) = 14 + 5 2 = 21.1 m (1 d.p.) and the minimum of h(t) = 14 – 5 2 = 6.9 m (1 d.p.) 7 3 7 Q4 a) H = 10 + 2 sin t – 2 cos t º 10 + R sin (t – a) º 10 + R sin t cos a – R cos t sin a 7 3 7 fi R cos a = 2 , R sin a = 2 396 Answers p tan a = 3 fi a = 3 2 7 3m 7 2 49 + 147 R = b2l +c = 4 4 2 196 = 4 = 49 = 7 p fi H º 10 + 7 sin bt - 3 l p b) Replace bt - 3 l with x to make the working clearer: h = 10 + 7 sin x – cos 2x Write cos 2x in terms of sin x, i.e. 1 – 2 sin2 x h = 10 + 7 sin x – (1 – 2 sin2 x) = 10 + 7 sin x – 1 + 2 sin2 x = 9 + 7 sin x + 2 sin2 x i.e. A = 9 and B = 2. Chapter 8: Integration 1. This is also true for a normally distributed random variable X. Q3 Find the sum of the moments about O, taking clockwise as positive: (13 × 3) + (X × 1) – (21 × 3) = X – 24 For the rod to rotate clockwise, the sum of the moments must be positive: X – 24 > 0 fi X > 24 Q4 Find the sum of the moments about O, taking clockwise as positive: 4(d + 1) + (38 × 2) – (25 × 1) – (24 cos 60° × 5) = 4d – 5 For the rod to rotate anticlockwise, the sum of the moments must be negative: 4d – 5 < 0 fi d < 1.25 Also, d must be greater than or equal to zero, otherwise the 25 N force will be applied to a point which isn't on the rod, and so the rod will rotate clockwise. This means that only one-to-one functions have inverses, as the inverse of a manyto-one function would be one-to-many, which isn't a function. Q5 Find f'(x) if f(x) = sin2 x. Cr The binomial coefficient of xr in the binomial expansion of (1 + x)n. 1 2 3 3 fg(1) = f(2(1)) = f(2) = + = 4 . p b) Hence find the equation of the normal to the curve at this point. Chapter 2 Algebra and Functions 9 Simply stating the identity at the end doesn't always answer the question. tan x ff First, work out what all the substitutions will be. ff The two tangents intersect when: x – p = 1 + p – x fi 2x = 1 + 2p 1 + 2p fi x= 2 ff Putting this value of x into one of the equations gives: 1 + 2p 1 1 y = a 2 k – p = 2 + p – p = 2 1 + 2p 1 ff So they intersect at a 2 , 2 k. Friction will act to try and prevent this motion. Use this information to find D, and hence r: 0i + 2j = 5(5e0 – 0)i + (0 – sin 0)j + D 0i + 2j = 25i – 0j + D fi D = –25i + 2j fi r = (5(5e0.2t – t) – 25)i + (11t – sin t + 2)j = 5(5e0.2t – t – 5)i + (2 + 11t – sin t)j Example 5 A particle's acceleration is connected to its velocity, PROBLE M dy 2 dx = 9 SOLVIN G = . Formula Sheet Contains all the Year 2 formulas you'll be given in the exams. a) Find q. dy dy = ax ln a dx Chapter 7 Differentiation 175 This method of rearranging and using implicit differentiation is also used to differentiate the inverse trig functions. But using the normal distribution (which is continuous) to approximate a discrete distribution is slightly awkward. b) ln 2x + x3 = 0 fi ln 2x = –x3 e- x fi 2x = e - x fi x = 2 e- x c) Using iterative formula xn+1 = 2 with starting value x0 = 0.4: You know the root is between 0.4 and 0.5, so it's a good idea to use one of these as your starting value. Example 2 Tip: Make sure you're comfortable working with vectors in all forms — as column vectors, using i, j, k notation, and as the movement between points given as Cartesian coordinates. # 6x 5 6 e x dx ff Here, u = x6 — it appears once differentiated (6x5) 6 and once within a function (e x). opp 60 a) From the triangle, sin q = = - 61 . Chapter 9 Numerical Methods 249 Exercise 5.1 Q1 Use the trapezium rule to find approximations of each of the following integrals. PQ = – OP + OQ J N J N KKK4OOO KKK2OOO = – KK3 OO + KK5 OO = KK0 OO KK4OO L P L P KJK2 - 4OON KKJ- 2OON KK - OO K O 5 3 = KK 2 OO KKK - OOO KK OO 4 4 0 L P L P y 2 P(4, 3, 0) Tip: Don't get confused between a point and its position vector. N 2.4 m R C B 25 N 10 N 28° A 28° F ff Taking moments about A: 2.4N = (10 cos 28° × 1.65) + (25 cos 28° × 3.3) fi N = 36.421... 1 s = ut + 2 at2 s = 8 cos 59° × 1.399... = 0.564 (3 s.f.) ff Both methods give the answer: P(84 ≤ X ≤ 106) = 0.564 (3 s.f.) Exercise 1.1 Q4 Hint: You'll often see the variance written as a number squared. Use the trapezium rule with 5 intervals to estimate the area of a cross-section of the wing enclosed by the curve y = ln x , the x-axis and the lines x = 2 and x = 7. ff If the function f(x) cannot be differentiated, then you won't be able to form an iteration formula to use. 19.3 1 Using s = ut + 2 at2: 0 38.6 o + 1 × (1.969...2) × e o s = (1.969...) × e 2 - 9.8 19.3 s=e 76.01... 1 -1 1 2 -2 = ^5 - 2xh 2 = 5 - 2 a1 - 5 x k 5 - 2x 1 1 2 2 k- 2 a1 - 5 x = 1 + a - 2 ka - 5 x k a - 12 k # a - 3 2k - 2 2 a 5 x k + ... q = tan–1 a 2.152... Example 2 A horizontal force of 30 N acts on a 4 kg block which is at rest on a rough horizontal plane. Beam A long, thin, straight, rigid body. The reaction force, R, at the hinge, acts vertically downwards. 1 x Example 2 For the functions f : x Æ 2x + 1, x d R and g : x Æ x2, x d R , solve gf(x) = 16. dV Find an expression for the rate of change of volume with respect dq to temperature. = 0.0387 (3 s.f.) 280 Chapter 12 The Normal Distribution 766 780 790 x Example 2 The times taken by 666 people to complete an assault course are normally distributed with a mean of 600 seconds and a variance of 105 seconds. dy 2 Q7 a) y = ln (3x)2 = 2 ln (3x) = 2 ln 3 + 2 ln x fi = x dx dy When x = –2, = –1 and y = ln 36 dx y = mx + c fi ln 36 = [(–1) × (–2)] + c fi c = ln 36 – 2 So the equation of the tangent is y = –x + ln 36 – 2 dy b) When x = 2, = 1 and y = ln 36 dx y = mx + c fi ln 36 = [1 × 2] + c fi c = ln 36 – 2 So the equation of the tangent is y = x + ln 36 – 2 dy f l (x) 2 Q8 a) y = ln (x + 6)2 = 2 ln (x + 6) fi =2 = + dx x 6 f (x) dy 2 When x = –3, = and y = ln 9 dx 3 3 1 So the gradient of the normal = – 2 = – 2 a3k 3 9 y = mx + c fi ln 9 = (– 2 × –3) + c fi c = ln 9 – 2 3 9 so the equation of the normal is y = – 2 x + ln 9 – 2 . You'd get the same answer. For the continuous random variable X ~ N(m, s²): ff X is a continuous random variable with mean m and standard deviation s. Find T, the tension in the rope pulling the rock. You'll often find there's more than one way to get to the answer. ff Often it's very useful to resolve forces into their horizontal and vertical components. 4 c) Rearranging the equation gives |3x – 4| = 1. Q14 Find the coordinates of the stationary points of the graph 1 2 3 3 y = 6x – 2xy. dt D2 dD The question tells you that the rate at which D decreases (i.e.) dt 2 is inversely proportional to D . ff So the area B = B1 – B2 = ^ 2 + ph - ^ - 2 + ph = 2 2 Sometimes you might need to add and subtract integrals to find the right area. gx 2 a) Show that y = x tan a – . Usually given as the set of values that x can take. For example, a falling parachutist would encounter significant air resistance, so the model should ideally allow for this. FA = μRA = 0.2 × 4g cos 50° = 5.039... ff As before, rearrange it and then simplify: dy 1 + 2 (2x - 3) 1 + 2 (2x - 3) m = e2x d n = e2x c dx 2x - 3 2x 3 = e 2x (4 x - 5) 2x - 3 Chapter 7 Differentiation 155 You might also see questions that ask you to rearrange the final answer to 'show that' it's equal to something, or to find the stationary points of a graph. q ff Find the magnitude (i.e. the length) of each side of the triangle. ff Using a calculator, P(780 < X < 790) = 0.038709... c) 2e4 – 3x # 2e 4 - 3x 2 dx = - 3 e4 - 3x + C Multiplying by 2 doesn't change the integration — the coefficient of x is –3, so divide by –3 and you're done. ff You put an approximate value of a root x into an iteration formula, and it gives you a slightly more accurate value. f(2.85) = 2.853 – (7 × 2.85) – 2 = 1.19... d fi 2x + 2x + y 2x – 2y = 10 + 4 dx dx dx Use the product rule dy dy dy fi 2x + 2x + 2y – 2y = 10 + 4 to differentiate 2xy. h 6y1 + y2 + ... = 4.79 m (3 s.f.) b) Resolving vertically, taking up as positive: s = s, u = 12 sin 37°, a = –9.8, t = 0.5 1 s = ut + 2 at2 1 s = (12 sin 37° × 0.5) + (2 × –9.8 × 0.52) = 2.385... a) Draw a diagram to show the forces acting on the block. c) Stating any assumptions you make, find the probability that both of the batteries in a car last at least 380 hours. Q14 Hint: Turn the fraction you're dividing by upside down and multiply. dx ff So substitute m = 2, x = 0 and y = –2p into y = mx + c, to find c: y = mx + c –2p = 2(0) + c fi c = –2p ff Putting c back into the equation gives: y = 2x – 2p or y = 2(x – p) c) Find the equation of the normal to C when t = p. The estimate is less than the real area. ff Only use each bit once unless something on the bottom line is raised to a power. 1 3 1 3-1 1 - 1× e o 3 3 = Exercise 6.2 — Simplifying, solving equations and proving identities sin ^ A - Bh cos ^ A - Bh sin A cos B - cos A sin B º cos A cos B + sin A sin B Divide through by cos A cos B... x terms fi –6 = –3C + D, so C = 3. This bit is d(x), the divisor. ff Now, x2 = (2k + 1)2 = 4k2 + 4k + 1 4k2 + 4k = 2(2k2 + 2k) is even because it is 2 × an integer fi 4k2 + 4k + 1 is odd. f –1f(x) = ff –1(x) = x. Bring down the –16x term just to make things clearer for the next subtraction. Proof by contradiction To prove a statement by contradiction, you start by saying "assume the statement is not true...". N ff Resolving vertically: F + 42 sin 22° = mg fi F = 50.902... Tip: You need to be careful that the interval you are looking at is not too large. 7-5 P(X < 7) = P a Z < 4 k = P(Z < 0.5) m = 5, s = 16 = 4 f(z) ff Draw a sketch showing the area you need to find. ff There are often lots of different ways to rearrange the equation, so in the exam you might be asked to 'show that' it can be rearranged in a certain way, rather than starting from scratch. Chapter 2 Algebra and Functions 21 Solving composite function equations If you're asked to solve an equation such as fg(x) = 8, the best way to do it is to work out what fg(x) is, then rearrange fg(x) = 8 to make x the subject. You're Reading a Free Preview Pages 433 to 458 are not shown in this preview. Assuming the variance has remained unchanged, test this claim at: a) the 5% significance level b) the 1% significance level Q2 Bree plays cards with her friends, and the amount of money she wins each week is normally distributed with a mean, m, of £24 and standard deviation, s, of £2. So for a positive number, the modulus is just the same as the number itself, but for a negative number, the modulus is its numerical value without the minus sign. Example 1 A particle is projected with initial speed 18 ms–1 from a point on horizontal ground, at an angle of 40° above the horizontal. Dividing the two equations gives: ar7 = 5 = 192 = r 32 & r = 5 32 = 2 6 ar 2 6 6 b) 3rd term = ar2 = 6 and r = 2, so a = 2 = 2 = 1.5 r 2 c) The sum of the first 15 terms is: a (1 - r15) 1.5 (1 - 215) = = 49 150.5 S15 = (1 - r) (1 - 2) 2nd term 3rd term Q6 a) Common ratio: 1st term = , so: 2nd term m = 2m - 21 + fi m² = (m + 10)(2m – 21) m m 10 fi m² = 2m² – 21m + 20m – 210 fi 0 = m² – m – 210 b) Factorising m2 – m – 210 = 0 gives: (m – 15)(m + 14) = 0, so m = 15 or m = –14, since m > 0, m = 15. Iteration is a numerical method for solving equations, like trial and improvement. a) Find the magnitude of the force which holds the particle at rest. Q5 The velocity of a particle at time t seconds (t ≥ 0) is given by v = [(t2 – 6t)i + (4t + 5)j] ms–1. sin 50° m= = = 1.70 kg (to 3 s.f.) g 9.8 b) Resolving horizontally: T = 8 sin 25° = 3.380... 0.85 = 11 997.50 to 2 d.p. When new, the car cost £11 997.50 Q6 a) The cost increases by 3% each year so multiply by 1 + 0.03 = 1.03 to get from one term to the next, so 2006 cost = 1.03 × (2011 cost) = 1.03 × £120 = £123.60 b) The costs each year form a geometric sequence with common ratio r = 1.03 and first term a = 120, so the total cost between 2011 and 2016 (including 2011 and 2016) is the sum of the first 6 terms: a (1 - r6) 120^1 - 1.036h = = 776.21 to 2 d.p. S6 = ^1 - 1.03h (1 - r) Nigel paid £776.21 (the cost to the nearest penny). dN dN c) μ N fi =k N dt dt 1 fi # dN = # k dt N fi 2 N = kt + C N = 20 at t = 0 fi 2 20 = 4 5 = C fi 2 N = kt + 4 5 N = 30 at t = 4 fi 2 30 = 4k + 4 5 30 - 2 5 fi k = 2 30 - 2 5 fi 2 N = t+4 5 2 30 - 2 5 So N = 1000 fi 2 1000 = t+4 5 2 4 1000 - 8 5 fi t = = 108.05 30 - 2 5 So the field will be over-run in 108 weeks. f(x) x a b Q7 h(x) = x + 1 , x d R . b) Find the range of j(x) = cos x on the same domain. g 2x2 x - 5 2x 3 - 7x 2 - 16x + 11 – (2x3 – 10x2) 3x2 – 16x ff Now divide the first term of the remaining polynomial (3x2) by the first term of the divisor (x) to get 3x (the second term in the answer). dx dy p b) Evaluate at q = 2 and hence obtain equations of the dx tangent and normal to the path at this point. # a b h a f (x) dx works like this: y Tip: If the number of strips is n, the number of y-values is n + 1. The beam makes an angle of 40° with the horizontal, as shown. You'll need to be able to swap between the forms. Tip: Or you can show: P(A|B) ≠ P(A), or P(A) × P(B) ≠ P(A Ç B). Q3 RT MODELL ING F + RC W T T 2000g 25° 800g A truck of mass 2 tonnes is moving along a rough, horizontal plane (m = 0.8), and towing a car of mass 0.8 tonnes up a smooth plane, angled at 25° to the horizontal, as shown in the diagram above. c) Use the addition formulas: sin (q – 30°) – cos (q + 60°) = 0 (sin q cos 30° – cos q sin 30°) – (cos q cos 60° – sin q sin 60°) = 0 c 3 sin q - 1 cos q m - c 1 cos q - 3 sin q m = 0 2 2 2 2 3 3 1 1 2 sin q – 2 cos q - 2 cos q + 2 sin q = 0 3 sin q – cos q = 0 3 sin q = cos q sin q 1 1 cos q = 3 fi tan q = 3 q = 30° and 30° + 180° = 210° p Q11 Use the sin addition formula for sin b x + 6 l: p p p sin b x + 6 l = sin x cos 6 + cos x sin 6 3 1 = 2 sin x + 2 cos x Now use the small angle approximations for sin and cos: 392 3 ≈ 2 x + 3 = 2 x + Answers 1 b1 - 1 x 2 l 2 2 3 - 1 2 1 - 1 2 1 2 4x = 2 + 2 x 4x 7. – T = 4a — call this equation 1 ff Resolve parallel to the plane ( ) for B: T – 3g sin 40° – FB = 3a T – 23.402... ff An object is in equilibrium if the resultant force on it is zero. ff You should now be left with something that's easier to integrate — just integrate as normal, then at the last step replace u with the original substitution. For Example 2: (x3 + 5x2 – 18x – 18) ÷ (x – 3) = x2 + 8x + 6 (i.e. remainder 0). MODELL ING Q1 The lengths and widths (in cm) of 8 leaves from a tree were measured. 2 dt dt V Tip: 'x is inversely proportional to y' means x is directly 1 proportional to y . Answers 375 Q3 a) f(x) = 3x + 4 with domain x d R has a range f(x) d R . ÷ 16.66 = 5.194... Example 3 Find dy if y = sin3 x. b) Hence find the rate of change in the water level, per minute, in a cylinder of radius 6 cm when the height of water is 4 cm. 1 1 Q6 y = 4 (x – 7)4, so let y = 4 u4 where u = x – 7 dy du fi = u3 = (x – 7)3, =1 du dx dy dy du = × = (x – 7)3 × 1 = (x – 7)3 dx du dy 1 1 When x = 6, y = 4 (6 – 7)4 = 4 and = (6 – 7)3 = –1 dx The gradient of the normal is –1 ÷ –1 = 1. Q4 Find the exact value of #1 1 2 Q5 Find # 2 tan3 x dx using the substitution u = sec2x. R There is no frictional force here, as there is no force which could cause the object to move along the surface. A particle of weight 25 N is placed at B. ff Using your calculator, the critical value is z = –1.645, P(Z = z) meaning the critical region is Z < –1.645. But, if you're finding angles, it's best to rearrange it: b2 + c2 - a2 cos A = 2bc See your Year 1 notes if you need a reminder. What was the particle's initial distance from the origin at time t = 0? x5 = 6 × 7.6739... Tip: Another set of numbers is C, the complex numbers, made up of 'imaginary' numbers — but you don't meet these in A-level Maths. So f –1(x) has domain x d R and range f –1(x) > 3. Chapter 11 Probability 263 Example 2 Horace is either late for school or on time for school, and when he gets to school he is either shouted at or not shouted at. + 1600g + 3331.10... s Answers 461 Resolving horizontally, taking right as positive: s = s, u = 3 cos 7°, a = 0, t = 1.770... Example 1 Use implicit differentiation to find dy for y3 + y2 = ex + x3. Resolving parallel to the plane () for the car: Fnet = ma T – 800g sin 25° = 800 × 0.0222... MODELL ING Q1 The weight of plums (in grams) from a tree follows the distribution N(42, 16). Q2 A cuboid block of sides 2x cm by 3x cm by 5x cm expands when heated such that x increases at a rate of 0.15 cm °C–1. dx dx 2 3 2 d 2 y (x) (- x) - (1 - 2 ln x) (3x) 6x 2 ln x - 5x 2 6 ln x - 5 = = = So x4 dx 2 x6 x6 Tip: A negative second derivative means a maximum and positive means a minimum. Then under H0, X ~ N(32, 52), 52 so X ~N a32, 20 k = N^32, 1.25h. seconds ff Now consider the horizontal and vertical components of motion together using the suvat equations in column vector form: 38.6 0 o o, a = e s = s, u = e - 9.8 , t = 1.969... This value is negative dt Use d because it's a rate of time. The modulus function lets you do this. 2m 20° A ff Here, the lamina's weight acts at 20° to its sides. The plane is inclined at an angle x° to the horizontal. You'll have to subtract integrals like you did in Example 2. Stretch horizontally by a y =|x + 1| y = 1 - |2x + 1| y = -3 sin 2x + 1 -3 Q15 a) Stretch horizontally by 7 a factor of 2 , stretch vertically by a factor of 3, reflect in the x-axis and 4 1 y = sin x x 2 1 factor of 2 , reflect in the x-axis and translate by 0 e o i.e. 1 up. c) f(1.8) = ln (1.8) – 21.8 + 3 = 0.105... Q9 a) sec2 x – 2 2 sec x + 2 factorises to (sec x – 2)2. Force An influence which can change the motion of a body (i.e.

cause an acceleration). Find the third force in vector form. For example, the degree of 4x5 + 6x2 - 3x – 1 is 5. (n - r + 1) r (1 + x)n = 1 + nx + x + ... -1 1 a - 2x + 2x 2 - k So ^9 + 4xh 2 = 1 9 27 ... S Sec The reciprocal of the cosine function, sometimes written as secant. The range is between f(2) and f(-1): ((2)2 - 4(2) + 5) ≤ f(x) ≤ ((-1)2 - 4(-1) + 5), which is 1 ≤ f(x) ≤ 10. ∴ (f(x) : 1 ≤ f(x) ≤ 10}. = 0.877 kg (3 s.f.) So Let x be the distance of the COM from A. Q4 A particle rests on a rough plane inclined at 45° to the horizontal. x 3 1 Replace f(x) with y: y = x 3 1 Rearrange: x = y 3 1 Replace with f -1(x) and x: f -1(x) = x - 3. ff You're given the test statistic for the sample: r = 0.76. different values of the product moment correlation coefficient. f (x) dx This is really just a change in notation, so that you don't have to write out the whole limit of a sum each time you want to write an integral. m xm 20 mm x mm a) Find an expression for the area of the end of the prism in terms of x. 2 4 The gradient of the normal is 4. Chapter 8 Integration 191 You might see questions where the numerator is a multiple of the derivative of the denominator just to confuse things. Differential equation An equation connecting variables with their rates of change. ff Put 2 and 3 into the equation and check for a sign change. Arcs and Sectors 2. Find AB to 1 decimal place. The position vector of B can also be described as: OB = AB + OA Substituting for OB : 2 OA = AB + OA 2 OA – OA = AB So AB = OA — as required. × 22) = 8.77 m (to 3 s.f.) Tip: Remember that A and B will have the same speed at any given moment in time. 3 In a large population, the mean weight of cereal boxes is 780 g. Find the probability that a randomly selected box of cereal contains less than 780 g of cereal. State the values of a and b. To get an answer in the correct interval, add p, since tan x repeats itself every p radians. There is evidence at the 2.5% level of significance to reject H0 and to support the alternative hypothesis that the length and width of leaves are positively correlated. ff Resolve to find two perpendicular components of the resultant force, then make each component equal to zero. k 1 2x 4x 2 = - 5 - 25 - 125 - ... You would get the same answer, but the working is a bit longer. 1 a1 - k 1 3 3 1 1 Q3 a) ^1 + 4xh3 = 1 + 3 (4x) + 1 # 2 (4x)2 + ... Acceleration The rate of change of an object's velocity with respect to time. 1) S tart by drawing a vertical line from x0 to the curve. 332 Chapter 15 Dynamics μ = 30 + 4g = 0.77 (to 2 d.p.) Example 3 10 ° Q A box of mass 10 kg is on the point of slipping across a rough horizontal plane. Write down the position vectors of R and S, giving your answers: a) as column vectors, and b) in unit vector form. A random variable Y following a normal distribution is to be used to approximate the probabilities below. Find the value of V. (1 -4 Hint: Remember, the phrase 'exact value' is usually a clue that the answer will include a surd or p. d sec x = sec x tan x. -4 m (N R < X) = (0.1 -1.1) fi P a Z < s k = 0.102 fi P a Z < s k = 0.102 m-4 m-4 fi P a Z > s k = 0.102 P # s k = 0.898 f(z) = 0.898 for z = 1.27 m-4 fi s 1.27 fi = 1.27 s 7-m P(R > 7) = 0.648 fi P a Z < s k = 0.648 m-7 fi P a Z < s k = 0.648 fi P(z) = 0.648 for z = 0.38 fi m - 7 = 0.38s 2 Subtracting equation 2 from equation 1 gives: m - m - 4 = (1.27 - 1.27s + 0.38s fi 3 = 0.89s fi s = 3.37 (3 s.f.) Putting s = 3.37 cm into equation 1 gives: m = 4 + 1.27 × 3.37 = 8.28 cm (3 s.f.) So m = 8.28 cm and s = 3.37 cm (3 s.f.) 2. You're going to need the rules for multiplying powers for this question. Domain The set of values that can be input into a mapping or function. You can apply different rates of change to a given situation, and if they have variables in common you say they're 'connected'. If they can take any value above or below a limit, use e.g. x ≥ 0. Find a suitable numerical approximation for X, giving m and s2. Projectiles 2. For the functions f : x Æ Tip: x means the positive root of x. 1 Using s = ut + 2 at2: 302 Chapter 14 Kinematics s = 10 × 0.6388... ff So the smallest possible value of x2 – 5 must be -5. i and j are the unit vectors directed due east and due north respectively. The remainder r(x) has degree 0. 2 - 1 : 14 - 9 - m - (–m) = 1.8494...s - 0.1495...s fi 5 = 1.699...s fi s = 2.9421... PROBLE M SOLVING Q Show that any point on the curve y ln x + x2 = y2 - y + 1 which satisfies y + 2x2 = 0 is a stationary point. one-tailed two-tailed ... cos q , d is the Greek letter delta, and dx just means "change in x". The significance level is a = 0.01, and it is a two-tailed test. Example 4 For y = 2 cos2 x + sin (2x), show that dy = 2(cos (2x) – sin (2x)). The normal reaction at C, N, acts perpendicular to the rod. Q5 The number of hours that Kylie spends studying each week is normally distributed with mean 32 and standard deviation 5. ff To divide by an algebraic fraction, you just multiply by its reciprocal. = 0 ff So the particle is directly above O when t = 1.10 s (3 s.f.) Tip: You need to check that the j-component of r is greater than 0 to make sure that the particle is above O (rather than at it or below it). Example 3 Find the following integrals: a) # 2 sec x dx Use the result for sec x above. Tip: Make sure to use the ANS button on your calculator when you're doing these iterations, so you avoid rounding errors. However, if the object is fixed by a hinge, the reaction force might not act perpendicular to the surface. Q1 Hint: If a surface is smooth, it means there's no frictional force. ff Be able to use these results to differentiate more complicated functions. Differentiation with Parametric Equations You saw in Chapter 4 — there are situations where x and y are defined separately in terms of a third variable. Integration by substitution Integration by substitution is a way of integrating a function of a function by simplifying the integral. It comes to rest after 4 seconds. Formulate a differential equation to represent this situation. ff Total moment is zero, so there is no overall turning effect about A. ^3t + 2h ^t + 1h Q6 Use the substitution u = x to find the exact value of 220 Chapter 8 Integration PROBLE M SOLVING # 2 Q4 A uniform beam, AB, has length 9 m and mass 8 kg. Help Center You're Reading a Free Preview Pages 15 to 44 are not shown in this preview. Tip: Another way to say this is that the trapezium rule gives an underestimate if the curve is concave and an overestimate if the curve is convex — see p.135 for more on convex and concave curves. # 2x ex dx = # u ddvx dx = uv - # v dduv dx # 2ex dx = 2xex – 2ex +C = 2xe - 2e + C x Don't forget the constant of integration. ff The total area under the curve Tip: You'll see how to find and use areas under the normal curve over the next few pages. ff This means that there is no resultant force in any direction — any forces acting on the body will cancel each other out. Learning Objectives: ff Be able to interpret Understanding the PMCC The product moment correlation coefficient (r for short) measures the strength of the linear correlation between two variables. Using the critical values table, the critical value is 0.7977, so the critical regions are r ≥ 0.7977 or r ≤ -0.7977. Then H0: m = 42 and H1: m > 42. cm3 s-1 (8) 49.324 2 ff So the volume is decreasing at a rate of 1650 cm3 s-1 (3 s.f.) Exercise 11.3 Q1 Hint: Don't be put off by the amount of information given in the question. f(0.01605) = 20.01605 – 1 – 4 (0.01605 = -0.00016... ^1 + 3xh6 So overall, the expansion of is valid ^1 + x3h2 for the interval Hint: 0 x N(100, 16), find P(X ≤ 105) to 3 d.p. ff Draw a sketch showing what you need to find. (A correlation does not necessarily mean a causal relationship.) Cosec The reciprocal of the sine function, sometimes written as cosecant. p x fi 2 = 2 fi x = p 1 Q4 a) sin 2x cos 2 2 sin x º cos x 2 2 sin x º 2 cos x 2 cos 2 x 2 sin x cos 2 x 2 2 1 1 º º º 2 sec2 x 2 cos 2x cos x 2 x 2 1 c) cos 2x º tan x – tan 2x 2^1 - tan 2 xh 1 º tan x – 2 tan x 1 - ^1 - tan 2 xh tan 2 x tan x º tan x sin 2x cos 2x 2 d) tan 2x + cot 2x º cos 2x + sin 2x cos 2x + 2 º sin 2x cos 2x cos 2x Use 2 sin 2x cos 2x º sin 4x º sin 4x = 2 cosec 4x ... 53.1° (to 3 s.f.) Resolving perpendicular to the plane: R = 10 cos x = 6 N 4 5 FN Substituting 1 into 2 : T cos 33° = R sin 12° + T sin 33° = sin 12° cos 12° cos 33° fi T sin 12° cos 12° ^ sin 33° k 15 T fi cos 33° = 4.41 N (to 3 s.f.) a tan 12° - sin 33° k b) Rearrange 1 and substitute the values for T and g (= 9.8 ms-2): T cos 33° = m g sin 12° = 1.82 kg (to 3 s.f.) Q8 SN 4F 20° 45° mg F a) Resolving parallel to the plane: F = mg sin 45° Resolving perpendicular to the plane: 25 = mg cos 45° cos 45° = sin 45°, therefore F = 25 N b) Resolving perpendicular to the plane: mg cos 45° = 25 25 m = (g cos 45° = 9.8 × cos 45° = 3.61 kg (to 3 s.f.) Q5 RN TN 30° 5g N ... ff So the normal reaction force is 21.7 N. Chapter 11 Parametric Equations 389 Q4 a) ... Magnitude of the tension in the string = T normal reaction force = R Resolving parallel to the plane: T cos 30° = 5g sin 10° ^5 × 9.8 × sin 10°h T = 9.83 N (to 3 s.f.) cos 30° b) Resolving perpendicular to the plane: R + T sin 30° = 5g cos 10° R = 5g cos 10° - T sin 30° = 43.3 N (to 3 s.f.) 468 Answers 50° W The string makes an angle of 70° – 50° = 20° with the plane. Formulate a differential equation that shows this information. a) Find the gradient of the curve, and the (x, y) coordinates, when t = p. ff If the sum of the moments about a particular point is zero, then the rod won't rotate about that point. Q8 A particle moves along a path given parametrically by x = s3 ln s, y = s3 - s2 ln s. c) The chain connecting the truck to the car snaps. r 0.5 d) r= Q6 a) = 6, 10 = S¥ = a- 61 r 1 r fi 1 - r = 6 = 0.6 fi r = 0.4 10 b) 5th term: u5= ar4 = 6 × 0.44 = 0.1536. Q15 a) y = 2 sin (2x) cos x fi y = 4 sin x cos2 x (from double angle formula sin (2x) º 2 sin x cos x) ff y = sin x (1 – sin2 x) (from identity sin2 x + cos2 x º 1) fi y = 4 sin x – 4 sin3 x b) First part: = 4 cos x dx Second part: y = 4 sin3 x = 4 (sin x)3, so let y = 4u3 where u = sin x fi = 12 sin2 x cos x dx du dx du fi dy = 12u2 cos x = 12 sin2 x cos x dx Answers 415 Exercise 4.2 — Differentiating by using the chain rule twice Q1 a) y = sin (cos 2x) ... y = sin u where u = cos (2x) dy = cos u du dx Using the chain rule again on given: du dx dx u = cos (2x) fi = -2 sin (2x) dx Putting it all together: dy dy du = × = cos u × (-2 sin (2x)) = -2 sin (2x) cos (cos 2x) dx du dx fi(x) b) y = 2 ln (x) fi y = e2 ln (x) fi 2 ln (x) fi y = 3e3x fi(x) = 3 ln (3x) - sin (3x) dy fi y = 2 cos (3x) = -6 tan (3x) dx fi(x) (5 s.f.) fi y = ln (tan2 x) = ln (f(x)) fi = dx f (x) fi(x) = y = u2 where u = tan x dv du = 2u = 2 tan x, = sec2 x dx du dx dv du dv dx Putting it all together: fi(x) = × = 2 tan x sec2 x dx du dx dy 2 tan x sec2 x 2 = = 2 sec x cosec x dx tan2 x Tip: You could've written ln (tan2 x) as 2 ln (tan x) using the laws of logs and then differentiated — you'd end up with the same answer. F B C 1m A 3m D a) Find the horizontal and vertical distances of the centre of mass of the lamina from A. 1 2 dx 1 3 ff Now, f(x) = x2 – 4x + 5. ff There's a peak at the mean, m. ff Taking moments about B for ladder: 176.4 sin 65° × 6 = (10g cos 65° × 3) + (50g cos 65° × x) fi x = 834.986... = 1.836... 3 2 Q2c) Hint: This one's a bit tricky! — the best way to do it is to consider the bits above and below the x-axis separately. so in fact, 8 is the point (-3, 0, -1). Equating coefficients: x4 terms fi 3 = A ... Putting this into y = mx + c gives: p2 p2 4 = 2(0) + c fi c = 4 p2 So the equation of the normal is y = -4x + 4 . ff Has an a rough plane (m = 0.1) with total mass 5 kg, that is ... so you need to be less than 1000 when n is greater than 15.425..., therefore u16 is the first term that's less than 1000. Exercise 11.1 Q1 R is the point (4, -5), and S is the point (-3, 0, -1). PROBLE M SOLVING Q1 The ... at 70° to the horizontal, and Q hangs vertically T below the pulley, as shown. metres and its vertical displacement is 0.5 m. TOP = mRB = 0.2 × 3g cos 40° = 4.504... 4 # Q3 a) Show that Q4 Show that # b a 15 ^5 + 3xh 1 dx = ln - Q3-4 Hint: Use the log laws from Year 1 (given on p.184). PROBLE M SOLVIN G f(z) Area = 0.5596 09– z ff Now, if you use a calculator to find z for which F(z) = 0.5596, you can form an equation in m and s. b) Find the probability that a battery lasts at least 380 hours. a) Find the critical region for a test of the doctor's claim at the 1% significance level. Q5 5 3 1 80 ms–1 40 m , 12 A bullet is fired horizontally at a speed of 80 ms-1 from the edge of a vertical cliff 40 m above sea level, as shown. ÷ 6.761... Q4 # b a 15 ^5 + 3xh dx :15 × 1 ln 5 + 3x D = 6ln 5 + 3x @ab 3 b = 5 ^ln 5 + 3a - ln 5 + 3b = 5 ln 5 + 3b h 5 + 3a + 3 = 5 ln d n = 5 ln 5 + 3a = ln 5 + 3a 5 3b 5 3b 5 + 3b a 5 Q5 Integrate the derivative to find f(x). N The block is in limiting equilibrium, so F = mR: F = 0.4 × 40.138... This fits in to 15%. ff She samples 10 students and finds that the PMCC is 0.76. N Resolving vertically (≠) for Q: Fnet = ma T – 20g = 0 fi T = 20g N Horizontal resultant force: Fnet = 412.63... Method 2 — algebraic long division You can also use long division to divide two algebraic expressions (using the same method you'd use for numbers). K = 217 N (3 s.f.) 2) Resolving vertically (≠) for P: Fnet = ma T – 20g – 20a 1 Resolving vertically (≠) for Q: T – 60g = 60a ... 2 Adding 1 and 2 : 60g sin 50° – 6g cos 50° – 20g = 80a fi a = 2.707... y = 1 - x x=1 But if you change the domain to x > 1, the mapping is now a function, as shown. y You could also restrict the domain by giving values that x can't be equal to, e.g. x d R , x ! 1. Q5 a) 2 cosec2 x = 5 – 5 cot x fi 2(1 + cot2 x) = 5 – cot x fi 2 cot2 x + 5 cot x – 3 = 0 This factorises to give: (2 cot x – 1)(cot x + 3) = 0 1 So cot x = 2 fi tan x = 2 or cot x = –3 fi tan x = - 3 Solving these over the interval –p ≤ x ≤ p gives: x = –2.03, –0.32, 1.11, 2.82 b) Solve 2 cot2 x + 5 cot x – 3 = 0 Q6 a) 2 cot2 A + 5 cosec A = 10 fi 2 (cosec2 A – 1) + 5 cosec A = 10 fi 2 cosec2 A + 5 cosec A – 12 = 0 This factorises to give: (2 cosec A – 3)(cosec A + 4) = 0 b) Solve 2 cot2 A + 5 cosec A – 12 = 0 This factorises to give: (2 cosec A + 4)(2 cosec A – 3) = 0 So cosec A = –4 fi sin A = - 4 3 2 or cosec A = 2 fi sin A = 3 The solutions (to 1 d.p.) are A = 194.5°, 345.5°, 41.8°, 138.2°. ff Are there other factors that have not been accounted for in the model? Q3 a) Suppose that there is a rational number x ≠ 0 and an irrational number y such that xy is rational. Q5 The path of a particle is given parametrically by x = q sin 2q, y = q2 + q cos q. The s.d. is unknown, so just leave it as s for now. y = |f(x)| = |5 – x| y ff Step 1: S Draw the graph of y = 5x – x2 using... MODELL ING Modelling assumptions and probability In most models, the probability of an event is based on some assumptions. Vectors 1. ff Next, turn the question into a probability for F. = 0.89 (2 d.p.) Answers Solving the simultaneous equations 1 and 2 : 5T – 0.7g cos 40° – g sin 40° = g sin 20° – 0.05g cos 20° + 0.7g cos 40° + g sin 40° fi T = 2.363... At time t = 0, the boat is at 2j m from the origin, moving in the positive j direction at a speed of 10 ms-1. p Q7 a) Using the trapezium rule with h = 6 , show that p p ^2 + 3 h 2 . c) y = 2 – log2(x) fi log2(x) = 2 – y fi x = 22 – y So f-1(x) = 22 - x f-1(x) has domain x ≥ 1, which means that log2(x) ≥ 0. The triangle has side lengths |v|, |p| and |p – v|. Find the area of the shaded grey region. 30° y 6.5 N 8N 61° 75° 10 N 80° 69° x x 47° 10 N 12 N Q2 A force of magnitude 18 N acts on a particle at 60° above the horizontal. Integrate each term separately using the results from above and p.187: # # cos 4x sin 4x # cos 4x - 2 sin 2x + sec 2 2 x 12 k sec 2 x dx = 11 tan x 12 k = 2 tan x 2 k 'Zj a 12 k 14 sin 4x + cos 2x + 2 tan 2 x + C # - 2 sin 2x + cos 2x = cos 2x Exercise 3.2 Q1 Find the following integrals: # 2 sec 2 x 1 dx a) # cosec 2 7x dx d) (3 cosec 2x + 3 cos 3x) dx = - cos1 dq q 2 q 2 2 (0) # 20 sec # 3 sec p d x cosec d z Write 2 3 2 2 3y dy ^ - 3x 4 b) Q2 Find the actual values of the integrals below. MODELL ING Q10 A particle is fired from a point 16 m above flat, horizontal ground. Q3 Let the mean mark of all the students in the school be m. Chain Rule Exercise 2.1 — The chain rule Q1 a) y = (x + 7)2, so let y = u2 where u = x + 7 dy du dy du fi = 2u = 2(x + 7), = 1 dx du dx dy dy du Putting it together: = × = 2(x + 7) × 1 = 2(x + 7) dx du dx b) y = (2x – 1)5, so let y = u5 where u = 2x – 1 dy du fi = 5u4 = 5(2x – 1)4, = 2 dx du dx dy dy du = × = 5(2x – 1)4 × 2 = 10(2x – 1)4 dx du dx c) y = 3(4 – x)8, so let y = 3u8 where u = 4 - x dy du dy fi = 24u7 = 24(4 – x)7, = –1 dx du dx dy du = × = 24(4 – x)7 × (–1) = -24(4 – x)7 dx du dx d) y = (3 – 2x)7, so let y = u7 where u = 3 - 2x dy du fi = 7u6 = 7(3 – 2x)6, = –2 dx du dx dy dy du = × = 7(3 – 2x)6 × (–2) = -14(3 – 2x)6 dx du dx e) y = (x2 + 3)5, so let y = u5 where u = x2 + 3 dy du fi = 5u4 = 5(x2 + 3)4, = 2x dx du dx dy dy du = × = 5(x2 + 3)4 × 2x = 10x(x2 + 3)4 dx du dx f) y = (5x2 + 3)2, so let y = u2 where u = 5x2 + 3 dy du fi = 2u = 2(5x2 + 3), = 10x dx du dx dy dy du = × = 2(5x2 + 3) × 10x = 20x(5x2 + 3) dx du dx g) f(x) = (4 - 7x2)3 ... fi(x) = -14x du dx du dy f(x) = (x2 + 5x + 7)6 ... fi(x) = 6(x2 + 5x + 7)5 × (2x + 5) = 6(2x + 5)(x2 + 5x + 7)5 (12x + 30) d) f(x) = (x + 4)–3, so let y = u–3 where u = x + 4 dy du fi = -3u-4 = -3(x + 4)-4, = 1 dx du dx f'(x) = × = -3(x + 4)-4 × 1 = -3(x + 4)-4 du dx du e) f(x) = (5 – 3x)–2, so let y = u–2 where u = 5 - 3x dy du fi = –2u-3 = –2(5 – 3x)–3, = -3 dx du dx dy du f'(x) = × = -2(5 – 3x)-3 × (-3) = 6(5 – 3x)-3 dx du dx f) f(x) = (5 - 3x)-4 where u = 5 – 3x dy du fi = -4u-5 = -4(5 – 3x)-5, = -3 dx du dx du dy f'(x) = × = -4(5 – 3x)-5 × (-3) = 12(5 – 3x)-5 dx du dx ... 12 f'(x) = × = -4(5 – 3x)-5 × (-3) = dx du dx (5 – 3x)5 ... 3 3 g) f(x) = (3x + 2)–1 ... so let y = u–1 where u = 3x + 2 dy du fi = -u–2 = -(3x + 2)-2, = 3 dx du dx f'(x) = × = -(3x + 2)-2 × 3 = -3(3x + 2)-2 du dx ... i) f(x) = 3x 2 - 2 ... 6x du 1 1 1 h) f(x) = (x2 + 3)-1 ... (x + 3)2 - 2 ... 2 (x 2 + 3) 2 ... When x = 1, = -3 = -3 (2 # 1) - 3 (2) h 4 2 1 2 h (2 # 1) ... 2x + 3 = (2 # 1) - 3 (2) h 4 2 1 2 h ... 2 1 2 h ... 3 y = 12(x + 6) + 3 ... Q3 - 2 ... 2 ... x + 1 ... 2 = 3 ... 2 (x2 + 1) ... 3 ... 3 ... So y = ... 12 x + 3 ... Remember it's ... b) y = 4 sin ... fi ... Q2 ... 4 ... 3 ... dy du ... f(x) = 2x + e(x) ... 5 ... 6x ... 3 ... Putting it all together: dy = -12 sin x cos x - 4 cos (2x) (4 - cos (2x)) ... Use double angle formula: 2 sin x cos x º sin 2x fi dy = -6 sin (2x) - 4 cos (2x) ... ff The coefficient of friction between the block and the surface is 0.35 and the tension in the rope is 70 N. From the double angle formula for sin: -sin (2x) = cos2 x – sin2 x cos x Q11 First part: y = 6 cos2 x = 6(cos x)2, so let y = 6u2 where u = cos x du fi dy = 12u = 12 cos x dx du dx du dy = -sin x du dx dx Putting it all together: dy = -12 sin x cos x dx Second part: y = 2 sin (2x), so let y = 2 sin u where u = 2x fi = 2 cos u = 2 cos (2x), =2 du dx du dx fi dy = 4 cos (2x) dx Putting it all together: dy = -12 sin x cos x - 4 cos (2x) Use double angle formula: 2 sin x cos x º sin 2x = -6 sin (2x) - 4 cos (2x) dx There is insufficient evidence at the 1% level to reject H0 in favour of the alternative hypothesis that the mean pigeon weight has gone up. back into one of the equations. ff If your steps are getting closer to the root, the sequence of iterations is converging. ff Points in three dimensions are given (x, y, z) coordinates. ff Resolving vertically: RV = 1.8g + 3.5 sin 80° = 95 cos 71° fi RV = 9.842... 234 ff Draw a sketch to help you picture the area. Frequency Density In real-life situations, the distribution of a set of data often follows a particular pattern — with most of the data values falling somewhere in the middle, and only a small proportion taking much higher or lower values. 2N Chapter 16 Moments 359 ff Resolving horizontally: RH = 16 sin 70° = 15.035... PROBLE M SOLVING Q7 In a 3D board game, players take turns to position counters at points inside a cuboid grid. n! Equal to d n = . This is because the function fx is not defined for negative values of x. The nth term is: un = ar n – 1 = 5(1.15)n – 1 So the 15th term is: 5 × 1.1514 = 35.3785... f(0.5) = (2 × 0.5) + 0.53 = 0.125 There is a sign change (and the function is continuous in this interval) so there is a root in this interval. The block is released from rest and begins to accelerate down the slope. W Weight The force due to a body's mass and the effect of gravity: W = mg. Q7 The upper and lower bounds are 2.75 and 2.85. Examples Simplify the following: x - 1 a) 2 + x 3x - 4 x 1 = (x - 1) (x + 1) 3x + 6 x 2 - 4 = = c) 1 x + 4 3 ^ 2h ^ x + 2h ^ x - 2h 3 x 2 x3 - 1 2x + 6x - 3 x ... ff Try factorising the denominator first. ff The p-value is P(Z ≤ –1.7888...) = 0.0368 < 0.05 a = 0.05 fi ff So the result is significant. (x) = f -1(f(x)) (f(x)) = fi = f -1 (f -1(x)) + f(x) ff x -1 cosec2 x dx × cot4 x + C Multiply everything by -2 to match the original integral: # 8cosec2 x cot3 x dx = -2 cot4 x + C Example 3 Find # ^ x - 2h 2 x - 4x + 5 dx. 3x Replace f(x) with y: y = f(x) + 1 Rearrange: y(x + 1) = 3x fi yx + y = 3x fi 3x - yx = y fi x(3 - y)x = y fi x = 3 y Replace y with f -1(x) and x: f -1(x) = 3 - x . Union (of events A and B) The set of outcomes corresponding to either event A or event B (or both) happening. There are several ways to answer this: ff The value of P0 is not given, so the model could be improved by finding a suitable value of P0. The nth term is arn - 1 = 12 × 1.03n –1, so the 10th term is: 12 × 1.039 = 15.7 miles (to 1 d.p.) Q7 a = 6000, d = 2000 and n = 12 (12 months in the year) n Sn = 2 62a + (n - 1) d @ 12 S12 = 2 [(2 (6000) + 11 (2000)] = 204 000 204 000 copies will be sold. ff For this example you can either split up the probability or calculate it directly. G When t = 2, the body is moving with velocity v = (-20i + 135j) ms-1. m 81.1 kg (3 s.f.) Example 4 A uniform ladder of length 6 m and mass 16 kg is held against a rough vertical wall by a horizontal force of magnitude 50 N, being applied at a distance of x m from the base of the ladder. dx ds For starters, we're going to need to find by implicit differentiation: dx 2 d d d d d x + 2xy = 10x + 4y = 21 dx dx dx dx dx d 2 d Differentiate x2, 1 0x and 8 (x + 2xy) = 10 + 4y dx dx 21 with respect to x. The beam is horizontal and in equilibrium. 6 a) Find the distance x. dy -1 x 5 5 fi 5 + 10 4 3 4 4 ... The graph of y = 5 - 3x + 1 ... has domain x ≥ 1 ... ff The first step is always to define the variable. Example 1 The first step is always to define the variable. e 2x x + 2 dx. ff Understand which functions will have inverses. e 2x x + 2 dx. Example 1 Tip: The first step is always to define the variable. fi(x) 5 4 y=x-5 fi = 4 3 2 1 3 e-x e - 0 .4 x1 = 2 = 0.4690..., x2 = 0.4509..., x3 = 0.4561..., x4 = 0.4547..., x5 = 0.4551..., x6 = 0.4550... 0 b y) 2 3 x 0 2 1 3 x0 A divergent staircase diagram. Given that B was initially released from a position 60 cm above the horizontal ground, find the speed of B when it hits the ground. The 30 N weight is at one end of the beam, as shown, whilst the other weight is a distance d from the midpoint, M. – 25 = 33.3 Nm clockwise (3 s.f.) ff Sum of moments = (6 × 0.5) – (10 sin 60° × 1) – (10 cos 60° × 11) = 3 – 8.6602... sec q + tan q 2 Q4 a) tan x = sec2 x + 2 fi sec2 x – 1 = 2 sec x + 2 fi sec2 x – 2 sec x – 3 = 0 b) Solve sec2 x - 2 sec x - 3 = 0 This factorises to give: (sec x - 3)(sec x + 1) = 0 ... 1 So sec x = 3 fi cos x = 3 and sec x = -1 fi cos x = -1 Solving these over the interval 0° ≤ x ≤ 360° gives: x = 70.5°, 180° and 289.5° (1 d.p.) ff Once you've got ... ff Let X be the number of ... just use the graph of cos x or the CAST diagram as usual to find all the solutions in the interval. Q10 Hint: The functions are equal where the two graphs cross. Using set notation, give a restricted domain which would make g a function. What is the turning effect about C? OA = 4.53 N (3 s.f.) fi ... 60° X H 0.6g Resolving horizontally: RH = T cos 60° = (4.526... c) cos 60° = 2.263... = 2.26 N (to 3 s.f.) ... ... 620 b) Find the number of people you would expect to complete the assault course in over 620 seconds. vx = ux = (23.811...) cos 45° = 16.837... dx 1 + 0 Putting this into y = mx + c gives: 3=0+c fi c=3 So the equation of the tangent is: y = x + 3 dy 2q + cos q - q sin q Q5 = dx sin 2q + 2q cos 2q When q = 2 : dy 1 p2 = - 2 , x = 0 and y = 4 dx 2 dy Therefore part a) you know that: Differentiation (3x + 4)5 15(3x + 4)4 Integration is the opposite of differentiation so: (3x + 4)5 + C Integrating 15^3x + 4h 4 This means: 15(3x + 4)4 dx = (3x + 4)5 + C gives you a constant term — you can call it C. The ball lands a horizontal distance of 50 m away. Normal Approximation to a Binomial Distribution Exercise 2.1 — Continuity corrections Q1 a) P(4.5 < Y < 5.5) b) P(11.5 < Y < 15.5) c) P(Y < 10.5) Q2 a) P(X(5 < Y < 50.5) b) P(Y < 299.5) c) P(Y > 98.5) Q3 a) P(X ≤ 200) is approximated by P(Y) ≤ 200.5) b) P(X ≥ 650) is approximated by P(Y ≥ 649.5). This result is the y-value, f(x), and its width is just dx. 2 3 When x = 3, 2 2 3 m = 2 3 m = 2 3 m = 2 3 + 12 0 4 4 0 2 4 3 9 q2x 3 d) 3 450 Answers 5. 1 5o a = 2 . MODELL ING ff This time the question tells you how V decreases — as time increases, the amount of charge in V with respect to t, or dV. Q3-5 Hint: Remember that every rational number can be written as a fraction where a and b are integers. This gives P(Z > -0.42) = 0.662757... The particles are released from rest with the string taut. Show that the particle accelerates up the slope at 20° to the horizontal. ff Once event C hasn't happened, there are 7 possible outcomes corresponding to event D happening and 13 possible remaining outcomes in total. C 1.5 m 8N D A ff Taking moments about A: 8 × 1.5 = F × 5 sin 18° F × 3 = 12 = 4 N If there are forces acting at angles to the sides of the lamina, it's usually easier to resolve them perpendicular and parallel to the lamina's sides. If the factor theorem holds. ff The points in three dimensions are given (x, y, z) coordinates. P(A Ç B) = P(A) × P(B) Example For two independent events A and B, P(A) = 0.5 and P(A È B) = 0.6. Find P(B). 5 + 3a 5 . Here, you can assume that a and b have no common factors because if they did, you could simplify the fraction to get a new a and b. 2.5.5 4.5 Y on the discrete probabilit0.5 you're trying The general idea is always the same, though — each discrete value b covers the continuous interval from b – 1 up to b + 1 . ff The easiest way to draw y = |f(x)| is to initially draw y = f(x), then reflect the negative part in the x-axis. ff In a fixed population. y = ax fi x2 = 3 ..."6 + m ... -1(x) 3 + m ... for y ≥ 0 (by ... domain is ... f(x) ... f(x) ... ff Find the set of ... Modell ing ... Q7 Model Use the equation ... Then you need to find x ≥ 7 (the range of f(x)). Logarithm A logarithm ... to which a base must be raised to give a number. The logarithm to the base a of a number x (written loga x) is the power to which a must be raised to give that number. Under H0, X ~ N(42, 16), so X ~ N a42, 25 k = 42, 0.64h. T1 and T2 are the tensions in the two strings. ff This is a one-tailed test — Mary thinks there is negative correlation, i.e. the PMCC is less than zero. Then you need to check that the particle is moving at 45° above i, not 45° below –i at this time. d) Find P(A|E'). Partial Fractions 3 12 19 23 27 33 39 Chapter 3 — Trigonometry 1. ff The mass is sliding, so F = mR: F = 0.4 × 27.626... 204 Chapter 8 Integration A2 is the area under the line y = 2 - x between 1 and 2, so integrate between these limits to find the area: # 2 2 2 # f1 ^ 2 - xh dx = ;2x - x2 E 2 = c2 # 2 - 2 2 m - c2 # 1 - 1 2 m = 2 - 1 2 = 1 2 ff Add the areas together to find the area of A: A = A1 + A2 = 3 + 1 2 2 Sometimes you'll need to find the area enclosed by the graphs of two functions — this usually means subtracting some area from another. This tells you what you're looking for and also helps check that your answer seems sensible. Points of inflection (S) ff 68% of the total area lies within one standard deviation of the mean. Add this to its initial position vector to find its new position vector. 5y = 5x + 1 ... 5 fi 5(4) = ... 0.736) = (5 fi) 4 = 4 × 5.6 fi (4 - 0.736 ... 0.47775... ... ... ff Find the velocity of the particle at this time. Q5 a) y = |5x + 10| y 10 x x -2 5 ... 2 1 –2 378 Answers –1 x b) There are two solutions to |f(x)| = |g(x)|, one where –2 < x < –1 and one where x < -2 (shaded). ff Write your conclusion. Solving differential equations ff Step 1: ff ff Write the differential equation in the form dy = f(x)g(y). ff Since the probability distribution is discrete, so if the random variable X follows this distribution, you can work out P(X = 0), P(X = 1), P(X = 2). Q2 - S he has assumed that the volunteer is equally likely to choose any of the 10 cards, but some people may tend to choose cards in the middle or from the ends more often. There is evidence at the 10% level of significance to reject H0 and to support the alternative hypothesis that test scores and hours spent revising are correlated. With thanks to Jan Greenway for the copyright research. Q6 f(x) = x2 – 9x – 20 a) Find an iterative formula for f(x) = 0 in the form xn+1 = pxn + q where p and q are constants to be found. (iii) This is quite tricky to find from the pulley to the wall, the distance in the first graph... ... ... (× 1 5 0.1 ... In this case the graph would be in two parts like in the first diagram. To 2 d.p. ... 0.15 ... ... 1.2 .3 4 ... ff Using lots of moments ... P ... the s ... ... 25 m ... initially 25 m from the pulley and 10 m towards the pulley in the first 30 seconds after starting from rest. = 128 + 64x – 9696x2 + ...) Using the product rule: if y = ... 0.0245 ... P(3) = 2 × (1.2 (3.5) ... 50 ... 0.20 ... 0.15 ... ... × 100 = 0.048% (2 s.f.) 5 5 6 Q4 a) (2 - 5x)6 = 26 + 1 2 x 375x 2 = 64 (1 - 15x + 4 - ... x 2 - 2x - 15 x 2 + 2x + 8 x 3x ^ + 3h^ - 5h ^x + 2h^ + 4h x ^x + 3h = ^x - 5h ^x + 4h Chapter 2 Algebra and Functions 2x a = x 92xx + 20 k Factorise the expressions in both fractions and cancel. o+e o e o - 19.00... It is found that 71.9% of bottles contain less than 506 ml of vinegar. Tips Lots of useful tips to help get your head around the tricky bits. 3x Q4 a) f(x) = + with domain x > –1 has a range x 1 f(x) < 3 — you can work this out by sketching the graph. When you're resolving forces on an object that is on an inclined plane, it's usually best to resolve parallel and perpendicular to the slope. So x = 62.4°, 117.6°, 242.4° and 297.6° (1 d.p.) Q4 a) sec x = 2 fi cos x = 0.5 fi x = cos-1(0.5) = 3 There is another positive solution in the interval fp 5p 0 ≤ x ≤ 2p at 2p - 3 ... 3 1 ... x = 3 p - ... ... Integrate ... for (x < 2) ... If you're given a definite integral to find using a substitution, it's important that you remember to change the limits to u. Don't forget to relate the answer back to the situation given in the question. y = (x - 2)3 between x = 2 ... x = 3 ... e.g. evaluate ... 2 3 + 2 ... ff You just need to remember that the limits of integration don't change ... within a = 1, b = –2 and n = 3 to work out the integral. Here's an example to show how you can use this technique to integrate functions of the form (ax + b)n. Example 1 Find the area enclosed by the curve y = x2, the line y = 2 – x and the x-axis. Q10 The volume, V, of a hemisphere of radius r cm, varies with its temperature, q, at a rate of k cm3 °C–1, where k is a constant that depends on the material of which the hemisphere is made. Example 1 Learning Objective: ff Be able to integrate rational expressions in which the denominator can be written as a product of linear factors, by splitting them into partial fractions. ff If the force which is trying to move the object is greater than the frictional force, the frictional force won't be large enough to prevent motion, and the object will move. 61 15 sin = – 17 17 15 8 Notice that 180° ≤ q ≤ 270° — this puts us in the 3rd quadrant of the CAST diagram so sin will be -ve, cos will be -ve and tan will be +ve. b) In sunflowers that measure 7.5 ft or taller are harvested. 1 9 = 1.9 ^ + 0.1 + 0.0694 1 This is the same as ^1 + 3xh4 with x = 0.1 0.3 < 3 so the approximation is valid. -2.5 b) y = |f(x)| y = |f(x)| y ff Also, f(x) = |f(x)| + |x| ... for x ≥ 0 ... ff (x) = ... |f(x)| ... Q) a) ... Range: f(x) ≥ 0 ... (the ... ... 3 2 1 |x - ... -5 ... -2 ... 0 ... y... S o y = |x - 1| ... Exercise 5.1 — The modulus function f(x) k x 4 3 5. 8^8 - 1h (1 + 2x)8 = 1 + 8(2x) + 1 # 2 (2x)2 ... 2 # 1 # 2 (2x) + ... Conditional Probability Exercise 1.1 — Conditional probability Q1 a) P(A') = 1 - P(A) = 1 – 0.3 = 0.7 b) P(A È B) = P(A) + P(B) - P(A Ç B), so P(A Ç B) = P(A) + P(B) – P(A È B) = 0.3 + 0.5 – 0.7 = 0.1 c) P(A Ç B') = P(A) – P(A Ç B) = 0.3 – 0.1 = 0.2 Q2 The probability that neither X nor Y occurs is P(X' Ç Y') = P(X È Y)' = 1 – P(X' Ç Y') Since X and Y are independent, P(X' Ç Y') = P(X) + P(Y) – P(X)P(Y) = 0.8 + 0.15 – (0.8 × 0.15) = 0.83 So P(X' Ç Y') = 1 – 0.83 = 0.17 P^G + Hh 0.24 2 = = = 8 3 P(G|H) = 0.63 0.63 21 P (H) You could give the answer as a decimal instead, but using a fraction means you can give an exact answer. Location of Roots Learning Objectives: ff Be able to locate the roots of f(x) = 0 by finding changes in sign of f(x) between two values of x. Find m and s2 in each case. fi b = 15.3 (3 s.f.) ff So the range of values is 10.77 N to 15.3 N Exercise 1.6 MODELL ING Q7 The lengths of time taken by a group of 56 blood donors to replace their red blood cells are modelled by a normal distribution with a mean of 56 days and a standard deviation of 6 days. 3 -1 1 3x 9x 2 (ii) (4 + 3x)-1 = 4-1 1 + 4 4 x + 16 - ... (- 0x -4 4 (4) x 4 (... Q4=4 + ... 4 - 2 d) Find ... since ... 4 ... ... 1 4 = ... (4 + 3x)-1 ... ... 3 ... 2 ... ... 3 ... -3(4 + 3x)... -1 ... 16 ... ... + Since vertical component of the resultant must be zero. + 2p 5.99 (2 d.p.) ff This shows that as one variable increases, the other decreases. So the equation to solve is: 3x + 8 = –x fi 4x = -8 fi x = –2 (valid since -2 < 0) So the region that satisfies the inequality is x < -2. Graphically, it means that the tangent at xn is horizontal, and so it won't cut the x-axis to find the next iteration. Newton's Laws of Motion Learning Objectives: ff Be able to use Newton's laws of motion in situations that require resolving forces. Points are given by (x, y, z) coordinates and position vectors are the movement needed to get to the point from the origin, given in the form xi + yj + zk. So the tangent will not cut the x-axis. You may also be asked to find probabilities when the mean and standard deviation have been given. Resolving horizontally, taking right as positive: v = u = 18 cos q = 9 So speed = ^"4.011...h2 + 9 2 = 9.85 ms-1 (3 s.f.) Tip: You can also have solved this by spotting that q = 60°, since tan 60° is one of the common values from p.53. So 3x 2 2x 3 + 1 3x - 2 2 ... Plane A flat surface. The sledge lies on a smooth slope inclined at an angle of 35° to the horizontal. There are a few tricky functions which are really easy to integrate — once you've learnt them, you'll be able to integrate loads of complicated-looking functions quickly. Find the magnitude and direction of force F, given that the resultant force on the particle acts in the x-direction. 2x - 1 f(x) = y= 2x - 1 y2 + 1 x = 2 Replace x with f−1(x) and y with x. Show that the bird hits the water in the interval 1.2 < t < 1.3 seconds. Implicit differentiation An 'implicit relation' is the mathematical name for any equation in x and y that's written in the form f(x, y) = g(x, y) instead of y = f(x). Q5 Divide 16x4 by 2x - 3, stating the quotient and remainder. (1 + 2x)-1 = 1 – 2x + 4x2 + ... Iteration A numerical method for solving equations that allows you to find the approximate value of a root by repeatedly using an iteration formula. The denominator is the difference of two squares. N Using Pythagoras' theorem: R = ^31.411...h2 + ^69.681...h2 = 76.434... Resolving Forces Exercise 1.1 — Components of forces Q1 a) Horizontal component (Æ) = 6.5 N Vertical component (≠) = 0 N b) Horizontal component (Æ) = 0 N Vertical component (≠) = 18 N c) Horizontal component (Æ) = 20 cos 40° = 15.3 N (3 s.f.) Vertical component (≠) = 20 sin 40° = 12.9 N (3 s.f.) d) Horizontal component (Æ) = 17 sin 25° = 7.18 N (3 s.f.) Vertical component (≠) = 17 cos 25° = 15.4 N (3 s.f.) e) Horizontal component (Æ) = 5 cos 10° = –4.92 N (3 s.f.) Vertical component (≠) = 5 sin 10° = 0.868 N (3 s.f.) f) Horizontal component (Æ) = 8 cos 60° = 4 N Vertical component (≠) = –8 sin 60° = –6.93 N (3 s.f.) Q2 a) 10.4 N b) 21.3 N c) 3.21 N 21 21 c) (25 cos 45°)i + (21 sin 45°)j b = 2 2 i + 2 2 j Q3 a) 7.21 ... Q3 a) 7.21 N (3 s.f.) ... 25 ... 22.4 ... 2.8 ... ... If ... When ... so ... r = 20 cos ... 1 s.f. ... 5 ... ... = 4.51... Q4 a) ... 2 ... as ... (a) dt2 = 3t2, dt2 = 2t, dt The equation ... 15.011... dx dy dx Q4 a) dt = 3t2, = 2t dt When t = p, dy 2 ... ... dy = 3p dx dx Q5 dx dy = 2t sin t + t2 cos t, dt = cos t – t sin t dt dy dt dy t3 cos t + t3 sin t dt dt dy t3 cos t + t sin t sin t + t2 cos t So dx = dt = 2t sin t + t2 cos t Q6 a) When t = p, dy = 3p dx dx fi = 2t + 2sin t cos t ... ff Then you need to use the answer to part a) ... the degree of the remainder must be less than the degree of the divisor (1) so it must be 0. So the product is 0 ... ... Using the result from part a), (sec q + cosec q)(sec q – cosec q) º (tan q + cot q)(tan q – cot q). ff She has assumed that the probability that it rains on one day is not affected by whether or not it rained the day before (i.e. that they are independent events) but this might not be true in real life. c) fi (x) = ... ff Find the value of x: ... ... x ... 4 ... ... , ... x ... 1 ... ... this ... 2 ... 1 ... ... Q f(x) = x and g(x) = ln (x + 1), both with domain {x : x > 0}. Each of the natural numbers has its own ... If there ... 1 1 ... Q. 2 = ... 1 + 8(2x) + 28(4x ... + 56(8x3) + ... Calculate W. The driving force of the truck ... Let X be the number of the pigeon weights. Hint: Leave the numbers from each ... that you can ... 2 ... 6 ... ... is ... between 1 and 2. Particles of mass MA kg and MB kg are placed at A and B respectively. Moments So far you've seen that a force applied to a particle can cause the particle to accelerate in a straight line in the direction of the force. Chapter 10 Vectors 251 Tip: Remember that to multiply by a scalar, each number in the column vector should be multiplied by that scalar. ^3x + 2h5 45 ·^3x + 2h4 18 +C Tip: Don't forget to rewrite your answer in terms of x again at the end. 254 Chapter 10 Vectors 2 b a + 2 ff OC is the distance from the origin ^ a2 + b2h + c2 = 2 y +C (a,b,c) O x z A2 X So the magnitude of any vector = ai + bj + ck is the same as the distance of point (a, b, c) from the origin: |v| = |ai + bj + ck| = a2 + b2 + c2 v The unit vector in the direction of vs is ... it is ... it is for 2D vectors. It grows by 2 × 10-2 mm for every 1 °C rise in temperature. Moments Exercise 1.1 — Moments Q1 a) Moment = Force × Perpendicular Distance = 3 × 2.5 = 7.5 Nm anticlockwise b) Moment = 10 sin 30° × 4 = 12 Nm clockwise c) Moment = 5 × 6 = 30 Nm anticlockwise d) Moment = 20 sin 70° × 0.5 = 9.40 Nm clockwise (3 s.f.) e) Moment = 15 × 6 = 90 Nm anticlockwise Q2 a) Taking clockwise as positive: (2 × 9) – (7 × 2) = 4 Nm clockwise b) Taking clockwise as positive: (19 × 5) + (5 × 3) = 110 Nm clockwise c) Taking clockwise as positive: (30° × 3) – (15 × 1.5) = 22.5 Nm anticlockwise ... = 12 Nm ... 12 Nm ... anticlockwise d) Taking clockwise as positive: (10 sin 30° × 3) + (8 sin 45° × 4) = (15 × 16 2) Nm clockwise ... = 37.6 Nm (3 s.f.) clockwise e) Taking clockwise as positive: (18 sin 25° × 13) – (24 sin 32° × 13) = -66.4 Nm (3 s.f.) anticlockwise f) Taking clockwise as positive: (6 cos 15° × 10) + (2 sin 50° × 8) – (3 sin 28° × 10) = 48.1 Nm (3 s.f.) clockwise ff You can use the ANS button on your calculator to speed things up. du dv dx and into the integration by parts formula: dx dx # x3 ln x dx = ln x x x4 4 – # 4 ln x 1 4 – # x dx = 4 ln x 1 4 – # x3 dx ... This is useful ... 1 x4 ... + C Chapter 8 Integration 215 Until now, you haven't been able to integrate ln x, but by writing it as 1 × ln x, you can integrate it using integration by parts. ^3x - 6h A + B / ^x - 4h^2x - 5h ^x - 4h ^2x - 5h ... x - 6 ... B(x - 4) Substitution: x = 4: 6 = 3A fi A = 2 ... 5 3 x 2 - 2 = 2 B fi B = -1 ... ... ^3x - 6h 1 2 So # dx dx / # ^x - 4h^2x - 5h ... -2x - 5 dx = 2 ln|x - 4| - 1 ... ln |2x - 5| + C ... In Q dx ... ... 1 8 ... 3 ... ... 4h2 ... 3 fi 2 ... 2 ... 3 ... ... 5 ... ... Answers dy dx = ... 1 ... ... So ... this ... ... 2 ... ... 4 ... ... 2 ... ... fi 2x ... ... 1 ... ... x ... ... Answers ... Q2 a) ... 1 ... = 3 ... 2 ... ... fi t ... ... y = 3 ... 6 ... When t = 3 ... x = ... , 2 ... ... so the question didn't specify the form of the answer. ff From the graph you can see that the required range is: {fg(x) : fg(x) > 0}. 80% of this population of 17-year-old boys are taller than 170 cm. Q1 (1 + 4x)4 ... 2 ... 2 ... ... It ... 1 ... ... 2 (–6x) · 3^ - 3 · 1h^ - 3 · 2h + (–6x)3 + ... Is this true? b) Show that 6x – x2 + 13 = 0 can be rearranged into the formula: x = 6x + 13 Tip: The key with the rearranging questions is being able to pick up hints that show how it needs to be rearranged. b) Sketch f(x) and f−1(x) on the same set of axes. 1 1 e tan ` p - p j tan `0 - p j o 4 2 4 11 =1 =2 = - 2c 1 · 1 m - 4 e tan ` p j tan ` - p j o 1 · 1h 4 2 = 2 Chapter 8 Integration 189 c) # 8 cosec ^2x + 1h cot ... 2 ... ... 2 d ... Take the constant outside the integral. Quotient rule A method of differentiating one function divided by another function. p p p p Q3 b = 2 + 3 = 6 , so the values are 0, 6 , 3 , 2 . 1 1 2 = 2 (x – 2)(x + 1)(x + 2) ... 4x ... + (2 + 4x)(x – 2) ... ... 6 ... ... 3 ... ... = 4 ... 2 ... 3 ... ... 6 ... ... x ... ... 16 ... ... fi 2x ... ... Q 2 h2 ... ... Factorise the denominator. The normal reactions at the wall and the ground have magnitude 22 N and 37.0 N (3 s.f.) The two ... 2 x 2 ... 0 ... 2 ... Chapter 14 Kinematics 309 Example 2 A stone is launched from a point 1.2 metres vertically above the point O, which is on flat, horizontal ground. ff P(499 ≤ X ≤ 501) is approximated by P(498.5 < Y < 501.5) e) P(250 ≤ X ≤ 750) is approximated by P(249.5 < Y < 750.5) f) P(100 ≤ X < 900) is approximated by P(99.5 < Y < 899.5) Exercise 2.2 — Normal approximation to a binomial distribution Q1 a) n is large (600) and p = 0.51 ≈ 0.5, so a normal approximation should be suitable. ff So (P + 1) isn't divisible by any of the prime numbers, meaning that it must also be a prime number. Q15 x - 1 3 6 Q16 + 2 + x 1 2x x 1 2 + x 2 Q19 Hint: To turn the 2 into a fraction just use '1' as the denominator. ff So the particle's horizontal displacement from its starting point is 24.8198... × C 3ff 2 (2x - 4) × 3(x – 2), so you need to divide everything by 3 to match the original integral: 198 Chapter 8 Integration # (x - 2) 3 1 x2 - 4x + 5 dx = 3 ... # (2x - 4) 3 (x2 - 4x + 5) ... Using the algebraic method, we solve: (2x + 3)2 = (4x – 1)2 fi 4x2 + 12x + 9 = 16x2 – 8x + 1 fi 12x2 – 20x – 8 = 0 fi 3x2 – 5x – 2 = 0 fi (3x + 1)(x – 2) = 0 fi 1 So x = - 3 . All you do is put these values into your equation and use them to find C (or k), so you won't be able to use calculator to work out a hypothesis test to check claims about the mean of a population that has a normal distribution. x-1 Use this to make an iteration formula and find the value of a root to 3 d.p. 3 2 1 3 dx (x + 3)3 (x2 + 2x) ... dx ... 0 ... ... (2x - 4)2 ... ... 2 ... 8 - ... ... 3 ... ... x ... 1 2 ... ... Your calculator ... ... So ... ... (2 - 3 2 ... 2 ... If the denominator ... ... = 1 ... ... 2 ... x ... There are ... y ... the normal ... ... So ... ... ff The '9' is on its own in the second fraction so try: x3 – x2 – 9 = (x – 3)(x2 + 2x + 3) + 9 (x - 3)(x2 + 2x + 3) ... x ... 2 ... 9 ... ff The LHS can be factorised here. Q9 (x + 1)(x - 1) ... ... Finally take the square root of both sides: x = 9 x-1 9 ff Using the iteration formula xn+1 = xn - 1 to find approximations of the roots. Small Angle Approximations Exercise 2.1 — Small angle approximations Q1 a) sin 0.23 = 0.23 From a calculator, sin 0.23 = 0.228 (3 d.p.) b) cos 0.01 = 1 - 2 (0.01)2 = 1 - 0.00005 = 0.99995 From a calculator, cos 0.01 = 0.999950 (6 d.p.) c) tan 0.18 = 0.18 From a calculator, tan 0.18 = 0.182 (3 d.p.) Q2 f(q) = sin q + cos q = q + 1 - 2 q2 1 = q + 1 - 2 q2 f(1.2) (3 d.p.) ... = 1.255 ... f(0.3) = 0.3 + 1 - 2 (0.3)2 = 1.255 f(0.3) = 1.2509 (4 d.p.) f(0.5) = 0.5 + 1 - 2 (0.5)2 = 1.375 f(0.5) = 1.3570 (4 d.p.) c) f(0.25) = 0.25 + 1 - 2 (0.25)2 = 1.21875 f(0.25) = 1.216316 (6 d.p.) 1 d) f(0.01) = 0.01 + 1 - 2 (0.01)2 = 1.00995 f(0.01) = 1.009950 (6 d.p.) Q3 a) sin q tan q cos q sin q = q2 sin q = q(5q) sin q = 5q3 1 4q b) 1 - 2 (2q) ... 2q2 ... (2q) 2 ... Q5 ... cos 2q ... 2 q 2 = 2b1 - 2 q 2 l = 2 – 2q2 ... 1 1 ... 2 (5q) ... 1 ... 5q 1 q2 cos q ... 2 ... 2 ... 1 ... x4 - ... (2 ... 2 ... q2 = q2 ... ... q sin q - cos q ... ... q2 = ... 2 ... q ... ... Since you're working in radians, q is in radians. Since you're given the probabilities ... ff Find the limits of (x) ... 1 ... P(X ... dx) = ... q2 ... 2 1 3 ... ... Q ... q ... ... 3 ... ... 2 ... sin 2q 2 ... 2 ... ... = q ... ... 1 1 1 (cos 3q + 3 sin q tan q) 1 (1 - 9 2 q 2 + 3q2) 3 1 2 q 2 = 1 + 2 q2 ... 2 ... 2 6 q 2 = 6q2 = 6 = 2 as required. 3 Q. For example: ff She has only taken data from the past 30 days, which might not be a large enough sample to give an accurate estimate, or might not take seasonal variations into account. b) m = ff Draw a diagram to show the forces on the ladder: 50 N ff Resolving forces vertically: R = 50 N 16g ff The ladder is in limiting equilibrium, so: F = mR fi F × 50 = 30 N R 70° ff Taking moments about the base of the ladder: (50 sin 70° × x) = (16g cos 70° × 3) fi ... fi x = ... (50 sin 70° × x) = (16g cos 70° × 3) + (30 × 70°... 0 ... 470.4 cos 70° ... x = 300 sin 70° + 180 cos 70° (50 sin 70°)x = 300 sin 70° + 180 cos 70° – 470.4 cos 70° x = 182.58... The graph shows that there are two solutions to |x – m| = |2x + 3|, one where both (x – 3) and (2x + 3) are negative (i.e. x < -3) and a < (x – 3) is negative and 2x + 3 is positive (-3/2 < x < 3). MODELL ING Values of r close to zero mean there is only a weak correlation. equations and inequalities involving the modulus. -1 Range +5+1 –4 0 1 1 6 12 21 ff The domain and range will often be the set of real numbers, R. The symbol 'ª' means 'approximately equal to'. Product Law A formula used to work out the probability of two events both happening. Describe the effect this has on the particle's motion. 1+x Q11 Hint: Use the binomial expansion, i.e. as a series of terms in x. ff For a body in equilibrium: Total Clockwise Moment = Total Anticlockwise Moment ff By resolving forces and equating clockwise and anticlockwise moments, you can solve problems involving bodies in equilibrium. Exercise 1.3 Q1 If X ~ N(106, 100), transform X to the standard normal variable to find: a) P(X ≤ 100) b) P(X > 122) Q2 If X ~ N(11, 22), transform X to the standard normal variable to find: a) P(X ≥ 9) b) P(10 ≤ X ≤ 12) Q3 If X ~ N(260, 152), transform X to the standard normal variable to find: a) P(240 < X < 280) b) P(232 < X < 288) Finding x-values You might be given a probability, p, and asked to find a specific range of x-values where the probability of X falling in this range is equal to p. Q2 Start with a sketch showing what you know about X. Resolving horizontally, taking right as positive: s = ?, u = 3 cos a, a = 0, t = 1 s = ut + 2 at2 fi 13 = (3 cos a) × t fi t = 3 cos a ... Resolving vertically, taking up as positive: s = 1, u = u sin a, a = –g, t = ... cos a fi 1 = ... fi ... 1 ... ... fi ... x 9.8 ... ... 2 ... 2 ... ... fi ... ... fi ... tan a - g, required 2 cos 2 a 2 Q16 a) Resolving horizontally for particle A, taking right as positive: s = 8, u = 8, a = 0, t = 2 fi ... s = ut + 2 at2 = ... (2) = 2U ... = 4U ... ff Resolving horizontally for particle B, taking right as positive: s = 2U ... ... = 4U cos q When the particles collide, their horizontal displacements will be the same: 2U = 4U cos q 1 cos q = 2 fi q = 60° b) Resolving vertically for particle A, taking down as positive: s = s, u = 0, a = 9.8, t = 2 s = ut + at2 1 = 0 + 2 × 9.8 × 22 = 19.6 m So the particles collide 19.6 m vertically below the point of projection. c) Particle A has fallen 19.6 m vertically down from a height of 45 m when they collide: 45 – 19.6 = 25.4 m above the ground ... = 45.3 N (to 3 s.f.) Inclined planes Tip: Example 3 on page 321 involved resolving forces parallel and perpendicular to a plane, so this shouldn't be completely new. Given that they saw a comedy film, what is the probability that they are under 20? The trick is to write ln x as (1 × ln x). C D 5 2 a) Find P(C|D). What is the maximum possible mass of the bucket of water for which the ladder remains in equilibrium? ff Finally, write out the division again, replacing A, B, C, etc. 1 ff Solving the following for y: ... 2 ... y = 3 x ... 3 x ... 3 Tip: You might be asked to explain why a relation doesn't have an inverse. ff Simple Probability ... ... 1 ... ... Q12 Rearrange the equation to get f(x) = e3xsinx – 5 = 0 f(0) = e3 × 0 sin 0 – 5 = -5 f(1) = e3 × 1 sin 1 – 5 = 11.9... Clipart from Corel® Text, design, layout and original illustrations © Coordination Group Publications Ltd. Example 1 a) Differentiate (3x + 4)5 with respect to x. 1 Replace f(x) with y: y = x + 2 - 1 Replace with f -1(x) and x: f -1(x) = x - 2 - 1 (dx) - k fi for some k = 0 dt D2 D2 The minus sign indicates that the depth of the water is decreasing. You start off as usual, but instead of ending up with two equations in two unknowns, so you end up with equations in two unknowns, so the you ... end up ... = 2 + 4 ... ... 3 5 ... 2 ... 4 ... ... 5 x ... 8 ... 12 ... 2 ... ... 4 ... ... 4 ... 10 4 ... ... 1 ... 16x ... 2 + 8 ... ... x 1 ... ... Q Be able to ... (x) ... x ... 1 ... ... 0 ... ... 1 2 ... (x) ... 2 ... ... 1 ... ... - 2 So the equation of the tangent is ... ... 0 ... ... A) ... ... y ... ... 45 ... ... 0 ... ... (2 ... ... 2 ... ... Q13 ... ff So the ... ... 3 ... ... the ... 1 ... ... 4 ... ff If ... ... z ... 4 ... 2 ... 8 ... 9 ... ... D ... ... 1 ... ... 3 ... ... 2 ... ... 4 ... ... 2 ... 2 ... 4 ... ... 2 ... x ... ... Q ... ... 2 ... 1 ... ... 1 ... (a ... 7 ... 9 7 ... ... fi b ... = ... - f ... x - 4 ... 7 = 2 ... 2 ... ... 0 ... ... ff Have you assumed that two (or more) events are equally likely? 1 ... j ... Find the unit vector (i.e. along the y-axis). Geometric Sequences and Series 5. dx dx dx 2y - 3 dy At the stationary points, = 0, so: dx 2x + 2 = 0 fi x = –1 3! 5 When x = –1, y2 - 3y + 1 = 0 fi y = 2 = 2.62 or 0.38 (2 d.p.) So there are 2 stationary points with coordinates (–1, 2.62) and (–1, 0.38). 2 ÷ 56.84 = 0.26 (2 d.p.) Example 3 A uniform ladder of mass 11 kg and length 4 m rests on a smooth vertical wall, and on a rough horizontal floor, at an angle of 58° to rough horizontal ground. W = 44.44... F mg sin q g mg mg cos q Tip: Remember — there will only be friction if the surface is rough. 2 + C Example 4 cos x dx. dy f(k) 2x2 Q2 a) ... ... dx = 2x + 2 ... + f(x) ... 1 ... ... = ... 2 ... 2 ... ... f(x) = ... 1 ... ... 2 ... ... 3 ... 9 ... 1 ... ... 0 ... ... 2 ... 4 ... 1 ... Q3 ... ... 1 ... ... 2 ... ... 4 ... ... 2 ... ... 1 ... ... 2 ... ... x ... ... 1 ... Q ... 2 ... ... 2 ... + ... (x - 4)2 ... 2 ... 1 ... 2 ... ... 2 ... 4 ... 3 ... 4 ... 2 4 ... ... - ... ... 2 ... 8 ... ... 2 ... 4 ... 2 ... ... 3 ... 3 1 ... Answers 413 1 2 1 Second part: f'(x) = 2 × = 2x + 1 2x + 1 Putting it all together: 6 (2x + 1) - 3x - 16 1 f'(x) = - = 6 ... (2x + 1) ... (2x + 1) 9x + 5 = (2x + 1) ... You could have left your answer as 2 ... fraction. f(4) = e4 – (10 × 4) = 14.598... Learning Objectives: MODELL ING Hypothesis tests of a population mean You'll be carrying out hypothesis tests of the mean of a normal distribution, based on a random sample of n observations from the population. ff Put the values of x you've found into the original equation to find the values of y. a) Show that when the rod is in equilibrium, the normal reaction at A must be zero. 7 - x3 b) Use the iteration formula xn+1 = with x0 = 1 to find values for x1, x2, x3 and x4 to 3 decimal places. ff Be able to apply ... ff Example 3 Find ... -7 So the two values ... ... the ... ... 3 ... ... # ... ... 1 ... ... n 2 ... ... Answers 427 0 0 # -3 16–72x = ... -7 × 12 ln 16 - 2x D -3 ... 3 ... 2 ... ... 3 x ... = ... 7 ln 16 – 7 ln 22 = 7 ln 16 ... 22 = 7 ln 8 11 ... fi x = 1 ... 11 ... in ... fi k2 2 2 12 11 ln 18 = 12 ... 0 ... in the interval 0.983 ≤ x ≤ 0.984 ... ... 2 ... ... 1 ... The ... 3 ... (x) ... 2x @ ... 1 ... a ... ... (x) ... ... 2 ... x ... 2 ... x - 1 ... x ... 2 ... 2 2 ... ... 4 ... 3 ... 1 ... = 13.5 ... ... 13.549. PMCC hypothesis testing To test whether a value of r is likely to mean that the two variables are actually correlated, you need to do a hypothesis test. 1.875 < 2.326, so the result is not significant. Q8 Divide consecutive terms to find the common ratio r: second term -5 = e.g. r = first term 2 Q9 a) Common ratio r = second term 16 = = 4 first term 4 ÷ 8) First term: a = 4 3 n-1 un = arn – 1 = 4 # b 4 l 7 8th term: u8 = 1 # b 3 l = 1 # 2187 = 2187 4 4 4 16384 65536 Q10 = 0.8. un = arn – 1 = (0.8)n – 1 7th term: 196.608 = u7 = a(0.8)6 = 750 196.608 = a(0.8)6 fi a = 196.608 0.86 second term = -2.4 = Q11 a) Common ratio r = 0.8 3 first term 5) Continuing the sequence gives -1.506, 1.2288, -0.98304. ff X is a continuous random variable that follows a normal distribution, you can describe the probability distribution of X using just two measures: its mean, m, and variance, s 2. Integration is a combination of the previous two. Integration Using Partial Fractions 219 11. 2 In # 0 3 , ff Is the probability based on past data? point) the object will not fall. -1 23 mm fi ff Resolving vertically, taking up as positive: s = 0 , u = ... = -9.8 Using v2 = u2 + 2as: 0 = (23 sin a)2 + (2 × -9.8 × 4.8) fi = 529 sin2 a = 94.08 MODELL ING 4.8 m Tip: When the ball reaches its maximum height, the vertical component of its velocity will momentarily be zero. fi sin2 a = 94.08 ÷ 529 = 0.1778... The product rule is a way of differentiating two functions multiplied together. Find the inverse of the function f(x) = 2x + 1 with domain {x : x ≥ 0}, and state its domain and range. 1 c) Both a) and b) are valid for |x| < 4 . But you can use the mean and standard deviation of the standard normal distribution, Z, and then use these to work out probabilities for the distribution. ff Then the hypotheses are: H0: m = 12 , H1: m < 12 So this is a one-tailed test. There's an (x – 1) on the top and bottom which will cancel. ff Draw a sketch to help you work out the ... for the distribution. 3 Q11 Original graph New graph y = 3(x – 4)3 + 5 y Sequence of transformations x Translate by e o 4 5 y = f(x + 4) i.e. 4 right and 5 up. 60° = 25 Q3 a) Prove that f(x) ≥ 4) Sketch the graph of f(x). Write f(x) = (x - 3) ... Q12 1 Original graph New graph 2. Q12 a) Differentiate y = e 2x using the chain rule. ff Draw a vertical line up from the x-value of your starting point (x0). Tip: In the exam you'll usually be given the graphs of y = x and y = f(x) and the x0 value, but in this case you're given x0 alone. ... Find the interval ... ... 2 ... ... 2 ... ... 2 ... ... The probability distribution of F, so show the range you need to integrate between. ff Find the mass of the particle. If two events are independent, it means you can multiply their probabilities together. 3 Q1 A 1 0 Tip: You need to find the x-coordinate of the intersection between the line and the curve, as well as the x-intercept of the curve, so that you know the limits to integrate between. ff Now find the ball's vertical displacement when it passes through the second target. Implicit Differentiation Exercise 10.1 — Implicit differentiation d d 3 d 2 d (y) + (y) = (x) + (x) dx dx dx dx 3y2 d d + 2y dy = 3x2 + 2x dx dx d dy dy = 3x2 + 2x fi (1 + 3y2) = 2x fi = 1 + 3y 2 Q1 a) dy 2x(1 + 3y2) dx ... dx 3x2 + 2x dx ... dy = 1 + 3y2 ... ... dy 2 ... 2 ... 2 ... ... 2 ... ... 2 ... 2 ... ... dx ... d) ... dx 2x + 3 dy dx ... ... 2x ... ... 2y ... ... dy ... dx ... ... 4y ... ... dx ... ... = 2 ... ... d ... ... dx ... ... 3 ... ... dy ... dx ... ... 2x ... 2 ... ... dx ... dy ... ... dy ... ... dx ... ... 2 ... ... x ... ... dy ... ... d ... dx ... x ... y ... dx ... dx ... ... 2 ... ... y ... ... dy ... ... dy ... ... dx ... ... cos ... ... y ... ... dx ... ... dx ... ... y ... ... 2 ... ... dx ... ... 1 ... 1 ... ... 1 ... ... LHS: e0 + 2 ln 1 = 1 RHS: 1 = 1 So (0, 1) is a point on the curve. You're Reading a Free Preview Pages 182 to 183 are not shown in this preview. Sequences 2. PROBLE M SOLVING Chapter 9 Differentiation 179 Chapter 8 Integration 1. Transformations of Graphs 7. 230 ff Be careful though — this only applies when the range of x you're interested in continues (no break or jump in the line of the graph). The normal reaction at the wall is 24 N. Tip: Doing it this way is a bit simpler than using the conditional probability formula. until it meets the curve y = f(x). ff So, rounding to the nearest second, the shortest day (t = 6) will occur when the x values are the same as the largest possible value. So the shortest day (t = 6) will occur when Ct + D) = -1 (i.e. when Ct + D = p. (1 + 6(0.01))-1 = 1 - 6(0.01) + 36(0.012) = 1 - 0.06 + 0.0036 = 0.9436 100 × 106 0.9436 d) 100 × 100 = 0.02% (1 s.f.) a 106 k In the answers that follow, the expansions will just be stated. Example 2 Simplify the following: 8 12 a) 5x ÷ 3x Turn the second fraction upside down. They give the heights of the sides of the trapezia. Together with the force vector form, these components form a right-angled triangle. Then x = 2x and y = –x – 1 ... x + 1 ... 2x2 + 1 ... x - 1 = 0 ... 2 ... ... x = ... 2 ... ... x - 1 ... x ... ... ff Don't forget — the point does not lie in the critical region and is not significant. . 1#2 2 2 x 2 = 1 - 9 # 2 2 ... x < 1 , the mean is 11, and the standard deviation is 2. ... - 1 d 2 y is negative. Q8 ff Find the distance from O to the point of the plane's destination from its starting point to the particle's starting point. ff Swap round the domain and the range of the function. The table below shows the results. It comes from writing sin x tan x = cos x (as shown on p.192). ff This is the probability P(C|W'). Chapter 15 Dynamics 347 Q4b) Hint: When the string is cut, the tension force K will disappear. The particle passes through the point with position vector (bi + 12j) m, where b is a constant. A force of magnitude 8 N acting along the line of greatest slope of the plane holds the particle in equilibrium. Chapter 2 Algebra and Functions Example 1 Divide x4 – 3x3 – 3x2 + 10x + 5 by x – 2. ff For the curve y = Q5 Hint: Remember from page 136 that if (a, b) is a point of inflection: d2 y = 0 (where x = a), dx 2 d y and changes sign at x = a. Q5 a) P(X < 75) = 0.15 456 0.15 0.73 61 68.75 0.15 u P(X > 75) = 0.85 75 – 68 = 7 fi P` Z < s j P` Z < s j = 0.85 Answers F(z) = 0.85 for z = 1.04 fi s = 7 = 1.04 fi s = 7 ÷ 1.04 = 6.73 (3 s.f.) Q6 a) P(M > 4.5) 0.5 P(M < 4.5) = 0.9192 fi P a Z > 4.5 m - m k = 0.9192 ... F(z) = 0.9192 for z = 1.40 (2 d.p.) 30 - m fi P(M < 1.4s = 4.5 - m ... m - 1.4s fi P(M < 36) = 0.9953 ff F(z) = 0.9953 for z = 2.60 ... 36 - m = 2.60s ... 0.0548 for z = –1.60 (2 d.p.) fi 1.4s = 36 – 2.6s fi s = 7.875 ... 6.5 fi P(X < 4.5 - m k = ... fi m = 4.5 + 1.4 × 6.5 ... 1.40 ... m = ... 4.5 ... 1.4 ... 2.60 ... = 0.9713 fi P a Z < s k = 0.9713 for z = 1.90 (2 d.p.) ... 4.1 ... 3.5 ... fi ... fi P(X > 600) = 0.0548 ... a 592 - m fi PaZ < s k = 0.2420 F(z) = 0.2420 for z = -0.70 (2 d.p.) 592 - m fi s = -0.7 fi m – 592 = 0.7s 2 Adding equations 1 and 2 gives: 696 – 592 = 1.96s + 0.7s fi 104 = 2.66s fi s = 39.097... Putting s = 40 into equation 1 gives: m = 0.7 × 40 + 592 = 620 Q8 P(X > 33) = 0.1056 fi P a Z > s k = 0.1056 fi P a Z < s k = 0.8944 F(z) = 0.8944 for z = 1.25 (2 d.p.) 33 - m fi s = 1.25 fi 33 – m = 1.25s 1 33 - m = 1.25s 1 21 - m P(X < 21) = 0.0401 0.0401 fi P a Z < s k = 0.0401 P(X > 21) = 0.9599 fi P a Z > s k = 0.9599 – 21 fi P a Z < s k = 0.9599 F(z) = 0.9599 for z = 1.75 (2 d.p.) m - 21 = 1.75s 2 Adding equations 1 and 2 gives: 33 – 21 = 1.25s + 1.75s fi 12 = 3s fi s = 4 Putting s = 4 into equation 1 gives: m = 33 – 1.25 × 4 = 28 Q9 P(X > 10) = 0.0322 fi P a Z > 10 - m k = 0.0322 fi P a Z < 10 - m k = 0.9678 F(z) = 0.9678 for z = 1.85 (2 d.p.) fi 10 – m = 1.85s 1 ... 8 ... ... fi P(X > 8) = 0.3632 fi P a Z < 8 - m s k = 0.6368 F(z) = 0.6368 for z = 0.35 (2 d.p.) 8 - m fi s = 0.35 fi 8 – m = 0.35s 2 Subtracting equation 2 from equation 1 gives: 10 – 8 = 1.85s – 0.35s fi 2 = 1.5s fi s = 1.33 (3 s.f.) Putting s = 1.33... into equation 1 gives: m = 10 – 1.85 × 1.33... = 7.53 (3 s.f.) So m = 7.53 and s = 1.33 (3 s.f.) Q10 ... There's a sign change (and the function is continuous in this interval), so there must be a root in this interval. ... Q6 ... ff You can use the iteration ... If you've shown that ... ... Hint: ... Exercise 11.1 ... 1 ... friction between the wall and the plank is 0.55, find: a) the value of M, b) the distance of the centre of mass of the plank from A. a) Find the general solution of this equation. So is sec2 q – cosec2 q. ... is the same as ... (sec q + cosec q)(sec q – cosec q). Given that the speed of projection of A is twice the speed of projection of B, find: a) the value of q, b) the speed of projection of each of the two particles. Show that the object will be at rest on the plane for a time of s m. Q4 a) Putting x = 0 into the equation gives: 1 + y2 = 5y fi y2 – 5y + 1 = 0 ... 5 ! 21 ... fi y = 2 ... y = 4.79 ... or 0.209 ... (3 s.f.) ... P(X > 75) (being with constant velocity). Q5 arr = 2, 3 12 = SY = a 1 - 12 - 1r = 1 r-1 The particle then moves with constant speed. Q8 a) Putting x = 1, y = 5 2 + 5 2, y ... plugging this into the second equation gives: 3 12 - 12r = fi rr 12 – 12r = 2 – 12r fi 0 = 4r2 - 4r + 1 = 2 (2 4r – 1) = 2 0 - 1 = 0 fi r = 2 ... 2 ... = 1 Equilibrium A state where there is no resultant force or moment acting on a body, so it remains at rest (or moves with constant velocity). Chapter 2 Algebra and Functions 11 13 fi ff Express the following as partial fractions: a) b) c) (x - 1)(x + 3) (x + 1)(x + 2) (x + 3)... d) ... 3 ... 2 ... = 3 ... 3 x ... 2 ... ... x ... ... 2 ... 2 ... 1 ... ... 3 ... ... 2 ... 5 ... Q1 ... 6 ... ... 2 ... ... 4 ... ... dx ... ... 2 ... 4 ... ... 3 ... ... 4 ... ... 1 ... ff ... fi ... 2 ... ... 3 ... ... Q2 ... ... 3 ... = 6 ... 2 ... ... 8 ... 2 ... ... 3 ... = ... 1 ... ... 1 ... ... 1 ... ... 3 ... ... Q ... ... 2 ... ... So ... 2 ... 2 ... x ... ... 3 ... 1 ... ... = ... 2 ... ... 1 ... ... 3 ... ... = 0 ... 2 ... ... 1 ... ... 1 ... ... 2 ... ... 3 ... 1 ... ... 1 ... ... 1 ... 2 ... ... 2 ... ... Product Rule Exercise 5.1 — Differentiating functions multiplied together ... y ... ... x ... + ... 1 2 ... x ... 2 ... ... Q1 ... ... 3 ... ... x ... 2 ... ... 2 ... ... 2 ... ... 5 ... 2 ... ... fi ... ... dx ... ... 2 ... 3 ... ... Q ... ... 2 ... ... 2 ... ... 4 ... ... 6 ... 3 ... 2 ... ... x ... 6 ... ... dx ... ... 2 ... 3 ... ... 1 ... ... x ... ... dx ... ... Use the chain rule ... You could also write this as ... = 4 ... ... 3 ... ... 370 Answers If x = 0 ... ... 2 ... 1 ... ... 3 ... ... ff ... ... 4 ... ... 3 ... ... 2 ... 1 ... ... 2 ... ... This could be turned into a function by restricting the domain to x ≥ 0 — see page 16. ff However, you'll nearly always be given a formula that converges to a certain root, otherwise there's not much point in using it. ... 3 ... 1 ... ... 2 ... ... 3 ... ... 27 ... 1 ... 2 ... ... so the ... ... x = 2 ... 1 ... 2 ... ... The R Addition Formulas Exercise 8.1 — Expressions of the form a cos q + b sin q Q1 3 sin x – 4 cos x º R sin (x – a) ... R sin (x – a) º R sin x cos a – R cos x sin a fi 3 = R cos a and 4 = R sin a ... fi tan a = 4 3 fi a = 53.1° (to 1 d.p.) R2 = 32 + 42 = 25 fi R = 5 ... 3 sin x – 4 cos x º 5 sin (x – 53.1°) ... Q2 ... R sin a = 2 ... R cos a = 1 ... fi tan a = 2 fi a = 63.4° (1 d.p.) ... R2 = 12 + 22 = 5 fi R = 5 ... ff Find the maximum value ... the minimum is cos 0 ... –1 ... in ... Q10 Particle M has ... ... 2 ... 2 ... ... 11 ... 2 ... fi ... ... = 13 fi R ... 2 ... ... 13 sin (x – 33.7°). Particles of mass M kg rests on the plank at B, as shown. The plank rests on supports 1 m from each end, at C and D. Q3 B 1.1 m A 20° C 1.6g A uniform beam, AB, of mass 1.6 kg and length 1.5 m, rests with A on rough ground. –1(x) : f -1(x) ≥ 0}. A) If the surface area of the snowball at all times t hours is A cm2, find dt when r = 5.5 cm. Q8 y y = f(x)y=x 3 1 (3 s.f.) y = f -1(x) y = f -1(x) ... ... (1.5 Q9 a) y = f(x) ... has a domain of x > 0, giving a range of y ... f(x) ... 3 x ... Q ... 3 ff Put ... f(x) fi 3 ... x ... Q7 ... 2 ... ... (1.5 ... 2 ... ... 2 ... ... x ... 0 ... ... x ... ... 2 ... ... x ... ... fi ... 2 ... 2 ... a ... x ... f(x) ... ... x ... ... -1 ... ... x ... ... y ... ... 0 ... ... 2 ... ... x ... ... x ... ... x ... x ... ... 1 ... ... 2 ... ... f -1(x) ... ... ff Show that the vectors are parallel and that the points are collinear. 4 ff Now solve for each factor's ... A ... ... 2 ... 3 ... ... 3 ... ... 3 ... 3 ... 1 ... ff Split up the area C: ff The area C1 is easy — it's the area under ... 1 ... ... 2 ... ... the curve and the ... 3. Correction to a discrete distribution to a continuous distribution. m So tan q = 23.557... Q6 Find the equation of the tangent to the curve x = ^ x + 2 h ^ x + 7 h at the point (0, 2). ff Taking moments about A: ... ... Using the double angle formula: ... ... 2 ... ... 2 ... 3 3 = # 3 sin 6x + sec 2 1 x dx = :- 3 cos 6x + 2 tan 1 2 6 2 0 0 ... Put in the limits. 2x 2 - 5x + 2 5 ff Evaluate # dx, expressing your 2x 2 - 5x + 2 answer as a single logarithm. This isn't really all that different from any of the stuff from the last few pages — the i-component of the vector describes the horizontal motion of the projectile, and the j-component of the vector describes the vertical motion of the projectile. Chapter 10 Vectors 253 2. So the area of each rectangle is F ≈ f(x) × dx. 9 9 1 1 3 cos y = cos x = 10 sin y = 10 Again, in a ... fi sin y = 10 ... y = 2 ... ... 2 ... ... answer ... 0 ... 1 ... ... 2 ... Exercise 1.2 Sketch all graphs in this exercise for -5 < x < 5 unless otherwise stated. A horizontal force P acts on the particle. Choosing Probability Distributions Learning Objectives: ff Be able to choose a suitable probability distribution for a given context. So to find if there's a root between two values 'a' and 'b', work out f(a) and f(b). dy 3 cos t - sin t Q3 = 2 cos 2t dx When t = p: dy 3 - 2 , x = 0 and y = -p. r = 9 2 = 3 ... 9 ... 2 ... 3 ... 2 ... 2 ... ... 6 ... ... 2 ... ... 446 Answers Q10 Rearrange the equation to get f(x) = 4x - 2x3 - 15 = 0 f(-2.3) = (4 × -2.3) - 2 × (-2.3)3 - 15 = 0.134 f(-2.2) = (4 × -2.2) - 2 × (-2.2)3 - 15 = -2.504 There is a sign change (and the function is continuous in this interval). So then multiply (x – 5) by (x + 5) ... 5. Mappings and Functions Exercise 2.1 — Mappings and functions Q1 2 1 2 3 6 12 18 24 1 2 3 4 5 6 7 8 9 10 ... 2 ... 23 13 1 35 20 62 21 1 x+1 b) ... 2 1 2 3 ... 1 ... 0.1 ... 0 ... Q5 f(1) ... 2 ... log10 ... 2 log10 ... 1 2 ... ... 24 ... 2 = ... (2 ... 2 ... 4 ... 0) ... 1 ... 0 ... 1.0 ... 2 ... 5 ... 1 ... 3 ... Q4 ... ... 3 ... 1 ... 1 ... ... 2 ... 2 ... ... 3 ... ... So minimum value of ... Show that ... y ... fi ... 46.084... Then multiply (x – 5) by (x – 5) ... you don't ... have to worry ... the ... ... 2 ... 3 ... ... Q ... ... 2 ... ... y ... ... = 4 ... Q6 a) Show that the equation ex – 10x = 0 has a root in the interval 3 < x < 4. ff Transform the first probability for X into one for Z. ... ... 2 ... b ... c ... from the ... ... a cos x ... A ... ... Exercise 1.2 ... Q1 ... ... When differentiating functions involving the sum of products ... 3 ... 7 ... 2 ... ... 1 ... ... 2 ... 5.472... Q5 Heat, H, is lost from a closed cylindrical tank of radius r cm and height 3r cm at a rate of 2 J cm-2 of surface area. ff First use Pythagoras on the green triangle. Taking moments about A: (1.8g × x) + (3.5 sin 80° × 7) = 95 cos 71° × 3.5 fi x = 84.123... The particles are held at rest with A 35 cm from the pulley. N ff Using Pythagoras' theorem to find the magnitude: |R| = RH2 + RV2 = ^15.035...h2 + ^ 2.492...h2 = 15.2 N (3 s.f.) ff Using trigonometry to find the direction: Tip: Remember that the direction of a force is usually measured anticlockwise from the horizontal. 1 = x - 1 ff Rearrange this equation to find x: 1 = x fi x = 1 + 1 fi x = 1 + y ... y = x - 1 ... y ... 2 ... y - 1 ... ... fi ... ... x - 1 ... 2 y ... So ... 2 ... ... 2 ... ... (x) ... 2 ... = np = 250 × 0.5 = 125 and s2 = np(1 - p) = 250 × 0.5 × 0.5 = 62.5 So Y ~ N(125, 62.5) Tip: If you used a continuity correction here, you'd work out P(Y > 130.5) instead, which is 0.243 (3 s.f.). Q5 a) Q2 fi x = - 1.08) = 17 fi x - 1.08 = 1.27 and (2p - 1.27) = 5.01 fi x = 2.35 and 6.09 (3 s.f.) Q2 g) (8 cos x + 15 sin x)2 = 172 cos2(x - 1.08) The function cos2 x has a maximum value of 1 (since all negative values of cos x become positive when you square them), so the maximum value of (x) = 289 cos2 (x - 1.08) is 289 × 1 = 289 . So the maximum value of the function is 289 (1.08 + ... cosec x ... ... n 2 ... ... So ... of 25.4 m above the ground. The same principle will apply to a lot of real-life differential equations questions. When t = 2, the particle has position vector (10i + 3j) m with respect to a fixed origin O. Tip: Don't forget to ... that the line of action of the force and its distance from the pivot are perpendicular to each other. Cobweb and staircase diagrams The instructions below show how to draw a cobweb or staircase diagram for the iteration formula xn+1 = f(xn). ff Then find the rate of change of A with respect to t is . Therefore, the particles collide at a height of 25.4 m above the ground. The same principle will apply to a lot of real-life differential equations questions. When t = 2, the particle has position vector (10i + 3j) m with respect to a fixed origin O. ... -3 ... 2 ... x -1 ... ... x ... fi ... ... 4 ... 2 ... 7 ... 3 ... 7 ... ... 3 ... ... a 2 ... ... 3 ... ... 15 ... 1 ... = ... ff You need to find ... 7 ... 7 ... 2 ... 9 ... 6 ... ... 2 ... ... = ... Find ... ... 7 ... 2 ... ... x ... 3 ... 2 ... 6 ... ... 2 ... ... 2 ... ... -3 ... 7 ... 2 ... ... ff You need to ... 4 ... of y = x2. Q1 ... Point P lies on a line ... Q3 a) i) ... ... 4 ... 7 ... x ... 2 ... ... of B ... ... So you ... ... KK4.5OO = 2.5i + 4.5j + 3k O K O K O KK0 OO KK 3 O K O+3 O K3 O L P L P L P Tip: Imagine a vector triangle: you get from O to R via P you go along OP and then along PR, hence OP + PR . dy d) y = ef(x) fi = f'(x)ef(x) dx dv du Putting it all together: dy = dv du sin ... 2 x sec2 x = 2 sec2 (2x) dx dx fi = 2 sec2 (2x) etan (2x) dx dy dy du = sin x = (sin x2), so let y = u4 where u = sin x2 dx du du For , set up another chain rule: dx ... u = sin v, v = x2 ... dv du dx ... = cos v = cos x2, = 2x ... dx dv dx du dx ... = 8x sin3 x2 cos x2 dx du dx

dy b) = f′(x)ef(x) dx f(x) = sin2 x = (sin x)2, so let y = u2 where u = sin x dy du f′(x) = × = (2 sin x) × (cos x) du dx = 2 sin x cos x dy sin2 x fi = 2e sin x cos x dx c) First part: y = tan2 (3x) = (tan (3x))2, so let y = u2 where u = tan (3x) dy = 2u = 2 tan (3x) du 416 Answers du set up the chain rule again: dx u = tan (3x), so let u = tan v where v = 3x du dv = sec2 v = sec2 (3x), =3 dv dx du du dv 2 fi = × = 3 sec (3x) dx dv dx dy dy du fi = × = 6 tan (3x) sec2 (3x) dx du dx Second part: dy = cos x dx Putting it all together: dy = 6 tan (3x) sec2 (3x) + cos x dx With practice, you should be able to do some of the simpler chain rule calculations in your head, d e.g. dx tan2 (3x) = 6 tan (3x) sec2 (3x). + 0.3410... 1 b) The gradient of the normal at t = 2 is 3 . ƒƒ Rearrange the equation to make x the subject. X - 70 So under H0, Z = ~N (0, 1) . ƒƒ Moments are either clockwise or anticlockwise. Integration of Trigonometric Functions Learning Objectives: ƒƒ Be able to integrate functions of sin x, cos x and sec2 x. Tip: This integral is of the form f′(x) × [f(x)]n (p.197). 1 a) b) –3 sin x c) –3 cos x – 3 sin x 7 cos x x d) sin 5x e) cos` 7 j f) 2 sin(–3x) p p g) 5 cos`3x + 5 j h) –4 sin`4x - 3 j i) cos(4x + 3) + sin(3 – 4x) 1 Q2 Integrate 2 cos 3q – sin q with respect to q. The frictional force can take a range of values, and will reach its maximum (F = mR) when the body is on the point of moving, or in ‘limiting equilibrium’. 4#3 4#3#2 Q9 a) (i) (1 + ax)4 = 1 + 4ax + 1 # 2 (ax)2 + 1 # 2 # 3 (ax)3 4#3#2#1 + 1 # 2 # 3 # 4 (ax)4 = 1 + 4ax + 6a2x2 + 4a3x3 + a4x4 -3 # -4 (ii) (1 – bx)–3 = 1 + (–3)(–bx) + 1 # 2 (–bx)2 + ... ƒƒ x2, x3 and all further iterations are the same when rounded to 2 d.p., so the root is x = 1.80 to 2 d.p. Tip: This doesn’t mean you’ll never see a diverging formula in an exam, but if you do it will usually be followed by a question like ‘what do you notice about the iterations?’. So 1 + 2 must be irrational. ƒƒ This value now gets put back into the formula to find x2: x1 = 1.8171..., so x2 = 3 x1 + 4 = 3 1.8171... Tip: You could just find the stone’s horizontal and vertical displacements separately, then write it in vector form at the end. dt dV V = pr2H, so = pr2. Chapter 10 Vectors 251 Glossary A Glossary All the definitions you need to know for the exam, plus other useful words. Q3 Evaluate the following definite integrals: p p 2 3 # sin x dx b) #p sin 3x dx c) a) 0 6 # -1 2 3 sin ^px + ph dx px Q4 a) Integrate the function y = 2p cos` 2 j with respect to x between the limits x = 1 and x = 2. For friction problems involving a moving object, see p.339-341. R = 43.5 N (3 s.f.) Q3 a) Resultant, R = (6i – 10j) + (4i + 5j) = (10i – 5j) N |R| = 10 2 + ^ - 5h2 = 125 N Fnet = ma 125 = 5a a = 125 ÷ 5 = 2.236... a) Find the acceleration of the particle as a vector. A force of 60 N acts on A at an angle of 40° to the rough, horizontal plane that A rests on, away from the pulley. The inverse of +7 is −7, so f (x) = x − 7. Vertical transformations will not affect how often the graph repeats itself. A machine which fills boxes of cereal is set so that the mass of cereal going into the boxes follows a normal distribution with mean 766 g and standard deviation 8 g. The block is on the point of sliding. b) If the player can play in goal, what is the probability that they are over 6 feet tall? Projectiles Exercise 1.1 — The two components of velocity Q1 a) Horizontal component: 10 cos 20° = 9.40 ms–1 (3 s.f.) Vertical component: 10 sin 20° = 3.42 ms–1 (3 s.f.) b) Horizontal component: 18 cos 65° = 7.61 ms–1 (3 s.f.) Vertical component: 18 sin 65° = 16.3 ms–1 (3 s.f.) c) Horizontal component: –6.8 cos 21.6° = –6.32 ms–1 (3 s.f.) Vertical component: 6.8 sin 21.6° = 2.50 ms–1 (3 s.f.) d) Horizontal component: 9.7 cos 19.7° = 9.13 ms–1 (3 s.f.) Vertical component: –9.7 sin 19.7° = –3.27 ms–1 (3 s.f.) e) Horizontal component: –24 cos 84° = –2.51 ms–1 (3 s.f.) Vertical component: –24 sin 84° = –23.9 ms–1 (3 s.f.) 460 Answers f) Horizontal component: 16 cos 123° = –8.71 ms–1 (3 s.f.) Vertical component: 16 sin 123° = 13.4 ms–1 (3 s.f.) Q2 Horizontal component: 8 cos 35° = 6.55 ms–1 (3 s.f.) Vertical component: 8 sin 35° = 4.59 ms–1 (3 s.f.) Q3 Horizontal component = 0 ms–1 Vertical component = 45 ms–1 Q4 22 kmh–1 ver a hor (22 × 1000) ÷ 602 = 6.1111... The question asks for exact values of x, so leave the second answer in terms of e rather than giving a rounded decimal from your calculator. Don’t forget the constant of integration (you only need one — not one on each side). c) You need to find h = 13, i.e. 9 + 7 sin x + 2 sin2 x = 13 2 sin2 x + 7 sin x – 4 = 0 This is a quadratic in sin x, so factorise: (2 sin x – 1)(sin x + 4) = 0 1 fi sin x = 2 or sin x = –4 (not valid since –1 2 000 000 k 2 + k - 2 000 000 > 0 Put the quadratic equal to zero and solve using the quadratic formula to get k = 1413.7... 3 3 b) x1 = 3 5 - x = 5 - 1.4 = 1.419, 0 x2 = 1.424, x3 = 1.425, x4 = 1.425, x5 = 1.425, x6 = 1.425 c) The last 4 iterations round to the same answer, so to 2 d.p. x = 1.43 Q4 a) f(5) = 5 – (5 × 5) – 2 = –2 f(6) = 62 – (5 × 6) – 2 = 4 There is a sign change (and the function is continuous in this interval) so there is a root in this interval. sin 2A º 2 sin A cos A 15 15 1 fi sin 2x = 2c - 4 × - 4 m = 8 15 sin 2x c) tan 2x = cos 2x = 7 Q4 a) Using the sin double angle formula: sin 3q cos 3q sin 6q º 6 3 b) Using the cos double angle formula: 2y 2y 4y sin2 a 3 k – cos2 a 3 k º –cos a 3 k c) Using the tan double angle formula: x 1 - tan 2 b 2 l 1 º tan x º cot x x 2 tan b 2 l Exercise 7.2 — Solving equations and proving identities Q1 a) Using the cos double angle formula involving sin: 4(1 – 2 sin2 x) – 14 sin x = 0 fi 4 – 8 sin2 x – 14 sin x = 0 fi 8 sin2 x + 14 sin x – 4 = 0 fi 4 sin2 x + 7 sin x – 2 = 0 fi (4 sin x – 1)(sin x + 2) = 0 1 So sin x = 4 or sin x = –2 (not valid) 1 Solving sin x = 4 in the interval 0 ≤ x ≤ 360°: x = 14.5° (1 d.p.) and (180° – 14.5°) = 165.5° (1 d.p.) b) Using the cos double angle formula involving cos: 5(2 cos2 x – 1) + 9 cos x + 7 = 0 fi 10 cos2 x + 9 cos x + 2 = 0 fi (2 cos x + 1)(5 cos x + 2) = 0 2 1 So cos x = - 2 or cos x = - 5 fi x = 113.6°, 120°, 240°, 246.4° (1 d.p.) c) Using the tan double angle formula: 4^1 - tan 2 xh 1 2 tan x + tan x = 5 fi 2(1 – tan2 x) + 1 = 5 tan x fi 3 – 2 tan2 x = 5 tan x fi 0 = 2 tan2 x + 5 tan x – 3 fi 0 = (2 tan x – 1)(tan x + 3) 1 So tan x = 2 or tan x = –3 fi x = 26.6°, 108.4°, 206.6°, 288.4° (1 d.p.) d) tan x – 5 (2 sin x cos x) = 0 sin x fi cos x = 10 sin x cos x fi sin x = 10 sin x cos2 x fi sin x – 10 sin x cos2 x = 0 fi sin x(1 – 10 cos2 x) = 0 1 10 Don’t forget to find cos–1 of both the positive and negative root... The coefficient of friction between the object and the plane is m. A particle of weight 2 N rests at B, and the rod is horizontal and in equilibrium. b) Given that a person catching the virus will initially be infected with 200 germs and that this will double to 400 germs in 8 hours, find the number of germs an infected person has after 24 hours. KJK x ONO xi + yj + zk or KKK y OOO so three-dimensional vectors can be written like this: KzO L P ƒƒ Three-dimensional vectors are used to describe things in three-dimensional space, e.g. an aeroplane moving through the sky. 2 - 4x + 3e x dx = 2x3 – 2x2 + 3ex + C On p.180 you saw how to integrate linear transformations of xn by differentiating with the chain rule and working backwards. e) f(x) has domain 0° ≤ x < 90° and range f(x) ≥ 0.

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